University of Sargodha

M.A/M.Sc Part-1 / Composite, 1st -A/2013

Mathematics-III Complex Analysis & Differential Geometry

Maximum Marks: 100   **Time Allowed: 3 Hours**

Note: **Objective part is compulsory. Attempt any four questions from subjective part.**

Objective Part

Q.No.1   Write short answers of the following questions on answer sheet in 2-3 lines only.   **(20)**

I.	State necessary condition for a function $f(z) = U(x,y) + iV(x,y)$ to be an analytic function.	
II.	Find the zeroes of $\sin z$.	
III.	Find the radius of convergence of $\sum_{n=0}^{\infty} \frac{z^n}{n!}$.	
IV.	Prove that $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$.	
V.	Define residue of a function.	
VI.	Define closed curve.	
VII.	State Cauchy's Integral Formula.	
VIII.	Define normal plane.	
IX.	Write down Serret-Fernet formulae.	
X.	Define radius of torsion.	

Subjective Part

Q. No. 2	a.	Derive Cauchy Riemann equations in polar form from Cartesian form.	10
	b.	Prove that $\left|\frac{az+b}{b\bar{z}+a}\right| = 1$ for $|Z| = 1$	10
Q.No. 3	a.	Prove that $U(x,y) = e^x \cos(y)$ is harmonic. Obtain its corresponding conjugate and the original function $f(z) = U(x,y) + iV(x,y)$.	10
	b.	Find the Laurent expansion of $f(z) = \frac{1}{(Z+2)(1+Z^2)}$ for $1 < |Z| < 2$.	10
Q.No. 4	a.	Evaluate $\int_{-\infty}^{\infty} \frac{x^4}{(a+bx^2)^4} dx$.	10
	b.	Find Residue of $f(z) = \frac{e^z}{z^2(z-\pi i)^4}$.	10
Q.No. 5	a.	Let $f(z)$ be analytic on and within the boundary of region C of a simply connected region D and let a be any point within C then $f'(a) = \frac{1}{2\pi i}\int_c \frac{f(z)}{(z-a)^2} dz$.	10
	b.	State and prove Rouche's theorem.	10
Q.No. 6	a.	Prove that $r''' = k'n - k^2 t + k\tau b$ for a curve r.	10
	b.	For a curve $x = 4a\cos^3 u$, $y = 4a\sin^3 u$, $z = 3c\cos(2u)$ Prove that $n = (\sin(u), \cos(u), 0)$, $k = \frac{a}{6(a^2+c^2)\sin 2u}$.	10
Q.No. 7	a.	Find k and τ if $X = a(u - \sin(u))$, $Y = a(1 - \cos(u))$, $z = bu$	10
	b.	Prove that there are infinite family of evolutes for the space curve C (involute).	10

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