University of Sargodha

M.A/M.Sc Part-I, 1st Annual Exam 2008

Mathematics- III **Complex Analysis & Differential Geometry**

Maximum Marks: 40 **Fictitious #:** ____________

Time Allowed: 45 Min. **Signature of CSO:** ____________

New Pattern ***Objective Part***

Note: Cutting, Erasing, overwriting and use of Lead Pencil are not allowed.

Q.1.A: Choose the correct option. **(5)**

i. The Primitive Period of e^z is:
   a. 2π b. πi c. $2\pi i$ d. $2K\pi i$

ii. A pole of order 'n' counted:
   a. $2n$ times b. n times c. $(n-1)$ times d. $(n+1)$ times

iii. An analytic function with constant modulus is:
   a. $\sqrt{x^2+y^2}$ b. Constant c. $\sqrt{U^2+V^2}$ d. Zero

iv. When two surfaces intersect then resultant is a:
   a. Point b. Curve c. Line d. Plane

v. The rate of turning of the binormal is denoted by:
   a. σ b. τ c. κ d. f

B: Mark true and false. **(10)**

i. $|Z-1| = |\bar{Z}-1|$ T F

ii. For any nonzero complex number Z, there are an infinite number of values for arg (Z) T F

iii. The domain of the function $f(Z) = \dfrac{1}{Z^2+i}$ is all complex numbers. T F

iv. The image of the circle $|Z - Z_0| = P$ under a linear mapping is a circle with a (possibly) different center, but the same radius. T F

v. If a complex function f is differentiable at point Z then f is analytic at Z. T F

vi. The Cauchy-Riemann equations are necessary conditions for differentiability. T F

vii. The function $Sin\bar{Z}$ is nowhere analytic. T F

viii. $\ell ni = \frac{1}{2}\pi i$ T F

ix. In first order magnitudes $F_1 = \vec{r}_1.\vec{r}_2$ T F

x. For a curve to be helix $\vec{t}.\vec{a}$ must not be constant. T F

C: Fill in the blanks. **(5)**

i. If Z is a point in the second quadrant, then $i\bar{Z}$ is in the ________________ quadrant.

ii. A complex function f is continuous at $Z = Z_0$, if $\lim_{Z \to Z_0} f(Z) =$ ________________

iii. If $f(Z) = \dfrac{1}{Z^2+5iZ-4}$, then $f'(Z) =$ ________________.

iv. If $e^{iz} = 2$, then $Z =$ ________________

v. The Principal value of i^i is ________________.

Q.2. Give short answers. (Answers should be One or Two lines) **(20)**

i. Consider the limit $\lim_{Z \to 0}\left(\dfrac{Z}{\bar{Z}}\right)^2$.

P.T.O

a. What value does the limit approach as Z approaches 0 along the real axis?

b. What does the limit approach as Z approaches 0 along the imaginary axis?

c. Do the answers from (a) and (b) imply that $\lim_{Z \to 0}\left(\frac{Z}{\overline{Z}}\right)^2$ exists? Explain.

ii. Define analytic function.

iii. Consider the multiple-valued function $F(Z) = (Z - 1 + i)^{1/2}$. What is the branch point of F? Explain.

iv. Find two complex numbers Z_1 and Z_2 so that $Ln(Z_1 Z_2) \neq Ln Z_1 + Ln Z_2$.

v. Differentiate between Pole and essential singularity.

vi. Define Meromorphic function.

vii. Differentiate between simply and multiply connected regions.

viii. Define Principal curvature.

ix. Differentiate between curvature and torsion.

x. Differentiate between evolute and involute.

University of Sargodha

M.A/M.Sc Part-I, 1st Annual Exam 2008

Mathematics- III **Complex Analysis & Differential Geometry**

Maximum Marks: 60 **Time Allowed: 2:15 Hours**

New Pattern ***Subjective Part***

Note: **Attempt three questions in all. Selecting at least one question from each section. All questions carry equal marks.**

Section- I

Q.3. (a) Derive Cauchy-Riemann equations in Polar form from Cartesian form. (12)

(b) Evaluate $\oint_C |Z|\, dz$ where contour 'C' is a unit circle having centre at $Z_0 = 1$ (8)

Q.4. (a) State and prove converse of Cauchy's Fundamental Theorem. (10)

(b) Find the Laurent's expansion of $f(Z) = \dfrac{1}{(Z+2)(1+Z^2)}$ for (10)

a. $1 < |Z| < 2$ b. $|Z| < 1$ c. $|Z| > 2$

Q.5. (a) Prove that (10)

$$\int_0^\infty \frac{x^\alpha}{x^2 - x + 1}\,dx = \frac{2\pi}{\sqrt{3}}\frac{1}{Sin\pi\alpha}Sin\frac{2}{3}\pi\alpha$$

Where α is a fraction.

(b) Define a bilinear transformation. Under the bilinear transformation $\omega = \dfrac{2Z+3}{Z-4}$, discuss the mapping of the circle $x^2 + (y-2)^2 = 4$. (10)

Section- II

Q.6. (a) Prove that the curve (10)

$x = aSin^2u,$ $y = a\sin u \cos v,$ $z = a\cos u$

Lies on the sphere and also verify that all the normal planes pass through origin.

(b) Prove that there are infinite family of evolutes for the space curve 'C' (involute) (10)

Q.7. (a) Show that the sum of the squares of the intercepts on the coordinate axes made by the tangent plane to the surface $x^{2/3} + y^{2/3} + z^{2/3} = a^{2/3}$ is constant. (10)

(b) Calculate the first and second curvature of the helicoids (10)

$x = u\cos v$, $y = u\sin v$, $z = f(u) + cv$

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