University of Sargodha

M.A/M. Sc. Part-1/Composite, 2nd-A/2015

Mathematics: II **Linear Algebra**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: **Objective part is compulsory. Attempt any four questions from subjective part.**

Objective Part (Compulsory)

Q.1. Write short answers of the following on your answer sheet. (2*10)

i. Define normal subgroup. **ii.** Show that centralizer of a subset X in a group G is normal. **iii.** Differentiate between order of a group and order of a element. **iv.** Explain why a group of order 47 cannot have proper subgroups. **v.** Let G be a group and $a \in G$. Let $H = \{a^K | K \in Z\}$, then prove that H is a subgroup of G. **vi.** Define integral domain and give example of it. **vii.** If R is a ring and $x^2 = x$, $\forall x \in R$, then prove that R is commutative. **viii.** Define ideal of a ring. **ix.** Define characteristic of a ring. **x.** Define subspace of a vector space.

Subjective Part

Q.2. a. Let $G = < a >$ be finite cyclic group generated by a and $0(a) = n$, then a^K also generates G if and only if $(K, n) = 1$. **(10)**

b. Let H and K be two normal subgroups of G, then prove that HK is also normal in G. **(10)**

Q.3. a. A group G is abelian if and only if the factor group $G/_{Z(G)}$ is cyclic. (10)

b. Let $(Z, +)$ and $(E, +)$ be the two groups. Define a mapping $\phi: Z \to E$ such that $\phi(n) = 2n \quad \forall b \in Z$ then show that ϕ is an isomorphism. (10)

Q.4. a. Prove that any two cyclic groups of same order are isomorphic to each other. (10)

b. State and prove Cayley's theorem. (10)

Q.5. a. A commutative ring R is an integral domain if and only if cancellation law under multiplication holds in R. (10)

b. Let R be a commutative ring with identity and P be an ideal of R, then P is prime ideal if and only if R/P is an integral domain. (10)

Q.6. a. If U and W are subspaces of a vector space V, then show that $U+W$ is the smallest subspace containing both U and V. (10)

b. The vectors $v_1, v_2 \in V$ are linearly independent if and only if the vectors $v_1 - v_2, v_1 + v_2$ are linearly independent. (10)

Q.7. a. If T is an isomorphism of V_1 onto V_2, then prove that T maps a basis of V_1 onto a basis of V_2. (10)

b. Let $T: U \to V$ be a linear transformation from an n dimensional vector space U to a vector space V over the same field F. then prove that $\dim N(T) + \dim R(T) = n$ (10)
