University of Sargodha

M.A/M.Sc Part- 1/Composite, 2nd-A/2014

Mathematics: II **Algebra**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Objective Part Compulsory

Q. 1	Answer the following Short Questions.	20
(i)	Define simple groups.	
(ii)	Define Characteristic subgroup.	
(iii)	Show that every subgroup of index two in a group is normal.	
(iv)	Define index of a subgroup.	
(v)	If H is a subgroup of a group G then show that $H \subseteq N_G(H)$	
(vi)	Define torsion element.	
(vii)	Show that $[a, bc] = [a, b][a, c]^b$	
(viii)	Find the dual basis ϕ_1, ϕ_2 corresponding to the basis $v_1 = (2,1), v_2 = (3,1)$.	
(ix)	Show that the annihilator of a subset W of a vector space V is a subspace of V^*.	
(x)	Show that annihilator of $W \subset V$ is a subspace of V^*.	

(Subjective Part)

Note: Attempt any four questions. All questions carry equal marks.

Q. 2	(a) State and prove Langrang's theorem.	10
	(b) If $\phi : G \to G'$ is an epimorphism, then show that G/K is isomorphic to G' where $K = Ker\phi$.	10
Q.3	(a) Prove that any two conjugate subgroups of a group are isomorphic.	10
	(b) State and prove Sylow's 2nd theorem.	10
Q.4	(a) If R is commutative ring with unity having $(0), R$ as its only ideals then show that R is a field.	10
	(b) If R is a ring in which $a^2 = a \ \forall a \in R$. Then show that R is a commutative Ring.	10
Q.5	(a) If V and W be two vector spaces over the same field F and T is an isomorphism from V to W then show that T mapps basis of V onto basis of W.	10
	(b) Suppose $\{v_1, v_2, \cdots\cdots\cdots, v_n\}$ is a basis of vector space V over K. Let $\phi_1, \phi_2, \cdots\cdots\cdots, \phi_n$ be the linear functionals defined by $\phi_i(v_j) = \delta_{ij} = \begin{cases} 1 if i = j \\ 0 if i \neq j \end{cases}$ then show that $\{\phi_1, \phi_2, \cdots\cdots\cdots, \phi_n\}$ is a basis of V^*.	10
Q. 6	(a) If $F : V \to W$ be a nonsingular linear mapping then the image of any linearly independent set is linearly independent.	10
	(b) State and prove Cayley Hamilton theorem .	10
Q. 7	(a) If V has finite dimension and $dimV = dimU$. Suppose $T : V \to U$ is any linear mapping then show that T is nonsingular iff it is isomorphism.	10
	(b) Suppose $v_1, v_2, \cdots\cdots\cdots, v_n$ are non zero eigenvectors corresponding to distinct eigenvalues $\lambda_1, \lambda_2, \cdots\cdots\cdots \lambda_n$. Then show that $v_1, v_2, \cdots\cdots\cdots, v_n$ are linearly independent.	10