University of Sargodha

M.A/M.Sc Part-1 / Composite, 2nd-A/2013

Mathematics: II **Algebra**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: **Objective part is compulsory. Attempt any four questions from subjective part.**

(Objective Part)

Q. 1	Answer the following Short Questions.	20
(i)	Define locally infinite group.	
(ii)	Show that the normalizer of any subset in a group is subgroup of that group.	
(iii)	Show that the number of elements in a conjugacy class of any element divides the order of that group.	
(iv)	Prove that a finite group G has a unique sylow p-subgroup H if and ony if H is normal in G.	
(v)	Prove that Boolean's ring must be commutative.	
(vi)	Define zero homomorphisim	
(vii)	If the vectors u, v, w are linearly independent, then show that $u+v, u-v, u-2v+w$ are also linearly independent.	
(viii)	Prove that a linear mapping T is singular if and only if $-T$ is singular.	
(ix)	Using usual inner product on $\Re^3$ find the angle between vectors $u=(2,3,5)$ and $v=(1,-4,3)$	
(x)	Find the minimal polynomial $m(t)$ of the matrix A $\begin{pmatrix} 4 & -2 & 2 \\ 6 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$.	

(Subjective Part)

Q. 2	(a) Show that any two cyclic groups of same order are isomorphic.	10
	(b) Show that any two sylow p-subgroup of a group are conjugate.	10
Q.3	(a) Let G be a group of finite order n then show that its subgroup H is isomorphic to its conjugate K.	10
	(b) State and prove third isomorphic theorem.	10
Q.4	(a) Show that the characteristic of an integral domain is either zero or a prime.	10
	(b) If R is a ring in which $a^2 = a \ \forall a \in R$. Then show that R is a commutative Ring.	10
Q. 5	(a) If $W = \{a + bt : a, b \in \Re\}$ be a vector space of polynomials of degree ≤ 1 and ϕ_1, ϕ_2 be the dual basis defined by $\phi_1(f(t)) = \int_0^1 f(t)dt$ and $\phi_2(f(t)) = \int_0^2 f(t)dt$, then find the corresponding simple basis v_1 and v_2.	10
	(b) If V is finite dimensional vector space over a field F, then show that $V \cong V^*$.	10
Q. 6	(a) State and prove Cauchy Schwarz inequality.	10
	(b) If U and W are subspaces of a vector space V, then prove that $(U+W)^\circ = U^\circ \cap W^\circ$.	10
Q. 7	(a) Show that the characteristic polynomial $\triangle(t)$ and the minimal polynomial $m(t)$ of square matrix A have the same irreducible factors.	10