University of Sargodha

M.A/M.Sc Part-1 / Composite, 2nd -A/2010

Math- II **Algebra**

Maximum Marks: 40 ***Objective Part*** **Fictitious #:** ____________

Time Allowed: 45 Min. **Signature of CSO:** ____________

Note: **Cutting, Erasing, overwriting and use of Lead Pencil are strictly prohibited. Only first attempt will be considered.**

Q. 1 (a) **Fill in the blanks.** 10

(i) For any two vector spaces U & V $F : U \to V$ is bijective then Ker f contains only __________.

(ii) For any vector space V over K and $\phi : V \to K$ is defined as $\phi(av + bu) = a\phi(v) + b\phi(u)$ is called __________ functional.

(iii) A ring R with mod p, where p is prime is called a __________ field.

(iv) A homomorphism of R into R' is said to be an isomorphism if it is a __________ mapping.

(v) For any square matrix A, λ is called __________ if for a vector v, Av= λ v.

(vi) A mapping $\alpha : G \to G'$ is automorphism if α is __________ and homomorphism.

(vii) An element x in a group G is self conjugate if x = __________.

(viii) A group which can be written in the power of a single prime p is called __________ group.

(ix) Every subgroup of index __________ is normal.

(x) A finite group G has a unique sylow p-subgroup H iff H is __________ in G.

Q. 1 (b) **Mark *T* for true and *F* for false.** 10

(i) The set of those elements of *G* which commute with *X* is called Centralizer of *X* in *G*. T/F

(ii) In vector spaces zero vector is always linearly independent. T/F

(iii) If $f : A \to B$ and $g : B \to C$ are bijective mappings then their composition mapping is gof. T/F

(iv) If *A* is real matrix then the root of its characteristic polynomial may be complex. T/F

(v) If *K* is a normal subgroup of a finite group *G* and *H* a sylow p-subgroup of *K* then *G* = *KN*, where *N* is normalizer of *H* in *G*. T/F

(vi) A field is a commutative division ring. T/F

(vii) If $V = R^3$ then $S = \{u_1, u_2, u_3, u_4\} \subset R^3$ may form basis of R^3. T/F

(viii) The non-zero row of a matrix in echelon form is linearly independent. T/F

(ix) The intersection of only finite number of subspaces is also a subspace of *V*. T/F

(x) If $u_1, u_2, \ldots, u_n$ span V and suppose one of $u's$ is zero, then $u's$ without zero span *V*. T/F

Q. 1 (c) **Answer the following short questions.** 20

(i) Define embedding of a group *G* into a group G'.

(ii) If $\phi : G \to G'$ be homomorphism for group G into G' then prove that $\phi(g^{-1}) = [\phi(g)]^{-1}$.

(iii) Define index of a subgroup.

(iv) Show that the centralizer $C_G(X)$ of a subset X in a group G is a subgroup of G.

(v) For a finite group G. Let $a \in G$ then show that order of conjugacy class divides the order of G.

P.T.O

(vi) Define zero homomorphism.

(vii) If V is finite dimensional vector space then show that any two bases of V have the same number of elements.

(viii) Show that the vectors $u=(1,1,2)$, $v=(2,3,1)$ and $w=(4,5,5)$ in R^3 are linearly dependent.

(ix) If W be a subspace of vector space V s.t $\dim W=m$ and $\dim V=n$ then show that $m=n$ if $W=V$.

(x) Let U and W be subspaces of vector space V then show that $U+V$ is also a subspace of V.

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Math- II **Algebra**

Maximum Marks: 60 **Time Allowed: 2:15 Hours**

Subjective Part

Note: **Attempt any three questions. All questions carry equal marks.**

Q. 2	(i) Show that the homomorphic image of a group is a group. (ii) State and prove Lagrange's theorem.	10 10
Q. 3	(i) Let G be a group of finite order n then show that subgroup H is isomorphic to its conjugate K. (ii) Let G be group, H be a subgroup and K a normal subgroup of G, then prove that (a) HK is a subgroup of G. (b) $H/_{H\cap K}$ is isomorphic to $HK/_{K}$.	10 10
Q. 4	(i) Show that a finite commutative ring with more than one element is a field. (ii) Show that the characteristic of an integral domain is either zero or prime.	10 10
Q. 5	(i) If a vector space V is internal direct sum of subspaces $U_1, U_2, \ldots, U_n$, then prove that V is isomorphic to external direct sum of these spaces. (ii) Show that two finite dimensional vector spaces are isomorphic iff they are of the same dimensions.	10 10
Q. 6	(i) If V and W are of the dimensions m and n respectively, then prove that $H(V,W)$ is of dimensions mn over F. (ii) For an eigen value λ of an operator $T: V \to V$ and V_λ is set of all eigen vectors of T of some eigen values λ. Then prove that V_λ is a subspace of V.	10 10