University of Sargodha

M.A/M.Sc. Part-I/Composite, 1st A-Exam 2016

Mathematics: II **Algebra**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Objective Part (Compulsory)

Q. 1	Answer the following Short Questions.	20
(i)	If G is a group under addition and H is its subgroup. Then define quotient group G/H.	
(ii)	Show that the commutator subgroup is fully invariant.	
(iii)	Define class equation.	
(iv)	Show that every subgroup of index two in a group is normal.	
(v)	Let R be a commutative ring with identity element I and J an ideal of R. If $I \in R$ then show that $J = R$.	
(vi)	Find the dual basis ϕ_1, ϕ_2 corresponding to the basis $v_1 = (2,1), v_2 = (3,1)$.	
(vii)	Show that the annihilator of a subset W of a vector space V is a subspace of V^*.	
(viii)	Prove that a matrix P is orthogonal if and only if P^T is orthogonal.	
(ix)	Find characteristic polynomial $\triangle(t)$ of the matrix $\begin{pmatrix} 2 & 5 \\ 4 & 1 \end{pmatrix}$.	
(x)	Show that the mapping $F(x,y,z) = (\lvert x\rvert, y+z)$ is not linear.	

(Subjective Part)

Note: Attempt any 4 questions. All questions carry equal marks.

Q. 2	(a) Every group of order p^2, where p is prime number is abelian.	10
	(b) Let G be a finite abelian group and p is a prime devisor of order of G, then show that G contains an elopement of order of G.	10
Q.3	(a)Show that any two cyclic groups of same order are isomorphic.	10
	(b) State and prove first fundamental theorem of isomorphism.	10
Q.4	(a) Show that the characteristic of an integral domain is either zero or a prime.	10
	(b) If R is a ring in which $a^2 = a \ \forall a \in R$.Then show that R is a commutative Ring.	10
Q. 5	(a) Show that every basis of a vector space V has the same number of elements.	10
	(b) If V is finite dimensional vector space over a field F, then show that $V \cong V^*$.	10
Q. 6	(a) If $F : V \to U$ be a non-singular linear mapping then prove that image of any linearly independent set is linearly independent.	10
	(b) Let W be a subspace of a finite dimensional vector space V, then show that $dim(V/W) = dimV - dimW$.	10
Q. 7	(a) For any vectors u and v in an inner product space V show that $\lvert\langle u,v\rangle\rvert \leq \lVert u\rVert \lVert v\rVert$.	10
	(b) If $F : V \to W$ be a non-singular linear mapping, then show that image of any linearly independent set is linearly independent.	10