University of Sargodha

Nida

M.A/M.Sc Part-1 / Composite, 1st-A/2015

Mathematics: II **Linear Algebra**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: **Objective part is compulsory. Attempt any four questions from subjective part.**

Objective Part (Compulsory)

Q.1. Write short answers of the following questions. (2*10)

i. define cyclic group. **ii.** If H and K are subgroups of a group G, then show that $H \cap K$ is also a subgroup of G. **iii.** Differentiate between centralizers and normalizers. **iv.** Define simple groups. **v.** Define conjugacy relation between elements of group. **vi.** Define zero divisor and give one example of it. **Vii.** Define homomorphism of ring. **viii.** Define vector space. **ix.** What do you know about eigen values and eigen vectors? **x.** Let $V = R^3$ be a vector space, Define $W = \{(W_1 + W_2 + W_3);\ W_1 + W_2 + W_3 = 0\}$ then prove that W is a subspace of R^3.

Subjective Part

Q.2. (a) Define and prove Lagrange's theorem. (10)

(b) Prove that every subgroup of a cyclic group is cyclic. (10)

Q.3. (a) Prove that every subgroup of index 2 in a group is normal. (10)

(b) Show that the conjugacy relation between the elements of a group is an equivalence relation. (10)

Q.4. (a) If $\phi: G \to G'$ be a homomorphism.If G is cyclic group, then show that $\phi(G)$ is also cyclic. (10)

(b) Let $\phi: G \to G'$be an epimorphism with $K_\phi = N$ where N is a normal subgroup of G, then prove that ${}^{G}/_{K_\phi} \cong G'$ (10)

Q.5. (a) Prove that centre of a ring is a subring of ring R. (10)

(b) Let R be a commutative ring with identity. Let M be an ideal in R, then M is a maximal ideal if and only if R/M is a field. (10)

Q.6. (a) If A and B are finite dimensional subspaces of a vector space V, then $A + B$ is finite dimensional and $\dim(A+B) = \dim A + \dim B - \dim(A \cap B)$ (10)

(b) Prove that a one-to-one linear transformation preserves basis and dimension. (10)

Q.7. (a) Any finite dimensional vector space contains a basis. (10)

(b) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$ (10)