University of Sargodha

M.A/M.Sc Part-1 / Composite, 1st-A/2014

Mathematics: II **Algebra**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Objective Part Compulsory

Q. 1	Answer the following Short Questions.	20
(i)	Define Hamiltonian group.	
(ii)	Show that centralizer of a subset X in a group G is normal.	
(iii)	Define simple groups.	
(iv)	If H is a subgroup of a group G then show that $H \subseteq N_G(H)$.	
(v)	If every element of a group G is its own inverse the show that G is commutative.	
(vi)	Define Characteristic of a ring.	
(vii)	Prove that Boolean ring is commutative.	
(viii)	Show that the set $\{U = (x, -x) : x \in \Re\}$ is a subspace of $\Re^2$.	
(ix)	If $\{v_1 = (2,1), v_2 = (3,1)\}$ be basis of $\Re^2$ then find the dual basis $\{\phi_1, \phi_2\}$.	
(x)	If S, T be two subsets of a vector space$V$, then show that $S \subseteq T$ implies $L(S) \subseteq L(T)$.	
	(Subjective Part)	
	Note: Attempt any four questions. All questions carry equal marks.	
Q. 2	(a) Show that the number of elements in a conjugacy class C_a of an element a in a group G is equal to the index of its normalizer in G.	10
	(b) Show that any two sylow p-subgroup of a group are conjugate.	10
Q.3	(a)If H and K be two subgroups of a group G then show that HK is also a subgroup of G if and only if $HK = KH$.	10
	(b) State and prove Sylow's 1st theorem.	10
Q.4	(a) Let G be a group with $\xi(G)$ as its centre and $I(G)$ the group of its inner automorphisims.Then prove that $G/\xi(G)$ is isomorphic to $I(G)$.	10
	(b) If J_1 and J_2 be two ideals of a ring R. Then show that $J_1.J_2 = \{r_1.r_2 : r_1 \in J_1, r_2 \in J_2\}$ is an ideal of R.	10
Q. 5	(a) Consider the vector space $P(t)$ with inner product $< f, g >= \int_0^1 f(t)g(t)$. Apply Gram-Schmidt algorithm to the set $\{1, t, t^2\}$ to obtained an orthogonal set $\{f_0, f_1, f_2\}$ with integeral coefficients.	10
	(b) Let A be a real positive definite matrix.Then show that the function $< u, v >= u^T Av$ is an inner product on $\Re^n$.	10
Q. 6	(a) (a) If $F : V \to W$ be a nonsingular linear mapping then the image of any linearly independent set is linearly independent.	10
	(b) State and prove Cayley Hamilton theorem .	10
Q. 7	(a) If V has finite dimension and $dimV = dimU$. Suppose $T : V \to U$ is any linear mapping then show that T is nonsingular iff it is isomorphism.	10
	(b) Suppose $v_1, v_2, \cdots\cdots, v_n$ are non zero eigenvectors corresponding to distinct eigenvalues $\lambda_1, \lambda_2, \cdots\cdots \lambda_n$.Then show that $v_1, v_2, \cdots\cdots, v_n$ are linearly independent.	10