University of Sargodha

M.A/M.Sc Part-1 / Composite, 1st -A/2013

Mathematics-II **Algebra**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: **Objective part is compulsory. Attempt any four questions from subjective part.**

Objective Part

Q.1. **Write short answer of the following.** (20)

i. If H is subgroup of a group G then show that $H \subseteq N_G(H)$. **ii.** Define torsion element. **iii.** Show that $[a, bc] = [a, b][a, c]^b$. **iv.** Define simple groups. **v.** Define Characteristic subgroup. **vi.** Find the dual basis ϕ_1, ϕ_2 corresponding to the basis $v_1 = (2, 1)$, $v_2 = (3, 1)$ **vii.** Show that the annihilator of a subset W of a vector space V is a subspace of V^* **viii.** If V be a vector space and S is a subset of V then show that $L(L(S)) = L(S)$ **ix.** Find characteristic polynomial $\Delta(t)$ of the matrix $\begin{bmatrix} 2 & 5 \\ 4 & 1 \end{bmatrix}$
x. Show that the mapping $F(x, y, z) = (|x|,\ y + z)$ is not linear.

Subjective Part

Q.2. a. If H and K be two subgroups of a group G then show that HK is also a subgroup of G if and only if $HK = KH$. (10)

b. State and prove Langrang's theorem. (10)

Q.3. a. Show that the number of elements in a conjugacy class C_a of an element a in a group G is equal to the index of its normalizer in G. (10)

b. If $\phi: G \to G'$ is an epimorphism, then show that G/K is isomorphic to G' where $K = Ker\phi$. (10)

Q.4. a. If V and W be two vector spaces over the same field F and T is an isomorphism from V to W then show that T mapps basis of V onto basis of W. (10)

b. Suppose $\{v_1, v_2, \ldots, v_n\}$ is a basis of vector space V over K. Let $\phi_1, \phi_2, \ldots \ldots \phi_n$ be the linear functionals defined by $\phi_i(V_j) = \delta_{ij} = \begin{cases} 1 if i = j \\ 0 if i \neq j \end{cases}$ then show that $\{\phi_1, \phi_2, \ldots \ldots \phi_n\}$ is a basis of V^*. (10)

Q.5. a. If V has finite dimension and $\dim V = \dim U$. Suppose $T : V \to U$ is any linear mapping then show that T is nonsingular iff it is isomorphism. (10)

b. Suppose $v_1, v_2, \ldots\ldots, v_n$ are non zero eigenvectors corresponding to distinct eigenvalues $\lambda_1, \lambda_2, \ldots \ldots \lambda_n$. Then show that $v_1, v_2, \ldots\ldots, v_n$ are linearly independent. (10)

Q.6. a. Consider the vector space $P(t)$ with inner product $< f, g >= \int_0^1 f(t)g(t)$. Apply Gram-Schmidt algorithm to the set $\{1, t, t^2\}$ to obtain an orthogonal set $\{f_0, f_1, f_2\}$ with integral coefficients. (10)

b. Let A be a real positive definite matrix. Then show that the function $<u, v> = u^T Av$ is an inner product on $\Re^n$. (10)

Q.7. a. If $F: V \to W$ be a nonsingular linear mapping then the image of any linearly independent set is linearly independent. (10)

b. State and prove Cayley Hamilton theorem. (10)