University of Sargodha

M.A/M.Sc Part-I, 1st Annual Exam 2008

Mathematics- II **Algebra**

Maximum Marks: 40 **Fictitious #:** ____________

Time Allowed: 45 Min. **Signature of CSO:** ____________

New Pattern ***Objective Part***

Note: Cutting, Erasing, overwriting, ink remover and use of Lead Pencil is strictly prohibited.

Q1.(a) Tick (✓) the correct choice in the following MCQ's. (5+10+5)

(i) The relation of conjugacy between elements of a group is an_____ relation.

1. anti-symmetric
2. order
3. equivalence
4. None of these

(ii) A group of all whose elements are of finite order is called a _____ group.

1. quaternion
2. dihedral
3. symmetric
4. periodic

(iii) The center of the group of quaternions is ___________ .

1. {I, -I}
2. {i, -i}
3. {j, -j}
4. {k, -k}

(iv) IF S, T are subsets of V, then L(SUT) = L(S) ____ L(T)

1. U 2. + 3. - 4. $\cap$

(v) Any two eigen vectors corresponding to two distinct eigen values of an orthogonal matrix are

1. Orthogonal 2. Bijective 3. Parallel 4. None of these

(b) Write true or false.

(i) A homomorphic image of a cyclic group is cyclic. T/F

(ii) Kelvin four group is the direct product of its subgroups A = < a : a^2 = 1> and B = < b : b^2 = 1>. T/F

P.T.O

(iii) If a subgroup H contains the normalizer of a sylow p-subgroup of a group G, then H is its own normalizer. T/F

(iv) If p is a prime number, then the ring of integers mod p, is a field. T/F

(v) If H and K are subgroups of a group G then HK is also a subgroup of G if and only if HK= KH. T/F

(vi) There is one-one correspondence between any two right cosets of H in G. T/F

(vii) If G be a group and H a singleton subset of G then $N_G(H)=C_G(H)$ where H={a} T/F

(viii) If V is a vector space over F then $\alpha .0=0$ T/F

(ix) If V is a finite dimensional and W is a subspace of V then d(V/W) = dimV - dim W T/F

(x) Any field F is an integral domain. T/F

(c) Fill in the blanks.

(i) The identity element in a group G is----------------------
(ii) A function φ: A→B is said to be --------------- if B φ (A) = B
(iii) The centre of a group G is always ----------- subgroup.
(iv) Let R be a commutative ring with unit element whose only ideals are (0) & R Itself then R is --------------------
(v) If U and W be the subspace of a vector space V then their intersection is --------------------- of V.

P.T.O

Q.2. Write the short answers. Answers should be one or two lines. (20)

i. Define torsion free group.

ii. Give an example of a non-abelian group all of whose sub-groups are normal.

iii. Show that the ring of integers mod 6 under addition and multiplication mod 6 is not a field.

iv. Give two proper normal subgroups of the group of quaternion.

v. Show that in the ring of integers Z, 6Z = {......- 12, -6, 0, +6, +12,......} is not a prime ideal.

vi. Prove that the annihililater A(W) of a sub space W of a vector space V is a subspace of V^*.

vii. Define zero devisors.

viii. Define Manimal ideal

ix. Every field is an integral domain.

x. If V is a vector space over F then $\alpha.0 = 0$

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Mathematics- II **Algebra**

Maximum Marks: 60 **Time Allowed: 2:15 Hours**

New Pattern *Subjective Part*

Note: **Attempt any three questions. All questions carry equal marks.**

Q3.a) If G is a finite group of composite-order, then show that G has non-trivial sub-groups. Prove that any non-commutative group has at least six elements. (10)

b) Let H, K be sub-groups of a group G. Show that HK is a sub-group of G if and only if HK= KH. (10)

Q4.a) Let H be a sub-group G and define a set N(H) by

$$N(H)=\{\ a\in G \mid aHa^{-1}=H\}.$$

Show that N(H) forms a sub-group of G and H is normal in G if and only if N(H) = G. (10)

b) Show that the relation of conjugacy between sub-groups of a group is an equivalence relation. Write down the conjugacy classes of the dihedral group. $D_4 = < a, b;\ a^4=b^2=(ab)^2 = 1 >$. (5+5)

Q5. a). Prove that every fully invariant subgroup is characteristic and every characteristic subgroup is normal in G. (5+5)

b). State and prove Sylow's third theorem. (10)

Q6.(a) If R is a commutative ring with unit element and M is an ideal of R, then M is a maximal ideal of R if and only if R/M is a field. (10)

(b) If U, V are ideals of a ring R and UV the set of all elements that can be written as finite sums of elements of the form uv where $u\in U$ and $v\in V$. Then prove that UV is an ideal of a ring R and $UV\subseteq U\cap V$. (10)

Q7. (a) Let W_1 and W_2 be sub-spaces of a vector space V(F). Show that

$$\dim (W_1 + W_2) = \dim W_1 + W_2 - \dim (W_1 \cap W_2)$$ (10)

(b) If V and W are of dimensions m and n, respectively, over F, then Hom (V, W) is of dimension mn over F. (10)