Roll No. ____________ **Subject: Math- I**

M.A/M.Sc: Part- I / Composite, 2nd -A/10

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University of Sargodha

M.A/M.Sc Part-1 / Composite, 2nd -A/2010

Math- I **Real Analysis**

Maximum Marks: 40

Time Allowed: 45 Min.

Fictitious #: ____________

Signature of CSO: ____________

Objective Part

Note: **Cutting, Erasing, overwriting and use of Lead Pencil are strictly prohibited. Only first attempt will be considered.**

Q.1.A: **Choose the most correct answer.** (5)

i. The function $f(x)=\frac{1}{x^2}$ is uniformly continuous on:

a. $(-\infty,+\infty)$ b. $(0,+\infty)$ c. $[1,+\infty)$ d. $(-1,1)$

ii. If $p^2=3$ then p is:

a. Natural number b. Integer c. Rational number d. Irrational number

iii. The series $\sum_{n=1}^{\infty}(-1)^{n-1}$:

a. Converges b. Diverges c. Converges absolutely d. Converges conditionally

iv. The only real solution of $\sin(x)=x$ is:

a. $x=0$ b. $x=\frac{\Pi}{2}$ c. $x=\Pi$ d. $x=\frac{3\Pi}{2}$

v. $\lim_{n\to\infty}((\frac{1}{n})\sin(nx+n)):(x\in R, n\in N)$ is:

a. 1 b. -1 c. 0 d. Does not exist

B: Write True or False. **(10)**

i. The extended real number system forms a field.

ii. The sequence $(\frac{\sin(n)}{n})$ converges to zero.

iii. A monotone bounded sequence may diverge.

iv. The infinite set N has no cluster points.

v. A continuous function on an interval takes on (at least once) any number that lie between two of its values.

vi. If $f: I \to R$ has a derivative at $c \in I$, then f is discontinuous at c.

vii. $f(x) = \tan(x)$ is Riemann integrable on $[0, \frac{\Pi}{2}]$.

viii. A continuous function is a function of bounded variation.

ix. A convergent improper integral may not be absolutely convergent.

x. $\int_0^1 \log(x).\log(1+x)dx$ is a proper integral.

C: Fill in the blanks. **(5)**

i. The set of rational numbers is … in R.

ii. A bounded sequence has a … subsequence.

iii. If $U(P,f,\alpha) - L(P,f,\alpha) < \varepsilon$ holds for some partition P of $[a,b]$, then it holds for every …of P.

iv. A bounded monotone function is a function of ….

v. $\int_a^b f(x)dx$ is called an improper integral of … kind if f(x) is unbounded at one or more points of $a \le x \le b$.

Q.2: Write short answers of the following: **(20)**

i. Prove that $\sqrt{5}$ is an irrational number.

ii. Let $f(x) = \frac{|x-3|}{x-3}$ for $x \neq 3$, $f(x) = 0$ for $x = 3$ then show that $\lim_{x \to 3} f(x)$ does not exist.

iii. Prove that a polynomial is continuous in every finite interval.

iv. If $z = x^2 \tan^{-1}\frac{y}{x}$, find $\frac{\partial^2 z}{\partial x \partial y}$ at (1,1).

v. Prove that $\lim(\frac{x^2 + nx}{n}) = x$ for $x \in R$.

P.T.O

University of Sargodha

M.A/M.Sc Part-1 / Composite, 2nd -A/2010

Math- I **Real Analysis**

Maximum Marks: 60 **Time Allowed: 2:15 Hours**

Subjective Part

Note: **Attempt any three questions. All questions carry equal marks.**

Q. 3.

a) State and prove "The completeness property of R". (10)

b) Prove that the set of rational numbers is dense in R. (10)

Q. 4.

a) Let $Y=(y_n)$ be defined inductively by $y_1=1$ and $y_{n+1}=\frac{1}{4}(2y_n+3)$ for $n\geq 1$. Show that $\lim Y=\frac{3}{2}$. (10)

b) Prove that a sequence of real numbers is convergent if and only if it is a Cauchy sequence. (10)

Q. 5.

a) Suppose f is a real differentiable function on some interval $[a,b]$ and suppose $f'(a)<\lambda<f'(b)$. Prove that there exists a point $x\in(a,b)$ such that $f'(x)=\lambda$. (10)

b) Let $f(x)=x^2\sin(\frac{1}{x})$ when $x\neq 0$ and $f(x)=0$ when $x=0$. Prove that f is differentiable at all points x, but f' is not continuous function. (10)

Q. 6.

a) Find the local maximum and minimum values and saddle points of $f(x,y)=x^4+y^4-4xy+1$. (10)

b) Find the points on the sphere $x^2+y^2+z^2=4$ there are closest to and farthest from the point (3,1,-1). (10)

Q. 7.

a) If f is monotone on [a,b] and α is monotonically increasing continuous function on [a,b], then prove that $f\in R(\alpha)$ on [a,b]. (10)

b) Let f be a positive decreasing function defined on $[a,+\infty)$ and assume that $f\in R(\alpha;a,b)$ for every $b\geq a$. Then prove that the integral $\int_a^\infty f d\alpha$ is convergent. (10)