University of Sargodha

M.A/M.Sc. Part-I/Composite, 1st A-Exam 2016

Mathematics: I **Real Analysis**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: **Objective part is compulsory. Attempt any four questions from subjective part.**

Objective Part (Compulsory)

Q.1. Write short answers of the followings on your answer sheet: [2*10]

(i) Evaluate $\lim_{x\to 0}\frac{|x|}{x}$.

(ii) Prove that every Cauchy sequence is bounded.

(iii) Discuss the behavior of the series $\sum(-1)^n$.

(iv) Prove that the differentiate function is continuous.

(v) Give an example of a function of bounded variation so that the derivative of function is bounded.

(vi) Define the Riemann-Stieltjes integral.

(vii) What do you meant by the limit of a convergent sequence of functions?

(viii) Define an improper integral of first kind.

(ix) What do you meant by refinement of a partition?

(x) Prove that for every real p, the integral $\int_0^\infty e^{-x}x^p\,dx$ converges.

Subjective Part (4*20)

Q.2. **a)** State and prove the Generalized Mean Value Theorem. [10]

b) Show that the set of natural numbers is unbounded above. [10]

Q.3. **a)** Prove that a continuous function on a compact set is uniformly continuous. [10]

b) Show that the series $1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}+\ldots$ diverges. [10]

Q.4. **a)** Discuss the continuity of the function $f(x,y)=xy/(x^2+xy+y^2)$ if $f(x,y)\neq(0,0)$ and $f(x,y)=0$ if $f(x,y)=(0,0)$. [10]

b) State and prove the Schawarz's Theorem. [10]

Q.5. **a)** Show that $\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$ exists if and only if m,n are both positive. [10]

b) If $z=f(x,y)$ has a continuous first order partial derivative in D, then z has total differential $dz=\frac{\partial z}{\partial x}\Delta x+\frac{\partial z}{\partial y}\Delta y$ at every point $(x,y)\in D$. [10]

Q.6. **a)** Prove that $U(p,f,\alpha)\geq U(p^*,f,\alpha)$ if p^* is a refinement of partition p. [10]

b) Let $f\in R$ on $[a,b]$. For $a\leq x\leq b$, put $F(x)=\int_0^x f(t)\,dt$, then F is continuous on $[a,b]$; furthermore, if f is continuous at point x_0 of $[a,b]$, then F is differentiable at x_0 and $F'(x_0)=f(x_0)$. [10]

Q.7. **a)** The product of two functions of bounded variation is of bounded variation. [10]

b) The function of bounded variation is expressible as the difference of two monotonical increasing functions. [10]