University of Sargodha

M.A/M.Sc Part-1 / Composite, 1st-A/2014

Mathematics: I **Real Analysis**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: **Objective part is compulsory. Attempt any four questions from subjective part.**

Objective Part

Q.1. Write short answers of the following. (2*10)

i. B, b are the bounds of a set S and B_1 , b_1 are bounds of a subset S_1 of S; show that $b \le b_1 \le B_1 \le B$. **ii.** Show that a polynomial is continuous for every value of x. **iii.** Find $\lim_{x\to 0+} \frac{|x|}{x}$ and $\lim_{x\to 0-} \frac{|x|}{x}$. Illustrate graphically. **iv.** Give the definition of a Cauchy sequence. **v.** Discuss the behavior of the series $\sum a_n$ where $a_n = (-1)^n$. **vi.** Give the geometrical interpretation of the partial derivatives $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ when $Z = f(x, y)$. **vii.** Prove that a necessary condition for f(x, y) to have a relative extremum (Maximum or Minimum) at (x_0, y_0) is that $f_x(x_0, y_0) = 0, f_y(x_0, y_0) = 0$. **viii.** Define the Riemann-Stieltjes integral and show that Riemann integral is a special case of the Riemann-Stieltjes integral. **ix.** Define the total variation of a function $f(x)$ over $[a, b]$. **x.** What is meant by the Limit of a convergent sequence of functions.

Subjective Part

Q.2. a. If x and y are real numbers with $x < y$, then prove that there exists an irrational number z such that $x < z < y$. (10)

b. Prove that any convergent sequence is bounded. (10)

Q.3. a. Define the Signum function and prove that $\lim_{x\to 0} Sgn\, x$ does not exist. (10)

b. Prove that the function $f(x, y) = x^2 - 2xy + y^2 + x^3 - y^3 + x^5$ has neither a maximum nor a minimum at the origin. (10)

Q.4. a. If f is continuous on $[a, b]$, then prove that $f \in R(\alpha)$ on $[a, b]$ where α is monotonically increasing on $[a, b]$. (10)

b. Prove that $\int_1^{\infty} \frac{\cos x}{x^2} dx$ converges. (10)

Q.5. a. Show that a function of bounded variation is expressible as the difference of two monotonically increasing functions. (10)

b. Show that the two series $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{|\underline{2n+1}}$ and $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{|\underline{2n}}$ are uniformly convergent in every interval $[a. b]$. (10)

Q.6. a. Prove that $f \in R(\alpha)$ on $[a, b]$ if and only if for every $\in > 0$ there exists a partition P such that $u(P, f, \alpha) - L(P, f, \alpha) < \in$. (10)

b. Suppose that the functions $w = f(x, y), z = g(x, y)$ are implicitly defined by (10)

$$2x^2 + y^2 + z^2 - zw = 0$$
$$x^2 + y^2 + 2z^2 + zw = 8$$

Find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$

Q.7. a. Show that $\int_0^1 x^{m-1}(1-x)^{n-1}dx$ exist if and only if, m, n are both positive. (10)

b. Suppose a and c are real numbers, $c > 0$ and f is defined on [-1, 1] by (10)

$$f(x) = \begin{cases} x^a \sin(x^{-c}) & if \quad x \ne 0 \\ 0 & if \quad x = 0 \end{cases}$$

Prove that: (i) f is continuous if and only if $a > 0$ (ii) $f'(0)$ exists if and only if $a > 1$.
