University of Sargodha

M.A/M.Sc Part-1 / Composite, 1st -A/2013

Mathematics-I **Real Analysis**

Maximum Marks: 100 **Time Allowed: 3 Hours**

Note: **Objective part is compulsory. Attempt any four questions from subjective part.**

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Objective Part

Q.1. **Write short answer of the following.** (20)

i. What do you mean by the least upper bound of a set? **ii.** Write a short note on the derivative of a function. **iii.** Define a continuous function and give an example of a continuous function on R. **iv.** Differentiate between differential and differential coefficient of a function. **v.** Give an example of a function which is undefined at a certain point but has got the limit at that point. **vi.** What is the difference between local maximum and absolute maximum of a function over an interval? **vii.** When a function of two variables is said to be differentiable at a point? **viii.** Define the terms "partition" of an interval [a, b] and "refinement of a partition" of an interval [a, b]. **ix.** Show that the total variation of a function f on an interval [a, b] is zero if f is a constant function. **x.** Define a convergent sequence of functions.

Subjective Part

Q.2. a. Give the definition of an ordered field and prove that the following statements are true in every ordered field. (10)

(i) If $x \neq 0$ then $x^2 > 0$, in particular $1 > 0$

(ii) If $0 < x < y$ then $0 < \frac{1}{y} < \frac{1}{x}$

Sandwich theorem

b. Suppose that $y_n \to \ell$ as $n \to \infty$ and $Z_n \to \ell$ as $n \to \infty$. If . $y_n \leq x_n \leq z_n$ $(n = 1,2, \dots\dots)$, then prove that $x_n \to \ell$ as $n \to \infty$. (10)

Q.3. a. Let $A: = R$ and let f be defined by $f(x) = \begin{cases} 1 \text{ if } x \text{ is rational} \\ 0 \text{ if } x \text{ is irrational} \end{cases}$ prove that f is not continuous at any point of R. (10)

b. If z be a function of two variables x and y and $x = r\cos\theta, y = r\sin\theta$, prove that (10)

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 z}{\partial \theta^2} + \frac{1}{r}\frac{\partial z}{\partial r}$$

Q.4. a. If P^* is a refinement of a partition P of an interval $[a, b]$, then prove that $L(P, f, \alpha) \leq L(P^*, f, \alpha)$ (10)

b. Show that $\int_0^\infty \sin x^2 dx$ is convergent. (10)

Q.5. a. Let $f(x) = \begin{cases} x\cos\frac{\pi}{2x}, & x \neq 0 \\ 0 & , x = 0 \end{cases}$ show that f is continuous but not of bounded variation on [0,1] (10)

b. Let $S_n(x) = x^n (0 \leq x \leq 1)$. Show that $x = 1$ is a point of non-uniform convergence of the sequence. (10)

Q.6. a. If f is continuous on $[a, b]$, then show that $f \epsilon R(\alpha)$ on $[a, b]$. (10)

b. If $x = u - v + w, y = u^2 - v^2 - w^2$ and $z = u^3 + v$, evaluate the Jacobian $\frac{\partial(x,y,z)}{\partial(u,v,w)}$. (10)

Q.7. a. Show that the series $\sum_{n=1}^{\infty} \frac{\cos nx}{n^4}$ is uniformly and absolutely convergent for all x. (10)

b. Prove Cauchy's generalized theorem of the mean. (10)