University of Sargodha

M.A/M.Sc Part-I, 1st Annual Exam 2008

Mathematics- I **Real Analysis**

Maximum Marks: 40 **Fictitious #:** ____________

Time Allowed: 45 Min. **Signature of CSO:** ____________

New Pattern ***Objective Part***

Note: Cutting, Erasing, overwriting and use of Lead Pencil are not allowed.

Q.1.A: Tick the correct option in the following. **(5)**

i. The sequence $\left\{\tan\frac{n\pi}{2}\right\}$ is:

a. Bounded b. Unbounded c. Convergent d. Divergent

ii. If f is continuous on $[a, b]$ and $f \in R(\alpha)$ on $[a, b]$ then

a. α is monotonically increasing b. α is monotonically decreasing
c. α is continuous on $[a, b]$ d. α is discontinuous on $[a, b]$

iii. In a field F if $0 < x < y$ then:

a. $0 > \frac{1}{y} > \frac{1}{x}$ b. $0 < \frac{1}{x} < \frac{1}{y}$ c. $0 > \frac{1}{x} > \frac{1}{y}$ d. $0 < \frac{1}{y} < \frac{1}{x}$

iv. If $a \in R$ is such that $0 \le a < \varepsilon$ for every $\varepsilon > 0$ then:

a. $a = 1$ b. $a = 0$ c. $a > 1$ d. $a = \varepsilon$

v. If f is real valued function defined on $[a, b]$ then $f \in R(\alpha)$ on $[a, b]$ if α is:

a. continuous on [a, b] b. Monotonically increasing on [a, b]
c. Monotonically decreasing on [a, b] d. Unbounded

B: Mark True or False. **(10)**

i. $\Sigma\frac{1}{n}$ is a convergent series. T F

ii. $f(x) = x^2$ is uniformly continuous on $]0, 1]$. T F

iii. For every real number x and every integer $n > 0$ there is only one and one y such that $y^n = x$ T F

iv. $f(x) = \begin{cases} x\sin\frac{1}{x} & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$ is differentiable at $x = 0$. T F

v. A function of bounded variation is not expressible as a sum of two monotonically increasing function. T F

vi. If f and α are continuous on $[a, b]$, then $f \in R(\alpha)$ on $[a, b]$ T F

vii. $\int_1^\infty x^p\,dx$ converges if $p < 1$. T F

viii. The infinite series $\sum_{K=0}^{\infty}\sin 2\pi k$ and the integral $\int_0^\infty \sin 2\pi x.dx$ behave alike. T F

ix. $\int_a^b \frac{dx}{(x-a)^n}$ converges if and only if, $n \ge 1$. T F

P.T.O

x. $\int_{a+}^{b} f\,dx$ converges if $\lim_{x\to a+} I(x)$ exists (finite) where $I(x)=\int_{x}^{b} f\,dx$, if $x\in]a,b]$. T F

C: Fill in the blanks. **(5)**

i. If $\{S_n\}$ converges to s then all of its ______________________ converges to s.

ii. Every polynomial is ______________________ everywhere.

iii. If $\int_{-\infty}^{a} f d\alpha$ and $\int_{a}^{\infty} f d\alpha$ both converges for some value of a then $\int_{a}^{\infty} f d\alpha$ is ______________.

iv. The extended real numbers system doesn't form a ______________________.

v. A point of continuity of a function $f(x)$ is also a point of continuity of $V(x)$ and ______________________.

Q.2. Write the short answers to the following: **(20)**

i. Let $p\in R, p\geq -1$ and $p\neq 0$, then for $n\geq 2$ show that $(1+p)^n > 1+np$.

ii. Prove that a function f defined as $f(x)=\begin{cases} x\cos x, & x\neq 0 \\ 0, & x=0 \end{cases}$ is continuous at $x=0$.

iii. Consider $f: R^2 \to R$ given by

$f(x)=\begin{cases} \dfrac{x^2y^2}{x^4+y^4}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$ Show that $\lim_{(x,y)\to(0,0)} f(x,y)$ doesn't exist.

iv. Let f be a real valued function defined on [a, b] and α be a monotonically increasing function on [a, b]. Show that $\int_{-a}^{a} f d\alpha \leq \int_{a}^{\bar{b}} f d\alpha$.

v. Show that the improper integral $\int_{a}^{b} \frac{dx}{(x-a)^n}$ converges if and only if, $n<1$.

vi. If $x\in R, y\in R$ and $x>0$ then there exists a positive integer n such that $nx>y$. Prove

P.T.O

vii. Test the convergence of $\sum \frac{1}{n}\sin^2\frac{x}{n}$.

viii. Show that the function f defined by $f(x)=\begin{cases} x\cos x & , x\neq 0 \\ 0 & , x=0 \end{cases}$ is continuous at $x=0$.

ix. Prove that if f is continuous on $[a, b]$ and α is monotonically increasing on $[a, b]$, then $f \in R(\alpha)$ on $[a, b]$.

x. Prove that for every real p, the integral $\int_1^\infty e^{-x}x^p\,dx$ converges.

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University of Sargodha

M.A/M.Sc Part-I, 1st Annual Exam 2008

Mathematics- I **Real Analysis**

Maximum Marks: 60 **Time Allowed: 2:15 Hours**

New Pattern ***Subjective Part***

Note: **Attempt any four questions.**

Q.3. (a) If n is a positive integer which is not a perfect square, then prove that $\sqrt{n}$ is an irrational number. (8)

(b) The field axioms imply the following (7)

i. $0\cdot x=0$, ii. If $x\neq 0$, $y\neq 0$ then $xy\neq 0$

iii. $(-x)y=-(xy)=x(-y)$ iv. $(-x)(-y)=xy$. Prove.

Q.4. (a) For each irrational number x, there exists a sequence $\{r_n\}$ of distinct rational numbers such that $\lim_{n\to\infty} r_n = x$ prove. (8)

(b) Let $a_n>0, b_n>0$ and $\lim_{x\to\infty}\frac{a_n}{b_n}=\lambda\neq 0$ (7)

Then prove that series $\sum a_n$ and $\sum b_n$ behave alike.

Q.5. (a) Suppose f is continuous on $[a, b]$ Prove that if $f(a)<0$ and $f(b)>0$ then there exists a point c, $a<c<b$ such that $f(c)=0$ (8)

(b) Given the transformation $x=u^2-v^2, y=2uv$ evaluate $\left(\frac{\partial u}{\partial x}\right)_y$ and $\left(\frac{\partial v}{\partial x}\right)_y$ (7)

Q.6. (a) Let $z=f(x, y)$, $x=u^2-v^2$, $y=2uv$ show that (8)

$$\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2=\frac{1}{4(u^2+v^2)}\left\{\left(\frac{\partial z}{\partial u}\right)^2+\left(\frac{\partial z}{\partial v}\right)^2\right\}$$

(b) Assume that f is continuous on $[a, b]$ and α is monotonically increasing on $[a, b]$. Prove that $f\in R(\alpha)$ on $[a, b]$. (7)

Q.7. (a) Prove that if f is continuous on $[a, b]$ and if α is continuous and monotonically increasing on $[a, b]$ then $f\in R(\alpha)$ on $[a, b]$ (8)

(b) Prove that if $f\in R(\alpha)$ on $[a, b]$ then $f^2\in R(\alpha)$ on $[a, b]$. (7)

Q.8. (a) Prove that a function of bounded variation on $[a, b]$ is expressible as a sum of two monotonically increasing functions. (8)

(b) Show that $\int_0^\infty \sin x^2\,dx$ is convergent. (7)

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