



UNIVERSITY OF THE PUNJAB

MathCity.org
Merging Man and maths

A/2014

Examination:- B.A./B.Sc.

Roll No.

Subject: Mathematics General
PAPER: B

TIME ALLOWED: 3 hrs.
MAX. MARKS: 100

Attempt SIX questions in all selecting TWO questions from Section-I, TWO questions from section-II, ONE question from section-III and ONE question from section-IV.

SECTION-I

Available at
www.mathcity.org

Q.1.a) Analyze and graph the conic represented by the equation,

$$xy = 1$$

9,8

b) Find the angle of intersection of curves $y^2 = 4ax$ and $x^2 = 4ay$ at the point other than (0,0).

Q.2.a) Find an equation of line tangent to $r = 1 + \cos\theta$ at $(1, \frac{\pi}{2})$ in rectangular coordinates.

9,8

b) Find equations of tangent and normal to the curve

$$x = 2a\cos\theta - a\cos 2\theta, y = 2a\sin\theta - a\sin 2\theta, \text{ at } \theta = \frac{\pi}{2}.$$

Q.3.a) Using vectors prove that medians of a triangle are concurrent.

9,8

b) Prove that the components of a vector $\vec{r}$ parallel and perpendicular to $\vec{C}$ in the plane of $\vec{C}$ and $\vec{r}$ are,

$$\frac{\vec{C} \cdot \vec{r}}{\vec{C} \cdot \vec{C}} \vec{C} \quad \text{and} \quad \frac{\vec{C} \times (\vec{r} \times \vec{C})}{\vec{C} \cdot \vec{C}}$$

Q.4.a) If $\vec{f}(t) = a\cos\omega t + b\sin\omega t$, then show that,

9,8

$$\vec{f} \times \vec{f}' = \omega \vec{a} \times \vec{b}$$

b) Find the directional derivatives of $\phi = 4xz^3 - 3x^2y^2$ at (2,-1,2) in the direction $2\vec{i} - 3\vec{j} + 6\vec{k}$.

SECTION-II

Q.5.a) Find the ratio in which the YZ-plane divides the segment joining the points A(-2,4,7), B(3,-5,9).

9,8

b) Find equations of the perpendicular from origin to the line

$$x + 2y + 3z + 4 = 0 = 2x + 3y + 4z + 5. \text{ Also find the coordinates of foot of perpendicular.}$$

Q.6.a) Find an equation of the sphere through the points (0,0,0), (0,1,-1), (-1,2,0) and (1,2,3). Also find its centre and radius.

9, 8

b) Transform the equation $x^2 + y^2 - z^2 = 9$ into spherical coordinates.

Q.7.a) Find the direction of Qibla at a place with Latitude $33^\circ 40' N$ and Longitude $73^\circ 8' E$.

9, 8

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b) Separate into real and imaginary parts of $\sin^{-1}(\cos\theta + i\sin\theta)$.

Q.8.a) Express $\sin^6\theta$ in terms of Sines or Cosines of multiples of θ .

9, 8

b) Prove that, $\coth^{-1}\left(\frac{z}{2}\right) = \sinh^{-1}\left(\frac{z}{\sqrt{4-z^2}}\right)$.

SECTION-III

Q.9.a) Let A and B be idempotent matrices, Show that if A^T is idempotent so is A. Is the sum of two idempotent matrices idempotent? Justify your answer.

8, 8

b) Solve the system of equations;

$$3x_1 + 2x_2 + 4x_3 = 7$$

$$2x_1 + x_2 + x_3 = 4$$

$$x_1 + 3x_2 + 5x_3 = 3$$

Q.10.a) Show that; $\begin{vmatrix} 1+x & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 1+y & 1 \\ 1 & 1 & 1 & 1-y \end{vmatrix} = x^2y^2$

8, 8

b) Express if possible the vectors (2,-5,3) in R^3 as a linear combination of vectors (1,-3,2), (2,-4,-1) and (1,-5,7).

SECTION - IV

Q.11.a) Solve: $dx + \left(\frac{x}{y} - \sin y\right) dy = 0$

8, 8

b) Find an equation of orthogonal trajectories of the curve:

$$r = a(1 + \sin\theta).$$

Q.12.a) Solve initial value problem:

8, 8

$$y'' - 8y' + 15y = 9xe^{2x}; \quad y(0) = 5, y'(0) = 10$$

b) Solve; $(x^3D^3 + 2x^2D^2 - 5xD - 15)y = x^4$