UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(B.A/B.Sc. Part-I)

# Mathematics B-Course (Paper-II)

Roll No: _ A-S ____

Time Allowed : 3 hrs
Max. Marks : 50
Pass Marks : 33%

Attempt **FIVE** Questions in all. Select **TWO** Questions from **Section-A** and **THREE** from **Section-B.**

## Section-A

**1.** a) Show that if $\vec{a}$, $\vec{b}$, $\vec{c}$ are three vectors, then $(\vec{a} + \vec{b}) . [(\vec{b} + \vec{c}) \times (\vec{c} + \vec{a})] = 2[\vec{a} . (\vec{b} \times \vec{c})]$. 5

b) Prove that $(\vec{a} \times \vec{b}) . (\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a}) = (\vec{a} . \vec{b} \times \vec{c})^2$ 5

**2.** a) If $\vec{f}(t)$ is a vector function of t and $\vec{a}$ is a constant vector, Differentiate $\dfrac{\vec{a} \times \vec{f}}{|\vec{f}|} + \dfrac{\vec{f}}{\vec{a} . \vec{f}}$ w.r.to 't'. 5

b) Solve $\vec{a} \times \dfrac{d^2\vec{v}}{dt^2} = \vec{b}$ ; $\vec{a}$ , $\vec{b}$ are constant vectors and $\vec{v}$ is a vector function of t. 5

**3.** a) if $\phi = r^2 e^{-r}$, where r is the magnitude of position vector of any point (x, y, z).

Show that $\nabla \phi = (2 - r) e^{-r} \vec{r}$ . 5

b) Find the value of m, so that the vector $(m\,xy - z^3)\,\underline{i} + (m - 2)\,x^2\,\underline{j} + (1 - m)\,x\,z^2\,\underline{k}$ has its curl equal to zero. 5

## Section-B

**4-** a) Two forces P and Q have a resultant R and the resolved part of R in the direction of P is of magnitude Q show that the angle between the forces is $2 \sin^{-1}\left(\dfrac{P}{2Q}\right)^{1/2} = \cos^{-1}\left(\dfrac{Q - P}{Q}\right)$ 5

b) If the forces $\ell\,\vec{AB}$, $m\,\vec{BC}$, $\ell\,\vec{CD}$ and $m\,\vec{DA}$ acting along the sides of a quadrilateral are equivalent to a couple, show that either $\ell = m$ or ABCD is a parallelogram. 5

**5-** a) The smallest force which can support a body of weight W on a smooth inclined plane, is of magnitude P. Show that the horizontal force necessary to support the same body on the same plane is of magnitude $\dfrac{PW}{\sqrt{W^2 - P^2}}$ 5

b) A circular disc of weight W and radius 'a' is suspended horizontally by two vertical strings each of length $\ell$ attached to the ends of a diameter. A horizontal couple applied to the disc turns it through the angle $\theta$. Find the moment of the couple. 5

**6-** b) A thin uniform rod passes over one peg and under another, the co-efficient of friction between each peg and the rod being $\mu$. The distance between the pegs is $\alpha$, and the straight line joining them makes an angle $\beta$ with the horizontal. Show that equilibrium is not possible unless the length of the rod is greater than $\dfrac{\alpha}{\mu}(\mu + \tan\beta)$. 5

b) A hemispherical shell rest on a rough inclined plane whose angle of friction is $\lambda$. Show that inclination of the plane base to the horizontal cannot be greater than $\sin^{-1}(2 \sin\lambda)$. 5

**7-** a) Find the C.G of the region bounded by the cardioide $r = a(1 + \cos\theta)$. 5

b) Find the C.G of a semi-circular lamina of radius r when its density varies as the cube of the distance from the centre. 5

**8-** b) Three equal rods each of weight W, are freely jointed together at one extremity of each to form a tripod, and rest with their other extremities on a smooth horizontal plane, each rod inclined at an angle of $\theta$ to the vertical, equilibrium being maintained by three light strings each joining two of these extremities. Prove that the tension in each string is $\dfrac{W \tan\theta}{2\sqrt{3}}$. 5

b) A regular octahedron of 12 equal rods each of weight w, freely jointed together, is suspended from one corner. Show that the thrust in each horizontal rod is $\dfrac{3\sqrt{2}}{2}$ W. 5