UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(B.A/B.Sc. Part-I)

Mathematics B-Course (Paper-I)

Roll No: ____________

Time Allowed : 3 hrs
Max. Marks : 50
Pass Marks : 33%

Attempt **FIVE** Questions in all. Select **TWO** Questions from **Section-A** and **THREE** from **Section-B.** All Question carry equal marks.

Section A

1- a) Let G be a group and $a \in G$ have order n, then for any integer K , $a^k = e$ if and only if $k = nq$, where q is any interger. 5

b) Let $C \setminus \{0\}$ be a group of all non-zero complex numbers under multiplication of complex numbers. Prove that the set $H = \{a + ib \in C \setminus \{0\}: a^2 + b^2 = 1\}$ is a subgroup of $C \setminus \{0\}$. 5

2- a) Let H.K be two subgroups of a finite group G. Prove that for any $g \in G$, $g (H \cap K) = g H \cap g k$. 5

b) Prove that every cyclic group is abelian. 5

3- a) Prove that the A_n of all permutations in S_n form a subgroup of S_n. 5

b) If G is an abelian group. Show that $(ab)^n = a^n b^n$ for all $a, b \in G$. 5

Section B

4- a) If the matrices A,B are conformable for sum A + B and the product AB, then prove that $(AB)^t = B^t A^t$. 5

b) Find rank of the matrix 5

$$A = \begin{bmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{bmatrix}$$

5- a) Show that the system of equations 5

$$2x_1 - x_2 + 3x_3 = a$$
$$3x_1 + x_2 - 5x_3 = b$$
$$-5x_1 - 5x_2 + 21x_3 = c$$

is inconsistent if $c \neq 2a - 3b$.

b) prove that 5

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc\,(a+b+c)^3$$

6- a) The matrix of a linear transformation $T : R^3 \to R^3$ is $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix}$ 5

Find T in terms of coordinates and its matrix with respect to the basis

$v_1 = (0, 1, 2)$, $v_2 = (1, 1, 1)$ $v_3 = (1, 0, -2)$

b) Let A and B be distinct n x n matrices with real entries if $AB^2 = BA^2$ and $A^3 = B^3$, show that $A^2 + B^2$ is not invertible. 5

7- a) Let V be a vector space over a field F and $S = \{V_1, V_2 \ldots V_n\}$ be a set of vectors in V, then

i) If S is linearly independent, then any subset of S is also linearly independent.

ii) If S is linearly dependent, then the set $\{V, V_1, V_2 \ldots V_n\}$ is linearly dependent for all $v \in V$. 5

b) Show that the complex numbers $2 + 3i$ and $2 - 3i$ generate the vector space C over R. 5

8- a) A set $S = \{V_1, V_2 \ldots V_n\}$ of n vectors $(n \geq 2)$ in a vector space V is linearly dependent if and only if at least one of the vectors in S is a linear combination of the remaining vectors of the set. 5

b) Let C be the vector space of complex numbers over R and $T : c \to c$ be defined by: $T(Z) = \overline{Z}$, where $\overline{Z}$ denotes the complex conjugate of Z. Show that T is linear. 5