UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(B.A/B.Sc. Part-I)

Roll No: ____________

Mathematics B-Course (Paper-II)

Time Allowed : 3 hrs
Max. Marks : 50
Pass Marks : 33%

Attempt **FIVE** Questions in all. Select **TWO** Questions from **Section-A** and **THREE** from **Section-B.**

Section-A

1. a) If $\vec{a}$ and $\vec{b}$ are two vectors, prove that $(\vec{a} \times \vec{b})^2 = (\vec{a})^2(\vec{b})^2 - (\vec{a}.\vec{b})^2$ 5

b) Show that $(\vec{a} + \vec{b}) . [(\vec{b} + \vec{c}) \times (\vec{c} + \vec{a})] = 2[\vec{a} . (\vec{b} \times \vec{c})]$ 5

2. a) If $\vec{r} = (\text{Cos } nt)\, \hat{i} + (\text{Sin } nt)\, \hat{j}$; where n is constant, show that $\vec{r} \times \frac{d\vec{r}}{dt} = n\hat{k}$. 5

b) If $\underline{f}''(t) = 4\hat{i}$ and if $\underline{f}(t) = 0$; when t = 0 and $\underline{f}'(t) = 4\hat{j}$; when t = 0, show that the tip of the position vector $\underline{f}(t)$ describes a parabola. 5

3. a) If $\underline{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$, then show that div (grad r^m) = $m(m+1)\, r^{m-2}$, where $|\underline{r}| = r$. 5

b) If ϕ be a scalar point function and $\underline{F}$ be a vector point function, then prove that curl $(\phi\, \underline{F})$ = (grad ϕ) x $\underline{F}$ + ϕ curl $\underline{F}$. 5

Section-B

4- a) If two forces P and Q act at such an angle that their resultant R = P, show that if P is doubled, the new resultant is at right angles to Q. ch 2 5

b) Forces of magnitude P, 2P, 3P, 4P act respectively along the sides AB, BC, CD, DA of a square ABCD, of side a, and forces each of magnitude $(8\sqrt{2})$ P act along the diagonals BD, AC. Find the magnitude of the resultant force and the of its line of action from A. 2.17 Example 1 5

5- a) A triangular lamina ABC is suspended from a point O by light strings fastened to the points A, B and hangs so that the side BC is vertical. Prove that, if α, β are the angles which the strings AO, BO make with the vertical, then $2 \cot \alpha - \cot \beta = 3 \cot B$. 5

b) A smooth circular cylinder of radius b is fixed parallel to a smooth vertical wall with its axis at a distance c from the wall. A smooth uniform heavy rod of length 2a rests on the cylinder with one end on the wall and in a plane perpendicular to the wall. Show that its inclination θ to the horizontal is given by $a \text{ Cos}^3 \theta + b \text{ Sin } \theta = c$ 5

6- a) Find the force necessary just to support a heavy particle on an inclined plane of inclination α $(\alpha > \lambda)$. 5

b) A thin uniform rod passes over one peg and under another, the co-efficient of friction between each peg and the rod being μ. The distance between the pegs is α, and the straight line joining them makes an angle β with the horizontal. Show that equilibrium is not possible unless the length of the rod is greater than $\frac{\alpha}{\mu}(\mu + \tan \beta)$. ch:5 5

7- a) Show that the C.G of uniform lamina bounded by a loop of the lemniscate $r^2 = a^2 \text{ Cos } 2\theta$ is on the initial line at a distance $\frac{\pi a}{4\sqrt{2}}$ from the pole . 5

b) Find the C.G of a uniform wire in the shape of the parabolic arc $y^2 = 4ax$ with ends as the extremities of the latus rectum. 5

8- a) Six equal rods AB, BC, CD, DE, EF and FA are each of weight W and are freely jointed at their extremities so as to form a hexagone. The rod AB is fixed in a horizontal position and the middle points of AB and DE are jointed by a string. Prove that its tension is 3W. ch: 6 5

b) A string of length a forms the shorter diagonal of a rhombus formed by four uniform rods, each of length b and weight w, which are hinged together. If one of the rods be supported in a horizontal position, prove that the tension in the string is $\frac{2w(2b^2 - a^2)}{b\sqrt{4b^2 - a^2}}$