UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(B.A/B.Sc. Part-II)

Mathematics A-Course (Paper-III)

Roll No: ____________

Time Allowed : 3 hrs
Max. Marks : 50
Pass Marks : 33%

Attempt **FIVE** Questions, selecting **TWO** questions form **Section-A**, and **THREE** from **Section-B.**

SECTION – A

1- a) Determine whether the following series converges or diverges:

(8.2 Exp) $\sum_{n=1}^{\infty} \frac{1}{(n+2)(n+3)} = \frac{1}{3.4} + \frac{1}{4.5} + \frac{1}{5.6} + \ldots\ldots,$ 5

b) Use Comparison Test to determine the convergence or divergence of the series $\sum_{1}^{\infty} \frac{\ell nn}{n}$ (8.2) 5

2- a) Test the series for convergence or divergence $\sum_{1}^{\infty} \frac{(n+2)!}{4!n!\,2^n}$ (8.3) 5

b) If $x > 0$, show that the series converges for $x < 4$ $\sum \frac{(n!)^2}{(2n!)} x^n$ (8.3) 5

3- a) Define ABSOLUTE CONVERGENCE. Test the given series for Absolute Convergence. $\sum_{0}^{\infty} \frac{(-2)^n}{3^n + 1}$ (8.4) 5

b) Use Integral Test to determine the Convergence or Divergence of (8.2 exampe) 5

$$\sum_{1}^{\infty} \frac{\text{arc}\tan n}{1+n^2}$$

Section – B

4- a) Solve the Boundary Value Problem $x\frac{dy}{dx} + 2y = 4x^2$ $\quad y(1) = 2$ where $y = x^2 + \frac{c}{x^2}$ is the general solution. (9.1) 5

b) Solve the Initial Value Problem $\frac{dy}{dx} = \frac{2x}{y + x^2 y}$ $y(0) = -2$ (9.2 Exp) 5

5- a) Solve $\frac{dy}{dx} = \frac{3x - 4y - 2}{3x - 4y - 3}$ (9.3) 5

b) Solve the Initial Value Problem $(2x\text{Cos}y + 3x^2 y)dx + (x^3 - x^2\,\text{Sin}y - y)dy = 0$ (9.4) $y(0) = 2$ 5

6- a) Solve $(x-1)^3 \frac{dy}{dx} + 4(x-1)^2 y = x + 1$ (9.6)(P=404) 5

b) Solve $(D^3 + D^2 - 4D - 4)\,y = e^{2x}\,\text{Cos}\,3x$ exmple 10.2 P=448 5

7- a) Solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2\,\text{Sin}(\ell n\,x)$ (10.4) 5

b) Find the family of Curves Orthogonal the family of Surfaces $x^2 + 2y^2 + 4z^2 = c.$ 5

8- a) Consider the function f defined by $f(t) = \frac{1}{t}$. Show that $\mathcal{L}\{\frac{1}{t}\}$ does not exist. 5

b) Evaluate $\mathcal{L}\left\{\int_{0}^{t} \frac{1 - \text{Cosh}\,au}{u}\,du\right\}$ 5