SECTION – A

1- a) Show that $a - b \mid a^n - b^n$ for all $n\ (\geq 0) \in Z$. 5

b) Prove that if n is any odd integer then $8 \mid n^2 - 1$. 5

2- a) Find the G.C.D. of the pair (243, 129) and express it as a linear combination of 243 and 129. 5

b) Show that (ma, mb) = m(a, b) where m is a positive integer. 5

3- a) Show that if (c, b) = 1, then (ac, b) = (a, b) 5

b) Find D O O W K H |0|.S U L P H V ” 5

SECTION – B

4- a) Define Interior of a set and prove that for any two subsets A and B of (X, τ) $(A \cup B)^o \supseteq A^o \cup B^o$ 5

b) Prove that a subset A of (X, τ) has empty frontier if and only if A is both open and closed. 5

5- a) Let A be a subset of (X, τ), then prove that $\bar{A} = A \cup A'$. 5

b) Let (X, τ) be a topological space. A collection $B = \{B_\lambda : \lambda \in \Lambda\}$ of sets in τ is a base for τ if and only if, for any open set U and any point x in U, there is a $B_\lambda \in B$ such that $x \in B_\lambda \subseteq U$. 5

6- a) Prove that $d(f, g) = \int_a^b |f(x) - g(x)|\,dx$ defines a metric on B [a, b], a set of real valued Bounded functions. 5

b) The diameter of a closed ball Š D U L Q D P H W U L F V S D F H ; G L V ” U 5

7- a) 3 U R Y H W K D W $d(f, g) = \int_0^1 |f(x) - g(x)|\,dx$ is a metrix on X which is a set of integral functions on [0, 1] 5

b) Let (X, d) be a metric space and x be a limit point of a subset A but not in A. Then every open ball B (x ; r) contains an infinite number of points of A. 5

8- a) Let $u, v \in R^2$, $u = (x_1, x_2)$, $v = (y_1, y_2)$ then prove that $\langle u, v \rangle = x_1y_1 - x_1y_2 - x_2y_1 + 3x_2y_2$ is an inner product on R^2. 5

b) Find the norm of $v = (3, 4) \in R^2$ w.r.t. the Euclidean Inner Product and the Inner Product defined in part (a). 5