UNIVERSITY OF GUJRAT
A WORLD CLASS UNIVERSITY

(B.A/B.Sc. Part-I)
Mathematics B-Course (Paper-II)

Roll No: ________

Time Allowed : 3 hrs
Max. Marks : 50
Pass Marks : 33%

Attempt **FIVE** Questions in all. Select **TWO** Questions from **Section-A** and **THREE** from **Section-B.**

SECTION-A

1- a) Prove that $\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} \times \vec{c}\right) \cdot \vec{d} = \left(\vec{a} \cdot \vec{d}\right)\left(\vec{a} \times \vec{b} \cdot \vec{c}\right)$ *5*

b) Establish the Identity: $\vec{a} = \frac{1}{2}[\underline{i} \times (\vec{a} \times \underline{i}) + \underline{j} \times (\vec{a} \times \underline{j}) + \underline{k} \times (\vec{a} \times \underline{k})$). *5*

2. a) Differentiate w.r. to t, where $\vec{r}$ is a vector function of scalar variable t and $\vec{a}$ is a constant vector $\frac{\vec{r} \times \vec{a}}{\vec{r} \cdot \vec{a}}$. *5*

b) Solve: $\frac{d^2\vec{r}}{dt^2} = \vec{a}$, where $\vec{a}$ is a constant vector also, it is given that when t = 0; $\vec{r} = 0$ and $\frac{d\vec{r}}{dt} = 0$. *5*

3- a) Prove that $\nabla^2 \left|\vec{r}\right|^n = n(n+1)\left|\vec{r}\right|^{n-2}$. Where n is constant and $\vec{r} = x\underline{i} + y\underline{j} + z\underline{k}$. *5*

b) Show that $\nabla \times (\phi\vec{A}) = (\nabla\phi) \times \vec{A} + \phi(\nabla \times \vec{A})$. *5*

SECTION-B

4- a) Forces $2\overrightarrow{BC}$, $\overrightarrow{CA}$, $\overrightarrow{BA}$ acting along the sides of a Δ ABC. Show that their resultant is $6\overrightarrow{DE}$, where D bisects BC and E is a point on CA such that $CE = \frac{1}{3}CA$. *5*

b) Forces $\vec{P}$, $2\vec{P}$, $3\vec{P}$, $6\vec{P}$, $5\vec{P}$ and $4\vec{P}$ act respectively along the sides AB, CB, CD, ED, EF and AF of a regular hexagon of side 'a', the sense of the forces being indicated by the order of the letters. Prove that the six forces are equivalent to a couple. *5*

5- a) A triangle lamina ABC is suspended from a point O by a light strings attached to points A and B and hangs so that the side BC is vertical. Prove that if α, β are due angles which the strings AO, BO make with the vertical, then $2\cot\alpha - \cot\beta = 3\cot B$. *5*

b) Two beads of weigh W and W′ can slide on a smooth circular wire in a vertical plane, they are connected by a light string which subtends an angle 2β at the centre of the circle when the beads are in equilibrium on the upper half of the wire. Prove that the inclination α of the string to the horizontal is given by $\tan\alpha = \frac{W - W'}{W + W'}\tan\beta$ *5*

6- a) The least force which will move a weight up the plane on inclined plane is of magnitude P. Show that the least force acting parallel to the plane which will move the weight upward is $P\sqrt{1+\mu^2}$, where μ is the coefficient of friction. *5*

b) A uniform rod of weight W placed with its lower end on a rough floor and upper end against an equally rough vertical wall. The rod makes an angle α with the wall and is just prevented from slipping down by a horizontal force P applied at its middle point. Prove that $P = W\tan(\alpha - 2\lambda)$, where λ is the angle of friction and $\lambda < \frac{1}{2}\alpha$. *5*

7- a) ABCD is a trapezium which bounds a uniform lamina. AB, CD are parallel and of length a, b respectively. Prove that the distance of the C.G of the lamina from AB is $\frac{1}{2}h\frac{a+2b}{a+b}$, where h is the distance between parallel sides. *5*

b) Find the centre of gravity of a uniform wire in the shape of parabolic arc $y^2 = 4ax$ with ends as the extremities of the latus rectum. *5*

8- a) A light thin rod, 12 ft. long, can turn in a vertical plane about on one of its points which is attached to a pivot. If weights of 3 lb and 4 lb are suspended from its ends, it rests in a horizontal position. Find the position of the pivot and its reaction on the rod. *5*

b) A hexagon ABCDEF, consisting of six equal heavy rods, of weight W, freely jointed together, hung in vertical plane with AB horizontal, and the frame is kept in the form of a regular hexagon by a light rod connecting the mid-points of CD and EF. Show that the Thrust in the light rod is $2\sqrt{3}$ W. *5*