

Mathematics A-Course (Paper-I)

Attempt FIVE Questions in all. Select THREE Questions from Section-A and TWO from Section-B.

Section-A

1. a) Solve $\frac{2x-5}{x-2} < 1$. 5
- b) Evaluate $\lim_{x \rightarrow \infty} \left(\frac{x + \sin x}{x} \right)$. 5
2. a) Show that the function $f(x) = |x|$ is continuous at every real number c. 5 *47*
- b) Evaluate $\lim_{x \rightarrow 0} \frac{\log_e (1-x^2)}{e^{\cos x}}$. 5
3. a) State and prove Leibniz's Theorem. 5
- b) If $f(x, y) = \frac{x^2 + y^2}{x + y}$, then prove that $(f_x - f_y)^2 = 4(1 - f_x - f_y)$. 5 *ok*
4. a) $\forall x > 0$ prove that $x - e^{\log(1+x)} > \frac{x^2}{2(1+x)}$. 5
- b) Find the Maclaurin's series of the function $f(x) = \cos x$. 5 *ok*
5. a) Evaluate $\lim_{x \rightarrow \infty} \frac{a^x - 1}{x}$, $a > 1$. 5 *ok*
- b) Prove that $\sqrt{2}$ is irrational. 5

Section-B

6. a) Compute $\int_a^b \cos x \, dx$ by definition. 5 *$-\frac{1}{3} - 1$*
- b) Evaluate $\int x^2 e^{ax} \, dx$. 5 *$-\frac{2}{3} + -3$*
7. a) Compute $\int_{-1}^8 \frac{dx}{x^{1/3}}$. 5 *ok*
- b) Let $I_n = \int_0^{\infty} x^n e^{-x} \, dx$ where n is a natural number.
 Prove that $I_n = n I_{n-1}$ and $I_n = n!$. 5
8. a) Evaluate $\int_0^{\pi} \frac{x \, dx}{a^2 \cos^2 x + b^2 \sin^2 x}$. 5
- b) Evaluate $\int_0^1 \frac{x^6 \, dx}{\sqrt{1-x^2}}$. 5