UNIVERSITY OF GUJRAT
A WORLD CLASS UNIVERSITY

(B.A/B.Sc. Part-II)

Mathematics B-Course (Paper-III)

Roll No: ________

Time Allowed : 3 hrs
Max. Marks : 50
Pass Marks : 33%

Attempt **FIVE** Questions, selecting **TWO** questions form **Section-A,** and **THREE** from **Section-B.**

SECTION – A

1- a) Prove that if 'n' is a positive even integer then $a + b \mid a^n - b^n$. 5

b) Show that forall Q ! / = $9/10^n + 3 . 4^{n+2} + 5$ 5

2- a) Write down the standard forms of 3180 and 575 and then find (3180, 575) and <3180, 575> 5

b) Show that none of the following (n-1) consecutive integers is prime.

n! + 2, n! + 3, n! + 4 ………. n! + n

Hence show that given a +ve integer N, it is always possible to find N consecutive composite integer 5

3- a) Find all the primes" |0|. 5

b) Determine whether 841 is a prime or not. 5

SECTION – B

4- a) Let (X, $\tilde{|}$) be a topological space and sets A and B are subsets of X. Then show that $\overline{A \cup B} = \overline{A} \cup \overline{B}$ 5

b) Let (X, $\tilde{|}$) be a topological space then show that ext (A) = $(\overline{A})^c$ 5

5- a) Let A be a subset of (X, $\tilde{|}$) then A is closed iff AD$\overline{A}$. 5

b) If X = {a, b, c} and $\tilde{|}$ = { n ^ D ` ^ E ` } be a topology on X. Then find A^d if A = {b, c}. 5

6- a) Prove that any OPEN BALL in a metric space is an OPEN SET. 5

b) Let X be the set of all continuous real function defined on [0, 1].

For f, g ÐX, defined $d(f, g) = \int_0^1 |f(x) - g(x)|$. Prove that (X, d) is a metric space 5

7- a) Prove that $\overset{o}{A}$ is the union of all open sets contained in A. 5

b) If (X, d) be a metric space. If F= {x} is a subset of X. Then F = X – {x} is open. 5

8- a) Every 'ORTHONORMAL' system $\{u_1, u_2 \ldots\ldots\ldots u_n\}$ is linearly independent.

Moreover for all v ÐV, the vector $w = v - \sum_{k=1}^{n} < V, U_k > U_k$ is orthogonal to each iU 1 " i " .Q 5

b) Let 'V, be the 'VECTOR SPACE' P(x) of polynomials over R.

Show that $< f, g > = \int_0^1 f(t)\, g(t)\, dt$ defines an inner product on 'V' 5