Section A

1- a) Let G be any group. Let $a \in G$ have order n. Then for any integer K, $a^k = e$ if and only if K = qn, where q is an integer. *5*

b) In a group G, let a, b and ab all have order 2. Show that ab = ba. *5*

2- a) Let H be a subgroup of a group G. Then the set of all left cosets of H in G defines a partition of G. *5*

b) Find all the subgroups of a cyclic group of order 12. *5*

3- a) Every permutation of degree n can be expressed as a product of transpositions. *5*

b) Let $Ŭ = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 5 & 4 & 1 & 7 & 2 & 6 \end{pmatrix}$. Find the inverse of Ŭ. *5*

Section B

4- a) Show that every Hermitian Matrix can be written as A + *i* B, where A is real and symmetric and B is real and skew symmetric. *5*

b) Find the inverse of the matrix $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 4 & 1 \\ 1 & 3 & 0 \end{pmatrix}$ *5*

5- a) Show that the system,

$$2x_1 - x_2 + 3x_3 = a$$
$$3x_1 + x_2 - 5x_3 = b$$
$$-5x_1 - 5x_2 + 21x_3 = c$$

is inconsistent if $c \neq 2a - 3b$. *5*

b) Find the matrix of each of the following linear transformations from R^3 to R^2 with respect to standard basis for R^3 $T(x_1, x_2, x_3) = (x_1, x_2, 0)$ *5*

6- a) Show that the inverse of a scaler matrix is a scaler matrix. *5*

b) Prove that $\begin{vmatrix} \text{Sin}^2Ŭ & \text{Cos}2Ŭ & \text{Cos}^2Ŭ \\ \text{Sin}^2б & \text{Cos}2б & \text{Cos}^2б \\ \text{Sin}^2ɔ & \text{Cos}2ɔ & \text{Cos}^2ɔ \end{vmatrix} = 0$ *5*

7- a) Let V be a vector space over a field F. Then prove that $a(u - v) = au - av$, for all $a \in F$, $u, v \in V$. *5*

b) Show that the vectors (3, 0, -3) (-1, 1, 2) (4, 2, -2) and (2, 1, 1) are linearly dependent over R. *5*

8- a) Verify that the polynomials $2 - x^2$, $x^3 - x$, $2 - 3x^2$ and $3 - x^3$ form a basis for $P_3(x)$. *5*

b) Let $V_1 = (1, 1)$ and $V_2 = (1, 0)$ be a basis of R^2. Find a formula for the linear transformation $T : R^2 \to R^3$ for which $T(V_1) = (1, 2, 1)$ and $T(V_2) = (-1, 0, 2)$. *5*