

Mathematics A-Course (Paper-II)

Attempt FIVE Questions in all. Select TWO Questions from Section-A and THREE from Section-B.

Section-A

1. a) Show that $z = a + ib$ is
 i) real if and only if $z = \bar{z}$
 ii) pure imaginary if and only if $z = -\bar{z}$ 5
 b) State and prove (De Moivre's Theorem). 5
2. a) Prove that $\cos \frac{\pi}{7} - \cos \frac{2\pi}{7} + \cos \frac{3\pi}{7} = \frac{1}{2}$ 5
 b) If $\tan(\theta + \phi i) = \tan \alpha + i \sec \alpha$, Prove that $e^{2\phi} = \pm \cot \frac{\alpha}{2}$ and $2\theta = n\pi + \frac{\pi}{2} + \alpha$ 5
3. a) Separate into real and imaginary parts $\tan^{-1}(x + iy)$. 5
 b) Evaluate the sum of the infinite series $\cos \theta - \frac{1}{2} \cos 2\theta + \frac{1}{3} \cos 3\theta - \frac{1}{4} \cos 4\theta + \dots$ 5

Section-B

4. a) Find the Pedal Equation of the Parabola $y^2 = 4a(x + a)$ 5
 b) Show that, the tangents at the end of a focal chord of a parabola intersect at right angle on the directrix. 5
5. a) Show that, In any conic semi-latus rectum is the harmonic mean between the segments of focal chord. 5
 b) Find an equation (in rectangular coordinates) of the tangent line to $r = (1 + \cos \theta)$ at $(1, \frac{\pi}{2})$ 5
6. a) Find the distance of the given point P from the given line L.
 $P(0, -2, 1)$ $L: \frac{x-1}{4} = \frac{y+3}{-2} = \frac{z+1}{5}$ 5
 b) Derive the equation of the plane $lx + my + nz = p$. 5
7. a) Find an equation of the plane containing the line $x = 2t, y = 3t$ and $z = 4t$,
 Also intersection of the planes $x + y + z = 0$ and $2y - z = 0$. 5
 b) Show that, the shortest distance between the lines $x + a = 2y = -12z$ and
 $x = y + 2a = 6(z - a)$ is $2a$. 5
8. a) Find the centre and radius of the sphere $x^2 + y^2 + z^2 - 4x + 2y - 6z - 11 = 0$ 5
 b) Find an equation of the torus obtained by revolving about y-axis and the circle in xy-plane with
 centre at $(a, 0, 0)$ and radius b , where $0 < b < a$. 5