UNIVERSITY OF GUJRAT

A WORLD CLASS UNIVERSITY

(B.A/B.Sc. Part-II)

Mathematics B-Course (Paper-IV)

Roll No: _____

Time Allowed : 3 hrs
Max. Marks : 50
Pass Marks : 33%

Attempt **FIVE** Questions in all, selecting **THREE** questions form **Section-A,** and **TWO** from **Section-B.**

Section – A

1. a) Find the tangential and normal components of the acceleration of a point describing the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with uniform speed 'v' when the particle is at (o, b) *5*

b) The position of a particle moving long an ellipse is given by $\vec{r} = a \cos t \hat{\wedge} + b \sin t \hat{j}$ if a > b

Find the position of the particle where its velocity has a maximum or minimum magnitude. *5*

2. a) A planet moves in an elliptic orbit with centre of attraction. The sun in one focus. If the greatest and least velocities of the planet are v_1 and v_2 find the eccentricity of the ellipse. *5*

b) Prove that the field of force $\vec{F}$ determined by $\vec{F} = (y^2 - 2xyz^3)\hat{\wedge} + (3 + 2xy - x^2z^3)\hat{j} + (6z^3 - 3x^2yz^2)\hat{k}$ is conservative and find its potential. *5*

3. a) Prove that the least velocity with which a particle must be projected so that it passes through the points 'P' and 'Q' at the heights hP and hQ respectively from the ground is given by $\sqrt{g(hP + hQ + P\theta)}$ *5*

b) A ball is dropped from the top of a tower of height h at the same moment another ball is thrown from a point of the ground at a distance 'k' from the foot of tower so as to strike the first ball at a depth 'd' show that the initial speed and the direction of projection of the second ball are respectively

$$\sqrt{\frac{g(h^2 + k^2)}{2d}} \text{ and } \tan^{-1}\left(\frac{h}{k}\right)$$ *5*

4. a) A particle describing simple harmonic motion has velocities 5ft/sec and 4ft/sec when its distances from the centre are 12ft and 13ft respectively. Find the time period of motion. *5*

b) Discuss the motion of a particle moving in a straight line if it start from rest at a distance 'a' from a point 'o' and moves with an acceleration equal to μ times its distance from 'o' *5*

5. a) A particle of mass m is moving under the action of the forces $F_1 = -mw^2x$, $F_2 = mF_0t$, $F_3 = -2m\mu x$ assuming that damping is small, set up and solve the equation of motion. *5*

b) The radial and transverse components of velocity of a moving particle are λr and μθ respectively. Find the radial and transverse components of its acceleration. *5*

Section – B

6. a) Use the Secont Method to find correct to 4 decimal places the root of the equation $e^x - 3x = 0$ starting values are $x_0 = 0.4$ and $x_1 = 0.9$ *5*

b) Use the Newton Raphson method find to 4 decimal places the root near 0.5 of the equation $e^{-x} - \sin x = 0$ *5*

7. a) Solve the following equations by Jacobi's method upto 4 iteration *5*

$$10x + y - 3z = 8$$
$$x + 9y - 4z = 6$$
$$x + y + 5z = 7$$

b) Find to 4 decimal places the root near 3 of the equation $e^x - \log e^x = 20$ using simple iteration. *5*

8. Evaluate the integral $\int_0^2 \frac{1}{\sqrt{1+x^2}}\,dx$

a) By Trapezium '5' point Rule *5*

b) By Simpson's '5' point Rule *5*