lopology And
Functional Analysis
Paper A/2011 SECTION I M. SCHMONES N. SCHM
Paper H/ZUII MATHEMATICA CHANGE STANGER STANGER
Paper H/2011 Section I M. Schrift and section # 1(a) s- Define a
sub-Base and a Base for a
topology on a set X. Let S be a non-empty collection of subsets of
X. Suppose that X = US. Then show
that S is sub-Base for some
topology on X. MATHEMATICA ACADEMY Offers
201 3- M.Sc(Maths) 0300-4285657 0305-5130982
Jub-Bases-let (X,T) be a
topological space & SCP(X), then S is said to be sub-base for this
(X,T), if the collection consisting of
all possible intersections of all possible finite sub-families of S forms a
base for (X,T).
Base: Let (X,T) be a topological
space. Then a collection & of subsets of X is said to be base for X
if:
ii) $\beta \subseteq T$ iii) For every $U \in T$, there is a
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subfamily 1. of B s.t. Let B be the collection of all possible finite intersections of members (Sub-families) of S. _i.e. β = { β : B = ∩ Si, Si ∈ S, 1 ≤ i ≤ n f $X = US \subseteq UB \subseteq X$ Moment of house, $X = US \subseteq UB \subseteq X$ M. Sc(PU) M. Phil(COMSATS)

Sortact: 0300-4285857 then → X = UB Further let B_1 , $B_2 \in \beta$ $A_1 \times E_1 \cap B_2$ some B_1 , A_2 are finite intersections of sub-families of S. Some sub-family of S \Rightarrow $B, \cap B_3 = B_3 \in S$ M.Sc(Math 0300-4285657 0305-513 Then $x \in B_3 \subseteq B_1 \cap B_3$. Then by a well known theorem X be a non-empty set. A family & of subsets of X is base for some topology T on X if and (i) $X = \bigcup_{\alpha \in I} \beta_{\alpha}$, $\beta_{\alpha} \in \beta$ (ii) For $B_1, B_2 \in \beta$ and $x \in B_1 \cap B_2$ then there exists some $\beta_3 \in \beta$ such that $x \in B_3 \subseteq B, \cap B_2$. is base for some topology on X.

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Hence S is sub-base for some
topology on X.
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(Juestion # 1 (b) 3-
(i) Give an enample of topological -
_ space which is not mexically
Space (X,T) is known as the Sierpinski
2015- let X = 2a, bs and 1=24, 1,292
Then I B a topology on the Sierpinski
Space (X, L) is known as
space. In this space, every subset of X - is either open or closed.
This space is not a merornance
topological space. That is no mexall
d on X can be defined such
that the topology of (1,01) is equal-
_ to_ l.
For suppose that d is a metric on
of (x,d) is T. Since {a} is open in
(X,T) and a E {a}, there is an open
La la so Contract his definition of
an open set. So for every $x \in B(a;r)$,
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
$+$ \times \times \times \times \times \times \times \times \times
Hence $d(a,b) \geq 8$
Consider now B(b; r). Then for all
Hence A & B(b; x), d(y,b) < x MATHEMATICA ACADEMY OHERS OHRS OHERS OHRS OHRS OHRS OHRS OHRS OHRS OHRS OH
Alence a \notin B(b; \tag{b}) MATHEMATICA Offers Offers ColMaths)
Thus $B(b; x) = \{b\}$ MATHEMOHERS OHERS $B(b; x) = \{b\}$ M.Sc(Maths) $B(b; x) = \{b\}$
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So that Eb3 is open in (X,d).
2 to Sh? is not open in (1,01,7)
But $\{b\}$ is not open in (X,T) . So (X,T) is not metrizable.
so (A, L) is not metrizable.
Question # 1(0):-
Question # 1(b):- (ii) Show that the internals
(0,1) and (a,b) are homeomorphic.
C,
Sols-
Here $f: X \rightarrow Y$ defined by
Here f: X -> Y defined by
$f(x) = \frac{x-a}{b-a} \text{ is homeomorphism.}$
6-9
because
i) f is continuous obviously.
(ii) f is bijective.
A 3434.
$x_1 - q = x_2 - q $ As a philicoms at the philicoms are set of the philipped are set of the ph
Let $f(x_1) = f(x_2)$ $\Rightarrow \frac{x_1 - a}{b - a} = \frac{x_2 - a}{b - a} \text{ AGAMAN Phillicoms ATS}$ $\Rightarrow \frac{x_1 - a}{b - a} = \frac{x_2 - a}{b - a} \text{ M.Sc(PU) M.Phillicoms AZ85857}$ Contect: 0300 4285857
x = 0
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=> ×, = ×2 MATHEMATICA ACADEMY Offers Offers 0,300-4285657 0305-5130982
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To f 15 H
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lot x6 y then we can sind
I am element $\pi(b-a)+a \in X$
8.t.
$\int \left[\chi(b-a) + \alpha \right] = \chi(b-a) + \alpha - \alpha$
6-9
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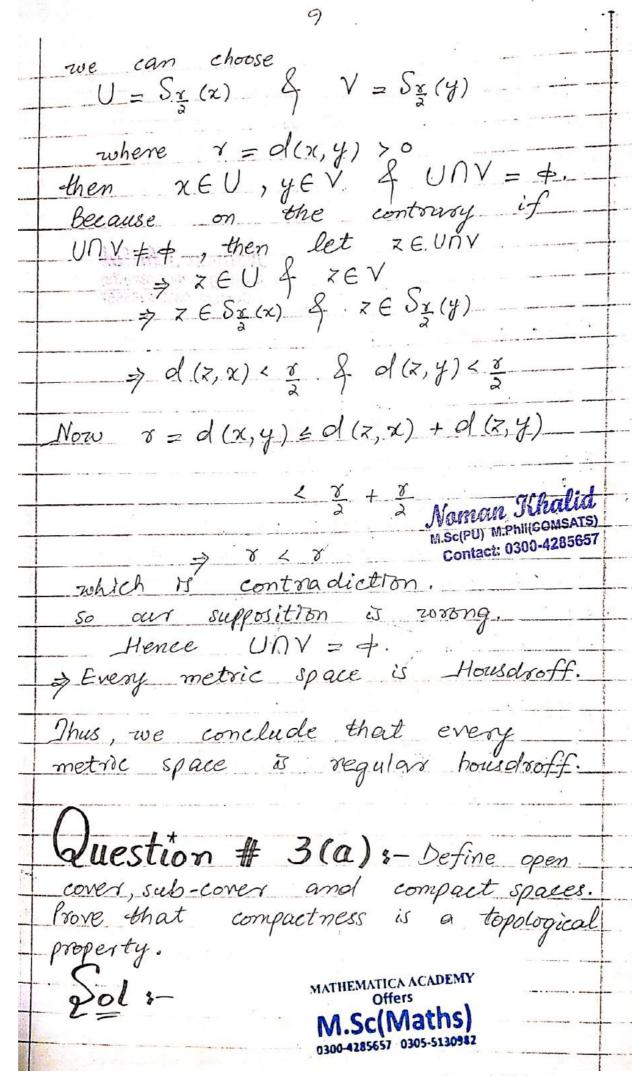
5
$\Rightarrow f[\chi(b-a)+a] = \chi$
=> f & onto.
Hence f 15 bijective.
(iii) Obviously f: Y -> X defined
f(y) = a(1-y) + by
exists and is continuered. Generally Hence $(a,b) \simeq (o,1)$. Normally Miscipul M. Phillipson 4285857 Question # 2(a) s- let x be
A CONTROL NO PHILCOMS SUST
Hence $(a,b) \simeq (0,1)$. M. Scipul C. 0300
Question # 2(a) s- let x be
an arbitrary topological space and Y be a Housdroff space let
Y be a Housdroff space. let
f: X -> Y be a continuous function
Then show that the graph $G = \frac{\{(x,y): y = f(x)\} \subseteq X \times Y \text{ is closed}}{}$
$\frac{1}{9} = \frac{3}{2} \left(\frac{\chi}{\chi}, \frac{\zeta}{y} \right) : \frac{1}{9} = \frac{3}{2} \left(\frac{\chi}{\chi} \right) =$
$\frac{1}{C}$ $\frac{1}$
Pol: For this we prove G is
epen in XxY.
let (xy) E G
then $(x, y) \notin G$
then $(x, y) \notin G$ $\Rightarrow y \neq f(x)$ $= \frac{1}{2} (x) + \frac{1}{2} (x)$ $= \frac{1}{2} (x) + \frac{1}{2} (x)$ $= \frac{1}{2} (x) + \frac{1}{2} (x)$
Ostra-
As 4 fix) EY & Y & To-space
As 4 fix) EY & Y & To-space
As y , $f(x) \in Y$ & Y is T_3 -space So \exists two open sets U and Y in Y s.t. $y \in U$, $f(x) \in Y$ and

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Now as $f(x) \in V$ so $\chi \in f(V)$ \Rightarrow $(\chi, y) \in f(V) \times U$ As V is open in $Y \notin f(X) \times Y$ \Rightarrow continuous so $f(V)$ is open in X . \Rightarrow $f(V) \times U$ is open in $X \times Y$. Now we prove $f(V) \times U \subseteq G$ Let $(\alpha, \beta) \in f(V) \times U \subseteq G$ \Rightarrow $f(\alpha) \in V$ \Rightarrow $\beta \in U$ \Rightarrow $f(\alpha) \in V$ \Rightarrow $\beta \in U$ \Rightarrow $f(\alpha) \in V$ \Rightarrow f	6
⇒ $f(v) \times U$ is open in $X \times Y$. Now we prove $f(v) \times U \subseteq G$ Let $(\alpha, \beta) \in f(v) \times U$ ⇒ $\alpha \in f(v) \times G$ ⇒ $f(\alpha) \in V$ ⇒ $f(\alpha$	
$\Rightarrow f(v) \times U \text{is open in } X \times Y.$ Now we prove $f'(v) \times U \subseteq G'$ let $(\alpha, \beta) \in f'(v) \times U$ $\Rightarrow \alpha \in f'(v) \text{?} \beta \in U$ $\Rightarrow f(\alpha) \in V \text{?} \beta \in U$ $\Rightarrow \beta \neq f(\alpha) \text{:: } U \cap V = \emptyset$ $\Rightarrow (\alpha, \beta) \in G \text{Unitare Straits}$ $\Rightarrow f'(v) \times U \subseteq G \text{Mathematica academy offers}$ $\Rightarrow (\alpha, \beta) \in G \text{Mathematica academy offers}$ $\Rightarrow f'(v) \times U \subseteq G \text{Mathematica academy offers}$ $\Rightarrow (\alpha, \beta) \in G \text{Mathematica academy offers}$ $\Rightarrow f'(v) \times U \subseteq G \text{Mathematica academy offers}$ $\Rightarrow G \text{is open in } X \times Y. \text{Osciolatis}$ $\Rightarrow G \text{is closed in } X \times Y. \text{Osciolatis}$ $\Rightarrow G \text{is closed in } X \times Y. \text{Osciolatis}$ $\Rightarrow G \text{is closed in } X \times Y. \text{Osciolatis}$ $\Rightarrow G \text{is closed in } X \times Y. \text{Osciolatis}$ $\Rightarrow G \text{is closed in } X \times Y. \text{Osciolatis}$ $\Rightarrow G \text{is closed in } X \times Y. \text{Osciolatis}$ $\Rightarrow G \text{is closed in } X \times Y. \text{Osciolatis}$ $\Rightarrow G \text{is closed in } X \times Y. \text{Osciolatis}$ $\Rightarrow G \text{is closed in } X \times Y. \text{Osciolatis}$ $\Rightarrow G \text{is closed in } X \times Y. \text{Osciolatis}$ $\Rightarrow G \text{is closed in } X \times Y. \text{Osciolatis}$ $\Rightarrow G \text{is closed in } X \times Y. \text{Osciolatis}$ $\Rightarrow G \text{is closed in } X \times Y. \text{Osciolatis}$ $\Rightarrow G \text{is closed in } X \times Y. \text{Osciolatis}$ $\Rightarrow G \text{is closed in } X \times Y. \text{Osciolatis}$ $\Rightarrow G \text{is closed in } X \times Y. \text{Osciolatis}$ $\Rightarrow G \text{is closed in } X \times Y. \text{Osciolatis}$ $\Rightarrow G \text{is closed in } X \times Y. \text{Osciolatis}$ $\Rightarrow G \text{is closed in } X \times Y. \text{Osciolatis}$ $\Rightarrow G \text{is closed in } X \times Y. \text{Osciolatis}$ $\Rightarrow G \text{is closed in } X \times Y. \text{Osciolatis}$ $\Rightarrow G osciolat$	$(x,y) \in f(V) \times U$ As $V \cap S \cap S \cap Y = f(X) \times S \cap S$
$\Rightarrow \alpha \in f'(v) \begin{cases} \beta \in U \\ \Rightarrow f(\alpha) \in V \\ \end{cases} \begin{cases} \beta \in U \\ \Rightarrow \beta \in U \\ \end{cases} $ $\Rightarrow \beta = f'(\alpha). \forall U \cap V = \emptyset$ $\Rightarrow (\alpha, \beta) \in G, Notion Finite of the property of the proper$	
$\Rightarrow \beta \neq f(\alpha).$ $\Rightarrow (\alpha, \beta) \in G.$ $\Rightarrow f'(x) \times U \subseteq G.$ $\Rightarrow (\alpha, y) \in f'(x) \times U \subseteq G.$ $\Rightarrow (\alpha, y$	Now we prove $f'(V) \times U \subseteq G'$ Let $(\alpha, \beta) \in f'(V) \times U$
$\Rightarrow \beta \neq f(\alpha).$ $\Rightarrow (\alpha, \beta) \in G.$ $\Rightarrow f'(x) \times U \subseteq G.$ $\Rightarrow (\alpha, y) \in f'(x) \times U \subseteq G.$ $\Rightarrow (\alpha, y$	$\Rightarrow \alpha \in f'(v) \xi \beta \in U$ $\Rightarrow f(\alpha) \in V \xi \beta \in U$
\Rightarrow $f(v) \times U \subseteq G$ \Rightarrow	$\beta + f(\alpha)$
⇒ (x, y) ∈ f (Y) × U ⊆ G MATHEMATICA ACADEMY M.Sc(Maths) M.Sc(Maths	$f'(v) \times U \subseteq G$ M.Sch 0300 Contact: 0300
Je is open in Xx Y. 0300-4285657 0305-52360627 Je G is closed in Xx Y. Question # 2(b) s- Show that every metric space is Regular Housdroff space. Pol s- First we prove every metric space is Regulars.	\Rightarrow $(x, y) \in f(v) \times U \subseteq G$ MATHEMATICA ACADEMY Offers Coloniation
Question # 2(b) s- Show that every metric space is Regular Housdroff space. Pol s- First we prove every metric space is Regular.	=> G is open in X x Y. 0300-4285657 0305-5130587
Show that every metric space is Regular Housdroff space. Sol :- First we prove every metric space is Regular.	= G is closed in XXI.
Show that every metric space. is Regular Housdroff space. Pol :- First we prove every metric space is Regular.	Question # 2(b) 5-
Pol :- First we prove every metric space is Regulars.	
	Show that every metric space.
For this, we first prive that every	Show that every metric space
the country of the state of the	Show that every metric space

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metric	space is completely regula	V8
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let (X,	d) be a metric space. To provi	2 X_
is c	ompletely regular.	
let	A be a closed set in 1	and
XEX	s.t. x ∉ A	
Now	define a function	
9:	s.t. $x \notin A$ define a function $X \to \mathbb{R}$ by $g(y) = d(y, B)$.	-
where	B is another closed set in	X
with.	ANB = + & XEB.	
-then	(i) $g(x) = d(x, B) = 0$	
	$(\ddot{u}) = g(A) = d(A, B) > 0$	
To the second se	Let $q(A) = d(A, B) = k$	
\$ · · ·	(iii) Now for E>0, we can de	1008e
Š.	8=E S.T.	
	whenever d(y, y') < 8 then	
		i
	g(y) - g(y) = d(y, B) - d(y, B)	
P0 -		- 1
	$\leq d(y,y)$	1 13-1
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	MATHEMATICA ACA Offers	DEM
, z	$\left[g(y)-g(y')\right] < \in MATHEM Offers$ $M.Sc(Ma)$ $M.Sc(Ma)$ $M.Sc(Ma)$ $M.Sc(Ma)$	ths
	M.Sc(Ma) 0300-4285657 030	5.51309ka
→	q is continuous	5
		7
Now	define $f: X \rightarrow [0,1]$	or 4
	by f(z) - 1 a(z) = 38hu	ILLUL ISATS)
	by $f(z) = \frac{1}{k}g(z)$ Noman General Solution of is continuous. Contact: 0300-4	285657
· the	n f is continuous. Contact. Usus	10
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with $f(x) = 1$ $g(x) = 1$ $(0) = 0$ k k k k $g(x) = 1$	\$\int_{(A)} = \frac{1}{k}g(A) = \frac{1}{k} \\ \$\int_{(A)} = \frac{1}{k}g(A) = \frac{1}{k} \\ \$\int_{(A)} = \frac{1}{k} \\ \$\int_{(8
\$\int f(A) = 1 g(A) = 1 (k) \times k \times	\$\begin{array}{c} f(A) = \frac{1}{k} g(A) = \frac{1}{k} \\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \	f(x) = 1 g(x) = 1 (0) = 0
\$\int f(A) = 1 g(A) = 1 (k) \\ k \\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \	\$\int f(A) = 1 g(A) = 1 (k) \(\frac{1}{k} \) \(\frac{Noman}{k}	k K
F(A) = 1 Naman Shillomsats M. Scievi M. Phillomsats Contact: 0300.4285657 Z X is completely regular. Now every completely regular space is regular. Let X be a completely regular. Let A be a closed set in X and X E X S.t. X f. A. Then as X is completely regular. So I a continuous function f: X -> [0,1] S.t. f(X) = 0 & f(A) = 1 Let U = [0, ½, [& Y =]½, i] Then U f V are open in [0,1]. B f is continuous, so f(U) & f(V) are open in X. And X E f(U), A C f(V) and f(U) O f(V) = † So X is regular. Now we prove every metric space is Housdraff. Let (X, &) be a metric space. To prove X is Housdraff.	⇒ f(A) = 1 Naman Michigomants Miscrew Miscr	A STATE OF THE PARTY OF THE PAR
Scontact: 0300-4285657 Space is regular. Let X be a completely regular. Let A be a closed set in X and X so F(x) = 0 f(x) f(x) = 4 Space is required. Let U = [0, 1] S.t. So A a continuous function And X E f(U), A E f(V) and A f(U) O f(V) = 4 So X is regular. Now we prove every metric space. Let (X, et) be a metric space. Let (X, et) be a metric space. Let (X, et) be a metric space.	Now every completely regular	k accorded
Scontact: 0300-4285657 Space is regular. Let X be a completely regular. Let A be a closed set in X and in X	Scontact: 0300.4285657 X	Naman January
Now every completely regular. Now every completely regular space is regular. Let X be a completely regular. To prove X is regular. Let A be a closed set in X and XEX S.t. X & A. Then as X is completely regular so I a continuous function f: X -> [0,1] s.t. f(x) = 0 & f(A) = 1 Let U = [0, ½ [& V =]½, i] Let U = [0, ½ [& V =]½, i] Let U = [0, ½ [& V =]½, i] And X & f(U) , A & f(V) are open in X. And X & f(U) , A & f(V) and f(U) O f(V) = 4 So X is regular. Now we prove every metric space is Housdroff. Let (X,d) be a metric space. To prove X is Housdroff.	Now every completely regular. Now every completely regular space is regular. Let X be a completely regular. Let A be a closed set in X and XEX S.t. X FA. Then as X is completely regular so I a continuous function f: X -> [0,1] s.t. f(x) = 0 & f(A) = 1 Let U = [0, ½ [& V =]½, i] Then U f V are open in [0,1]. And X E f(U), A E f(V) and f(U) O f(V) = 4 So X is regular. Now we prove every metric space is Housdroff. Let (X,d) be a metric space. To prove X is Housdroff.	M.Sc(PU) M.Phillips
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To prove X is regular. Let A be a closed set in X and $X \in X$ s.t. $X \notin A$. Then as X is completely regulars so $X = X = X = X = X = X = X = X = X = X $	To prove X is regular. Let A be a closed set in X and $X \in X$ s.t. $X \notin A$. Then as X is completely regulars so $X = X = X = X = X = X = X = X = X = X $	Now every completely regular
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let A be a closed set in X and x \in X \in S.t. \times \in A. Then as X is completely regular so \(\frac{1}{3}\) a continuous function f: \(X \rightarrow [\circ\), \(\frac{1}{3}\) s.t. f(\times) = 0 \(\frac{1}{3}\) f(\(\frac{1}{3}\)) = 1 then \(U = [\circ\), \(\frac{1}{3}\) \(\frac{1}{3}\) \(\frac{1}{3}\) then \(U = [\circ\), \(\frac{1}{3}\) \(\frac{1}\) \(\frac{1}{3}\) \(\frac{1}{3}\) \(\frac{1}{3}\) \(\	let A be a closed set in X and $X \in X$ $S.t.$ $X \notin A$. Then as X is completely regular So \exists a continuous function $f: X \to [o,1]$ $S.t.$ $f(x) = 0$ $f(A) = 1$ Let $U = [o, !, [& V =]'_2, i]$ then U f Y are open in $[o,i]$. And $f(U)$ $f(V)$ $f(V)$ and $f(U)$ $f(V)$ $f(V)$ and $f(U)$ $f(V)$	let 1 be a completely organis-
Then as X is completely regulars so \exists a continuous function $f: X \to [o, 1]$ s.t. $f(x) = 0$ & $f(A) = 1$ but $U = [o, !, [$ & $V =]!_2, 1]$ then U & V are open in $[o, 1]$. B f is continuous, so $f(U)$ & $f'(V)$ are open in X . And $x \in f'(U)$, $A \subseteq f(V)$ and $f'(U)$ Of $f'(V) = f(V)$ So X is regular. Now we prove every metric space is Housdroff. Let (X', d) be a metric space. To prove X is Housdroff.	Then as X is completely sigulars so \exists a continuous function $f: X \to [o, 1]$ s.t. $f(x) = 0$ & $f(A) = 1$ bet $U = [o, 1] [a] [a] [a] [a]$ then $U = [a, 1] [a] [a] [a]$ then $U = [a, 1] [a] [a]$ $f(x) = 0$ & $f(x) = 0$ $f($	To prove X is regular.
Then as X is completely regulars so \exists a continuous function $f: X \to [o, 1]$ s.t. $f(x) = 0$ & $f(A) = 1$ but $U = [o, !, [$ & $V =]!_2, 1]$ then U & V are open in $[o, 1]$. B f is continuous, so $f(U)$ & $f'(V)$ are open in X . And $x \in f'(U)$, $A \subseteq f(V)$ and $f'(U)$ Of $f'(V) = f(V)$ So X is regular. Now we prove every metric space is Housdroff. Let (X', d) be a metric space. To prove X is Housdroff.	Then as X is completely sigulars so \exists a continuous function $f: X \to [o, 1]$ s.t. $f(x) = 0$ & $f(A) = 1$ bet $U = [o, 1] [a] [a] [a] [a]$ then $U = [a, 1] [a] [a] [a]$ then $U = [a, 1] [a] [a]$ $f(x) = 0$ & $f(x) = 0$ $f($	Let A be a closed set in X and
So \exists a continuous function $f: X \rightarrow [o, 1]$ s.t. $f(x) = 0$ & $f(A) = 1$ Let $U = [o, 1]$ & $V = [o, 1]$ hen $U \notin V$ are open in $[o, 1]$. And $X \in f(U)$, $A \subseteq f(V)$ and $f(U) \cap f(V) = f(V)$ And $X \in f(U) \cap f(V) = f(V)$ Now we prove every metric space is Housdroff. Let (X, d) be a metric space. To prove X is Housdroff.	So \exists a continuous function $f: X \to [o, 1]$ s.t. $f(x) = 0$ $\mathcal{F}(A) = 1$ Let $U = [o, !, [$ $\mathcal{E}(X) =]!_{2}, 1]$ Then $U = [o, !, [$ $\mathcal{E}(X) =]!_{2}, 1]$ Then $U = [o, !, [$ $\mathcal{E}(X) =]!_{2}, 1]$ And $U = [o, !, [$ $\mathcal{E}(X) =]!_{2}, 1]$ And $U = [o, !, [$ $\mathcal{E}(X) =]!_{2}, 1]$ And $U = [o, !, [$ $\mathcal{E}(X) =]!_{2}, 1]$ And $U = [o, !, [] = [v,]]$ And $U = [v, [] = [v,]$ And $U = [v, [] = [v,]]$ And	$1 \propto \in X$ s.t. $\propto \notin A$.
So \exists a continuous function $f: X \rightarrow [o, 1]$ s.t. $f(x) = 0$ & $f(A) = 1$ Let $U = [o, 1]$ & $V = [o, 1]$ hen $U \notin V$ are open in $[o, 1]$. And $X \in f(U)$, $A \subseteq f(V)$ and $f(U) \cap f(V) = f(V)$ And $X \in f(U) \cap f(V) = f(V)$ Now we prove every metric space is Housdroff. Let (X, d) be a metric space. To prove X is Housdroff.	So \exists a continuous function $f: X \to [o, 1]$ s.t. $f(x) = 0$ $\mathcal{F}(A) = 1$ Let $U = [o, !, [$ $\mathcal{E}(X) =]!_{2}, 1]$ Then $U = [o, !, [$ $\mathcal{E}(X) =]!_{2}, 1]$ Then $U = [o, !, [$ $\mathcal{E}(X) =]!_{2}, 1]$ And $U = [o, !, [$ $\mathcal{E}(X) =]!_{2}, 1]$ And $U = [o, !, [$ $\mathcal{E}(X) =]!_{2}, 1]$ And $U = [o, !, [$ $\mathcal{E}(X) =]!_{2}, 1]$ And $U = [o, !, [] = [v,]]$ And $U = [v, [] = [v,]$ And $U = [v, [] = [v,]]$ And	Then as X is completely negular
$f: X \rightarrow [o, 1]$ s.t. f(x) = 0 & $f(A) = 1let U = [o, 1] & V = [o, 1].then U \neq V are open in [o, 1].And V \neq V is regular.And V \neq V \neq V is regular.Now we prove every metric space is Housdraff.Let (X, A) be a metric space.$	f: $X \rightarrow [o, 1]$ s.t. f(x) = 0 & f(A) = 1 Let $U = [o, \frac{1}{2}, E]$ & $V = \frac{1}{2}, E$. Then $U = [o, \frac{1}{2}, E]$ & $V = \frac{1}{2}, E$. Then $U = [o, \frac{1}{2}, E]$ & $V = \frac{1}{2}, E$. Then $U = [o, \frac{1}{2}, E]$ & $V = \frac{1}{2}, E$. To prove every metric space. To prove $X = [o, 1]$ & $X = [o, 1]$. To prove $X = [o, 1]$ & $Y = [o, 1]$. To prove $X = [o, 1]$ & $Y = [o, 1]$. To prove $X = [o, 1]$ & $Y = [o, 1]$. To prove $X = [o, \frac{1}{2}, E]$. To prove $X = [o, \frac{1}{2}, E]$.	So I a continuous function
thet $U = [o, \frac{1}{2}, [a] = [a, \frac{1}{2}, \frac{1}{2}]$ then $U \neq V$ are open in $[o, 1]$. As f is continuous, so $f'(U) \neq f'(V)$ are open in X . And $x \in f'(U)$, $A \subseteq f'(V)$ and $f'(U) \cap f'(V) = \phi$ So X is regular. Now we prove every metric space is Housdraff. Let (X, d) be a metric space. To prove X is Housdraff.	tet $U = [o, !, [$ $\{ \{ \{ \{ \{ \{ \} \} \} \} \}]]$ $\{ \{ \{ \{ \{ \{ \{ \} \} \} \} \} \} \} \}$ $\{ \{ \{ \{ \{ \{ \{ \{ \} \} \} \} \} \} \} \} \}$ $\{ \{ \{ \{ \{ \{ \{ \{ \{ \{ \{ \{ \{ \{ \{ \{ \{ \{ \{$	$f: X \to [0,1]$ s.t.
Let $U = [o, !, [$	Let $U = [o, !, [$	f(x) = 0 & $f(A) = 1$
then $U \notin V$ are open in $[0,1]$. B f is continuous, so $f(U) \notin f(V)$ are open in X . And $x \in f(U)$, $A \subseteq f(V)$ and $f(U) \cap f(V) = \phi$ So X is regular. Now we prove every metric space is Housdraff. Let (X,d) be a metric space. To prove X is Housdraff.	then $U \notin V$ are open in $[0,1]$. As f is continuous, so $f(U) \notin f(V)$ are open in X . And $x \in f(U)$, $A \subseteq f(V)$ and $f(U) \cap f(V) = \phi$ So X is regular. Now we prove every metric space is Housdraff. Let (X,d) be a metric space. To prove X is Housdraff.	
then $U \notin V$ are open in $[0,1]$. B f is continuous, so $f(U) \notin f(V)$ are open in X . And $x \in f(U)$, $A \subseteq f(V)$ and $f(U) \cap f(Y) = \emptyset$ So X is regular. Now we prove every metric space is Housdraff. Let (X, \emptyset) be a metric space. To prove X is Housdraff.	then $U \notin V$ are open in $[0,1]$. As f is continuous, so $f(U) \notin f(V)$ are open in X . And $x \in f(U)$, $A \subseteq f(V)$ and $f(U) \cap f(V) = \phi$ So X is regular. Now we prove every metric space is Housdraff. Let (X,d) be a metric space. To prove X is Housdraff.	Let 11-10, 101 & V=]/2, 1]
As f is continuous, so $f'(U)$ $f'(V)$ are open in X . And $x \in f'(U)$, $A \subseteq f'(V)$ and $f'(U) \cap f'(V) = \phi$ So X is regular. Now we prove every metric space is Housdroff. Let (X,d) be a metric space. To prove X is Housdroff.	As f is continuous, so $f'(U)$ $f'(V)$ are open in X . And $x \in f'(U)$, $A \subseteq f(V)$ and $f'(U) \cap f'(V) = \phi$ So X is regular. Now we prove every metric space is Housdraff. Let (X,d) be a metric space. To prove X is Housdraff.	
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and f(U) Of(Y) = \$ So X is regular. Now we prove every metric space is Housdroff. Let (X,d) be a metric space. To prove X is Housdroff.	and f(U) Of(V) = ¢ So X is regular. Now we prove every metric space is Housdroff. let (X,d) be a metric space. To prove X is Housdroff.	The Table Continuous of the X
and f(U) Of(Y) = \$ So X is regular. Now we prove every metric space is Housdroff. Let (X,d) be a metric space. To prove X is Housdroff.	and f(U) Of(V) = ¢ So X is regular. Now we prove every metric space is Housdroff. let (X,d) be a metric space. To prove X is Housdroff.	are open in A. AC f(V)
Now we prove every metric space is Housdroff. Let (X,d) be a metric space. To prove X is Housdroff.	Now we prove every metric space is Housdroff. Let (X,d) be a metric space. To prove X is Housdroff.	
Now we prove every metric space is Housdroff. Let (X,d) be a metric space. To prove X is Housdroff.	Now we prove every metric space is Housdroff. Let (X,d) be a metric space. To prove X is Housdroff.	$\frac{1}{2} \frac{1}{2} \frac{1}$
space is Housdroff. Let (X,d) be a metric space. To prove X is Housdroff.	space is Housdroff. Let (X,d) be a metric space. To prove X is Housdroff.	Loo V is reduing.
space is Housdroff. Let (X,d) be a metric space. To prove X is Housdroff.	space is Housdroff. Let (X,d) be a metric space. To prove X is Housdroff.	
Let (X,d) be a metric space. To prove X is Housdroff.	Let (X,d) be a metric space. To prove X is Housdroff.	1 00
To prove X is Housdroff.	To prove X is Housdroff.	
Let for every x, y \in X s.t. x+y	To prove X is Housdroff. Let for every x, y E X s.t. x+y	let (X,d) be a metric space.
Let for every x, y ∈ X s.t. x+y	let for every x, y ∈ X s.t. x+y	To prove X is Housdroff.
1 1 1		let for every x, y E X S.t. x+y



10
Joen Covers-let (X,T) be a
Open Covers-let (x,T) be a topological space and $A\subseteq X$, then a collection $Y = \{O_\alpha : \alpha \in I\}$ of open sets
collection $\sqrt{=\{0_{\alpha}: \alpha \in I\}}$ of open sets
is said to be open cover for -
A if
$A \subseteq \bigcup_{\alpha \in I} O_{\alpha}$. Noman Khalid
Ocartest: 0300-4285657
Sub-Covers-9f & is an open - cover for A & B is a subfamily of & s.t.
1000-COVC A & B is a subfamily
of χ s.t.
$A \subseteq U\beta$ then β is called
open subcover for A.
Compact Space: A topological
Compact Space: A topological space (X,T) is said to be compact
if every open cover for X has a
finite subconer.
Non
To prove compactness is a topological
property.
we prove homeomorphic image of
compact space is compact.
Let $f: X \rightarrow Y$ be a homeomosphism
from a compact space X
to a topological space Y. To prove Y=f(X) is compact.
Let $\{U_{\alpha}: \alpha \in I\}$ be an open cover
for Y, for all $\alpha \in \mathbb{I}$, U_{α} are open
Subsets of Y.
$=$ $Y = U U_{\alpha}$
$\sim \epsilon I$

1.1
Since f, being a homeomosphie is a
_ continuous function and Ux are open
Subsets of Y, so $V_{\alpha} = f'(U_{\alpha})$ are
open subsets of X.
Now $\forall \alpha = f(U_{\alpha})$
$\Rightarrow \bigcup_{\alpha \in I} \forall \alpha = \bigcup_{\alpha \in I} f'(U_{\alpha})$ $\Rightarrow \bigcup_{\alpha \in I} \forall \alpha = f'(\bigcup_{\alpha \in I} U_{\alpha})$
THE VALL OF YELL OF YE
The state of the s
Noman Khalid
$\Rightarrow \qquad \bigvee_{\alpha} = \times \qquad \qquad \text{M.Sc(PU) M.Phil(COMSATS)}$ $\Rightarrow \qquad \text{Contact: 0300-4285657}$
∠ ∠ ∈ I
This shows that { Va: a E I} Is an
onen cover of X
Since X is compait, so this open cover
must have a finite subcover.
let the finite subcover be
$\frac{5}{\sqrt{\alpha}}, \frac{1}{2} = 1, \frac{2}{3}, \frac{3}{3}, \dots \frac{3}{7}$
i.e. / n
$X \ni UV\alpha_i$
$X = U f(U\alpha_i)$
THEMATICA ACADEMY
$X = f(U)_{\alpha_i} \xrightarrow{\text{MATHEM Offers}}$
0300-A285657 0305-5130982
$Y = \bigcup \bigcup \alpha_i : f(X) = Y$
This shows that { Ux; : i=1,2,3,ns
18 a frite subcorer of Y.
Thus, an open cover of Y has
a finite subcover of Y.
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Hence, Y of compact space.
Thus, the homeomorphic image of a compact space is compact.
Thus, the homeomorphic image of a compact space is compact.
a compact space
The state of the second
Hence, compactness & a
topological property Naman MenicomsATS)
topological property Naman Thalid M.Sc(PU) M.Phil(COMSATS) Contact: 0300-4285657
Hence Proved.
Tience Proved.
Question # 3(b) 3- Define
Quescion in a more that
connected spaced and prove that
connected if and
There does not
emtinuous tunction T Jobin T
two point discrete space.
C
2015-0
2015-Connected Spaces s-
A topological space (X, T) is
Said to be connected if
said to be connected of
there exist no non-empty disjoint
open sets A and B. s.t.
AUB = X MATHEMATICA ACADEMY Offers
M Sc(Maths)
let X is connected, 0300-4285657 0305-5136762
thank door not exect
to prove allest out of fine town
a continuous Surjective function from X to discrete two
I from X to discrete con
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13	71.4
point space Y={a,b}	
Suppose on the contrary there exists a function f:	that_
there exists a function f:	X-jy
which is continuous and sur	jective
Now as f is surjective	
So $f(X) = Y$	- 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
As Y={a,b} is discrete	<u> </u>
le sof shy are open in	Y
As f is continuous so t	(303)
and f'(863) we open in X	·
1 01	
f ({a3}) Uf ({b3}).	
= f ({a} v {b})	
	1
= f (Y) Noman giful Noman philoms M. Sc(Pi) M. Philoms N. Sc(Pi) M. Philoms Option: 0300-4285	(TS)
= X M.Sc(PU) M.Phil(COMS) Contact: 0300-4285	1001
$ \begin{cases} f(\{a\}) \cap f(\{b\}) \\ f(\{a\} \cap \{b\}) \end{cases} $ MATHEMATICA A Offers	CADEMY
$f\left(\frac{5a^{2} \cap 5b^{2}}{4}\right)$ MATHEMATICAL Offers M.Sc(N) 2.5657 03	aths)
$ \frac{1}{2} \int_{0.300-4285657} \frac{M.Sc(IV)}{0.300-4285657} $	05-5130932
1 > X is disconnected	
which is a contradiction	
· X is connected	
So, our supposition is rorong.	
Hence, there does not enist	Lan
continuous surjective function	7.8011)
X to a discrete two poi	nc
space Y.	21 22+
Conversely assume there does	100
exist a continuous surgective	lisexoto
function f from X to a	usc o Cic
MALKANDER MILLS A TO SEE SELECT A FROM 1895	

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two point space Y = {a,b}.
To prove X is connected.
Suppose on the contrary X is
disconnected.
Then, there exist two non-empty
men, there A & B in X s.t.
open sets A & B in X s.t.
$\frac{1}{1} \frac{1}{1} \frac{1}$
Now define f: X -> Y by
C. S 2 0 (18) Sh2
$f(A) = \{a\} \{f(B) = \{b\}\}$
Then $f(X) = f(A \cup B)$
$= f(A) \cup f(B)$
= {a} U {b}
f(x) = y
=> f II susjective
Also as open sets in Y are
\$, \{a\frac{2}{3}, \{b\frac{2}{3}\}, \forall \\
C(1) -/ & mon with
with $f(4) = 4$ Noman Ishalid f(3a3) = A Noman Ishaliconsats) f(3a3) = B M.SciPU) M.Phillconsats f(3a3) = B M.SciPU) M.Phillconsats
$f(\lbrace 2 \alpha \rbrace) = A $
f(y) - x
I each open
Inverse image of each open
set is open.
of is continuous.
which is a contradiction.
: there does not exist a
continuous surjective function
So our supposition is rorong.
Hence X is connected.
MATHEMATICA ACADEMY Offers
an crimaths)
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and the second s

15
Question # 4(a):- let x be
a countably compact space. Show that every infinite subset of X
that every infinite subset of X
has a limit point in X.
<u> </u>
Pols-let A be an infinite subset
Tof X.
To prove A has a limit point. Suppose on the contrary A has no
limit point
Then every subset of A also has
l and limit point
Let $B = \{x_1, x_2, x_3, \dots \}$ be a countably
Let $B = \{x_1, x_2, x_3, \dots \}$ be a countably infinite subset of A. Then B has
no limit point.
Now consider Consider new new
- n n+ - n+
Then, $\forall n \in \mathbb{N}$, $D(C_n) = \phi_{NOMAN J(halid)}$ $\Rightarrow D(C_n) \subseteq C_n$ $\longrightarrow D(C_n) \subseteq C_n$
$\frac{ hen }{hen}, \forall n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\Rightarrow D(C_n) \subseteq C_n$ $\frac{ hen }{hen}, \forall n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \forall n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \forall n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \forall n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \forall n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \forall n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \forall n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \forall n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \forall n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \forall n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \forall n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \forall n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \forall n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \forall n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \forall n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \forall n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \forall n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \forall n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \forall n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \nabla n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \nabla n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \nabla n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \nabla n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \nabla n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \nabla n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \nabla n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$ $\frac{ hen }{hen}, \nabla n \in \mathbb{N}, D(C_n) = \mathcal{P}_{Normalisons A157}$
$\Rightarrow \forall n \in \mathbb{N}$, C_n is closed. $\Rightarrow \{C_n : n \in \mathbb{N}\}$ is a class of closed
=> {Cn:nEN} is a class of closed
sets which satisfy finite intersection
property.
Because for every finite subcollection
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$\bigcap_{i=1}^{n} C_{n_i} = C_{n'} \neq \emptyset$ M.Sc(Maths) 6300-4285657 0305.5136982
$i=1$ $n_i = n_i + p_i = 0.300-4285657 0305-5130982$
where $n' = \max(n_1, n_2, \dots, n_r)$

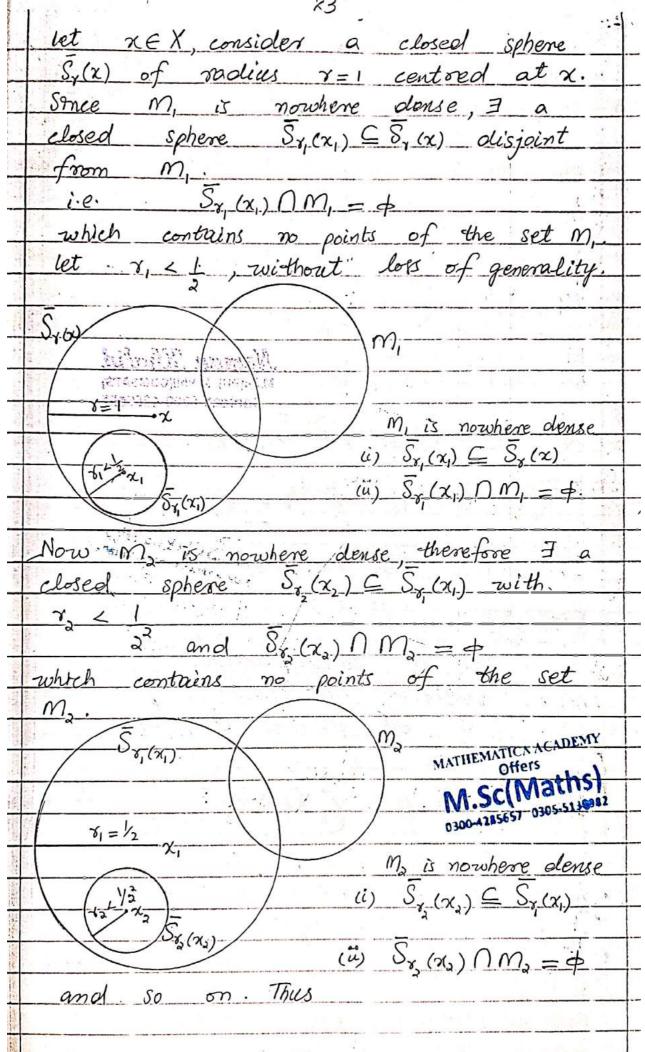
10
(ii) let A be the connected
subsmee of X.
Subspace of X. To prove A is contained in only one component of X.
mly me component of X.
only one component
Let $X = \{C_{\alpha} : \alpha \in I\}$ be a collection of all connected subspaces of X
f all emperted subspaces of X
thiming A
then Silly ()
nen 0 C 1 1 8 118 = U Cx = C
$\frac{\bigcap C_{\alpha} + \phi}{\alpha \in I} = \frac{\bigcup C_{\alpha} = C}{\alpha \in I}$
which is connected subspace of X.
Olco ACC
Also ACC > C is connected subspace of X
Also C = UV = U Ca Namen Thalid Also C = UV = U Ca Nomen Thalid M.sc(PU) M.Phil(60MSATS) Contact: 0300-4285657
M.Sc(PU) W.Fimio-4285657
So C is such maximal connected.
a basses of X.
is component of X containing
is component of A confounting
Now we show C is the only
component of X containing A.
- component of *
F this lot (by another component
For this, let C* be another component
of X containing A.
Now as C* is maximal connected
Now as C* is maximal connected subspace of X containing A and
Now as C* is maximal connected subspace of X containing A and C is connected subspace of X
of X containing A. Now as C* is maximal connected subspace of X containing A and C is connected subspace of X containing A So C E C*.
Now as C* is maximal connected subspace of X containing A and C is connected subspace of X containing A so C = C*. Further also as C* is connected
of X containing A. Now as C* is maximal connected subspace of X containing A and C is connected subspace of X containing A so C \(\) C \(\) \\(\) \(
Now as C* is maximal connected subspace of X containing A and C is connected subspace of X containing A so C = C*. Further also as C* is connected

-	Sols-let (R,d) -be -the real
	lone with usual metric on R
	and 7/ = \\ \frac{1}{2} \cdot
	is a set of integers as a subset
	is a set of integers as a subset of the real line IR. Since
1	
6	$Z = \mathbb{R} \setminus Z = U(n, n+1)$
†	$\pi \epsilon Z$
1	is open being union of open intervals
†	
t	So Z is closed in R. Noman Schalid
1	and hence $Z = Z$ Contact: 0300-4285657
T	Also Z contains no open ball
r	i.e. $B(x, x) \nsubseteq \overline{Z}$, $\forall x \in \mathbb{R}$
-	Hence Z is nowhere dense in R
H	Flerice 2 10 moners
	(ii) Show that Q is of the First
_((U) Show that Q is of the First
•	category.
-9	Pol 3- As Q is coventable.
1	& Q = U {9} countable union of
	9 E Q
	singleton {9} subsets of Q.
	Now we show that each singleton
	subset {9} of Q is nowhere dense.
	OK THE PROPERTY OF THE PARTY OF
7	$\{q\} = R \setminus \{q\} = (-\infty, q) \cup (q, \infty)$
_	which is open: being inion of open:
- ;	ntervals. Therefore each singleton [93]
	s closed. So Section So Section Sectin Section Section Section Section Section Section Section Section

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Also {9,} contains no open ball. Therefore each singleton is nowhere
Therefore each singleton is nowhere
1 / -1 - 1 / -1
Hence Q = U {9} is the countable.
9 C Q
union of nowhere dense subsets.
Hence Q is of first category.
Question # 5(b) 3- Show that I
is complete space.
C is complete of ace.
Pol: The space l'consists of all
sequences $X = \{x_i\}$ of real or complex
numbers such that Noman Khalid
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Contact: 0300-4285657
then $ x = (\sum x_i ^i)^i$
(i = 1 o p
Thus X={xi} is in l > / [xil converges.
To prove l'is complete.
let { x 4 be a cauchy sequence
in l, then for every Exo, = a
the integer no such that
11 cm) (n) 11 c C
$\frac{11 \times -1 \times 11 \times E}{2}, \frac{\sqrt{m}}{m}, \frac{\sqrt{m}}{m} = \frac{11 \times 11}{m}$
$\int_{-\infty}^{\infty} \frac{(m)}{(n)} \frac{(n)}{[p]} \frac{p}{p}$
$\Rightarrow \frac{ \chi_i - \chi_i }{ z } = \frac{ \chi_i - \chi_i }{ z } = \frac{ \chi_i - \chi_i }{ z }$
12 ^(m) (n) 1 (G) 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
\Rightarrow $ \alpha - \alpha \alpha \alpha \alpha \alpha \alpha \alpha \alpha \alpha$
168:

$\frac{\Rightarrow \{\chi_i^{(n)}\}}{\text{Since } \mathbb{R}} \text{ is a cauchy sequence in } \mathbb{R}.$ $\frac{\text{Since } \mathbb{R}}{\text{xi}} \Rightarrow \text{xi} \in \mathbb{R}, \text{ so when } n \to \infty$
Since R is complete so
$\chi^{(n)} \to \chi_i \in \mathbb{R}$, so when $n \to \infty$
then from 1
0-1/0-1/0-1
$\int_{i-1}^{\infty} \chi_i^{(m)} - \chi_i ^{p-1/p} \angle \in \forall m \ge n_0$
i=i :
$\Rightarrow \ \chi - \chi\ \angle E, \forall m \ge n_0$
$\chi = (\chi_1, \chi_2, \dots, \chi_n)$
$\Rightarrow \chi \rightarrow \chi$
Now $x = x - (x^m) - x \in \ell$
) D/ / /
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> Complete. MATHE Offers aths)
3 Company Company
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MATHEMOHERS MATHEMOHERS M.SC(Maths) 0300-4285657 0305-5135982
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Question # 6 (a) s- State 4
Question # 6(a) s- State
Question # 6(a) s- poore Baire's Category théorem.
Question # 6(a) s- Prove Baire's Category théorem.
Question # 6 (a) s- State 4 poore Baire's Category theorem. Sol: Statement: A complete
Question # 6(a) s- poore Baire's Category théorem.
Question # 6 (a) s- prove Baire's Category theorem. Sol: Statement: A complete metric space is of second category 1.+ V be a complete metric space.
Question # 6 (a) s- prove Baire's Category theorem. Sol: Statement: A complete metric space is of second category 1. t. V. ba a complete metric space.
Question # 6 (a) s- prove Baire's Category theorem. Sol: Statement: A complete metric space is of second category Let V be a complete metric space.
Question # 6 (a) :- State f prove Baire's Category theorem. Pol: Statement: A complete metric space is of second category let X be a complete metric space. To prove X is of second category. Suppose on contrary X is of
Question # 6(a) = State 4 poore Baire's Category theorem. Pol: Statement: A complete metric space is of second category let X be a complete metric space. To prove X is of second category. Suppose on contrary X is of first category.
Question # 6(a): State f prove Baire's Category theorem. Sol: Statement: A complete motive space is of second category let X be a complete metric space. To prove X is of second category. Suppose on contrary X is of first category. i.e.
Question # 6 (a) :- prove Baire's Category theorem. Sol: Statement: - A complete metric space is of second category let X be a complete metric space. To prove X is of second category. Suppose on contrary X is of first category.



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$S_{r_1}(x_1)$, $S_{s_2}(x_2)$,, $S_{s_n}(x_n)$ is a nested sequence of closed spheres such that
sequence of closed spheres such that
(i) $S_{\chi_{k+1}}(\chi_{k+1}) \subseteq S_{\chi_k}(\chi_k)$, $k = 1, 2, 3, \dots$
(ii) $\overline{S}_{\kappa}(x_{\kappa}) \cap M_{\kappa} = +, k = 1, 2, 3, \dots$
By Canton's intersection theorem, since dia $(S_{r_k}(x_k)) \rightarrow 0$ as $k \rightarrow \infty$
$formula = \frac{\sin(S_{r_k}(x_k))}{\cos(S_{r_k}(x_k))} \rightarrow 0$ as $k \rightarrow \infty$
$\int_{\gamma_{K}}^{\infty} (\chi_{K}) = \{\chi\} \neq \int_{M.Sc(PU)-M.Phil(GOMSATS)}^{\infty}$
$\frac{110_{7k}(\lambda k) = 721 + 9}{\text{K=1}} = \frac{\text{M.Sc(PU)-M.Phil(COMSATS)}}{\text{Contact: 0300-4285657}}$
where $x \in X \Rightarrow x \in S_{r_k}(x_k)$
The second secon
Strice $S_{\gamma_k}(x_k) \cap M_k = +, k=1,2,3,\cdots$
\Rightarrow $x \notin M_k$ for $k=1,2,3,\cdots$
$\Rightarrow x \notin X$
which is a contradiction.
So our supposition is wrong. Hence X is of second category.
Hence X is of second category.
Question # 6(b) = Show that
QUESTION # Olds Show that
a normed space N is a
Banach space if every absolutely
convergent series converges.
DO 15- Lat NI L
Pols-let N be a normed linear
space in ruhich every absolutely
convergent series converges.
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25 -
Here we have to show that N is
is Banach space. For this we prove
that N is complete.
in N. Then for each natural number
in N. Then for each natural number
k, 3 an integer mk s.t.
$\forall m, n : m, n \geq n_k$
$\Rightarrow \ x_m - x_n\ < 2^{-n}$
without any loss of generality, we can assume that for k=1,2,3,
$\frac{n_1 < n_2 < n_3 < \dots}{\sum_{i=1}^{n} 2^{i}}$
Now we form as subsequence {xn,}
of {xn} as follows.
let $S_k = \chi_{n_k}$ $y = s$, and $y_k = s_k - s_{k-1}$ for $k \ge 2$.
7 K K -1
$\mathcal{Y}_{3} = S_{3} - S_{1} = S_{3} - \mathcal{Y}_{1} \implies S_{3} = \mathcal{Y}_{1} + \mathcal{Y}_{2}$
$y_3 = S_3 - S_3 = S_3 - (y_1 + y_2) \Rightarrow S_3 = y_1 + y_2 + y_3$
So y + y + · · · · + y MATHEMATICA ACADEMY
Sk = 4 + 4 + · · · + 4 MATHEMOHERS
Thus $ y = S_k - S_{k-1} $ 0300-1285657 0305-5138582
$= \ \chi_{n_k} - \chi_{n_{k-1}} \ < 2^{-(k-1)}$
A STATE OF THE PARTY OF THE PAR
By the choice of number Xnx.
Hence (k-1)
Hence $\sum_{k=1}^{\infty} \ y_k\ < \sum_{k=1}^{\infty} \frac{1}{2} < \infty$
But and accumpation & 4 is convergent.
By our assumption $\sum_{k=1}^{\infty} y_k$ is convergent.
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Hence {s	x} is conve	ingent i	<u>n_</u> N
1 4 5			<u> </u>
let x	= Lim Sx		
	k→∞	Noma	r Khalid
then n	$= \lim_{k \to \infty} \chi_{n_k}$	M.Sc(PU) M	Denii(COMSATS)
	K → 00 K	Gomadia	-0300-4203031
So that	{xnk} conv	erges to	χ.
But then	$\frac{1}{2}x_n^2$ co	nvenges 1	to x.
Hence ex	ery cauchy	sequene	ce in
N. conver	ges to a	point o	f N.
Therefor	N is con	plete.	
Hence	N 15 Bana	oh soace	2.
in the second	· · · · · · · · · · · · · · · · · · ·		
0			-
Question	n # 7(a) 3- Let	T:N->M
by a su	rijective ld	end one	a tow
Then pron	no that	ar gon	
	sts if ano	0	$\mathcal{Q}(T)$
implies		-oney of	/!x_=0
in F.C T	x = 0 is bijective		/s N in
then show	that m		$\lim_{N \to \infty} N = n$
The state of the s		also mas	
dimension	<i>n</i> :		
2-1.1		?	
4000	e (onto) le exists, th	$: N \to M$	be
8urjectiv	re (onto) l	mear op	entor.
Let T	exists, th	en T'is	linear
Take Tx	=0		
But To	20		
we ha	ve $T_{21} = T_{0}$		- 4.
		lying T	on 6.5.
) of off	J. J. J	
(- v	$T^{\dagger}T_{x} = T^{\dagger}T$	$o \Rightarrow I_{\alpha}$	= I.
	> X 20	move and radius of familiary and an artist	- Annie Caban Annie -
I To be the second			

×7 -
Conversely suppose that Tx =0 => x =0
To show that T' exists, it is
sufficient to prove that T is bijective
ii) T is given to be surjective (onto).
in) T is injective (one-one)
Let $T_{x_1} = T_{x_2}$
$\Rightarrow T_{x_1} - T_{x_2} = 0$
$\Rightarrow T(x,-x_2) = 0$: T is linear
$\Rightarrow x_1 - x_2 = 0$
$\Rightarrow \chi_1 = \chi_2$
> T is injective. That
Hence T is bijective. Noman M. Phill COMSATS) W. Scipul M. Phill COMSATS) W. Scipul M. Scipul
Hence T is bijective. Noman M. Phillicoms At 15 Therefore T exists. Contact: 0300-4285657
(ii) s- Suppose that T: N>m be a
high the linear operator hen olm (N) =n
Let B= se, ez,, en be a basts of N. we show that B= STe, Te,, Ten 3
we show that B= {Te, Te,, Teng
forms a basis of M.
100 B* is linearly independent.
To move STO TO. To. Ten ? 15
linearly independent, there enists
and a contract of a
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The oxput q_{i-2} $\forall i=1,2,3,\ldots n$
we prove $q_{i} = 0$, $\forall i = 1, 2, 3, \dots n$
Sonce T is lonear, so
and the second of the second o
(010-700) (00) (00) (00) (00) (00)
CE Price Co
The state of the s

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2	
$\frac{1}{2} \sum_{i=1}^{2} q_i e_i = 0$	A
Since B & a basts of N	
so the vectors e, e, en must	3 -
be linearly independent.	C
i.e. $q_{i=0}$, $\forall i=1,2,3,,n$	
., ., ., ., ., ., ., ., ., ., ., ., ., .	1/4
(b) B* spans m.	•
or B* is spanning set of m.	
Since B & basis of N, so for	
each XEN, there is a unique linear	
_ combination of the form	
$x = \sum_{i=1}^{n} a_i e_i$	_
$\lambda = 24.ei$	
Also for each XEN, I yEM	-
HISO for each xEN, I yEM	
= T (Saiei) MATHEMATICA ACADEMY	
ta (nacths)	
=> y = \(\frac{1}{2} \) \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \	Di Caranta
1=1	
Thus y can also be written as a	
Thus y can also be written as a unique elmear combination of vectors in B*. Hence B* is basis of M.	
in B*. Hence B* is bass of M.	
$: \dim(N) = n = \dim(m).$	
J'ence Proved.	
Noman Khalid	
M.Sc(PU) M.Phil(COMSATS)	
Contact: 0300-4285657	

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	The state of the s
every Anite obs	(b) - Proper that
event a in ale	- Poble that
general finite our	nensional. normed
Salt-1+	
pols- let m be dimensional su normed space (N	a finite
domens tonal su	bspace of a
normed space (N	(, 11.11).
<u>let {x, x,, x, 3</u>	be a basts of
M.	
Let $\{x^{(m)}\}\$ be a	cauchy sequence h x is of
in m. Then each	h 2 (m)
the form	
$\chi^{(m)} = q^{(m)} \chi + q^{(m)}$	(m)
	$\frac{(m)}{a_{i}} \times \frac{f \cdot \cdot \cdot + q_{n} \times n}{(m)} \times \frac{(m)}{a_{i}} \times \frac{f \cdot \cdot \cdot + q_{n} \times n}{(m)} \times \frac{(m)}{a_{i}} \times \frac{f \cdot \cdot \cdot + q_{n} \times n}{(m)} \times \frac{(m)}{a_{i}} \times \frac{f \cdot \cdot \cdot + q_{n} \times n}{(m)} \times \frac{(m)}{a_{i}} \times \frac{f \cdot \cdot \cdot + q_{n} \times n}{(m)} \times \frac{(m)}{a_{i}} \times \frac{f \cdot \cdot \cdot + q_{n} \times n}{(m)} \times \frac{(m)}{a_{i}} \times \frac{f \cdot \cdot \cdot + q_{n} \times n}{(m)} \times \frac{(m)}{a_{i}} \times \frac{f \cdot \cdot \cdot + q_{n} \times n}{(m)} \times \frac{(m)}{a_{i}} \times \frac{f \cdot \cdot \cdot + q_{n} \times n}{(m)} \times \frac{(m)}{a_{i}} \times \frac{f \cdot \cdot \cdot + q_{n} \times n}{(m)} \times \frac{(m)}{a_{i}} \times \frac{f \cdot \cdot \cdot + q_{n} \times n}{(m)} \times f \cdot \cdot $
	the state of the s
χ χ $= \sum_{m} Q_{m} \chi_{m}$	Noman Khalid
i=1	M.Sc(PU) M.Phil(COMSATS) Contact: 0300-4285657
	ny E>0, 7 a
$m 0 \ge m \Rightarrow \ \chi^{(m)} - 1 \ $	S.t. ∀ m, ρ:
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	
By Imeanly indepen	don't lamma Ham
is a real number	c>o s.t.
	100
$E > \ x^{(m)} - x^{(p)} \ - \ $	$\frac{\sum_{i=1}^{n} a_{i}^{(m)} \chi_{i}^{2} - \sum_{i=1}^{n} a_{i}^{(r)} \chi_{i}^{2}}{i=1}$
	$= 1 \qquad \qquad i = 1$
11 2	$(a_i - a_i) \chi_i$
= \(\frac{1}{\lambda} \)	(4; -4; /x; 1
	n- (m) (p)
> C 2	$\geq a_i^{(m)} - a_i^{(p)} $
	MATHEMATICA ACADEMY
So that	Outers
	M.Sc(Maths)

	.30
Ĩ	$m, \rho \geq n_{\rho} \Rightarrow \sum_{i=1}^{n} a_{i}^{(m)} - a_{i}^{(\rho)} \leq \epsilon$
+	$m, \rho \geq n_0 \Rightarrow (-1)^{i}$
+	
1	that 15 (m) (p) (m) (p) (p)
	$\frac{2nat}{m, \rho \geq n_o} \Rightarrow \frac{ a_i^{(m)} - a_i^{(p)} }{ a_i^{(m)} - a_i^{(p)} } \leq \frac{n}{2} \frac{ a_i^{(m)} - a_i^{(p)} }{ a_i^{(p)} - a_i^{(p)} } \leq \frac{n}{2}$
*	
1	Hence {ai } is a cauchy sequence.
	in E. Sonce Fis R or C.
1	So F 95 complete, aim > ai, say as m > 00
1	for each i=1,2,3,,n
-	
1	Take n
	$\chi = \sum_{i=1}^{n} a_i \chi_i i \cdot e \cdot \chi \in \mathcal{M}$
-	$\frac{(m)}{\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ $
1111	then $\ \chi - \chi\ = \ \geq (a_i - a_i) \chi_i \ $
	1=1
1.0	$\leq \sum_{i=1}^{n} \frac{(m)}{ a_i - a_i \cdot x_i }$
	i=1
-	$\Rightarrow \ \chi^{(m)} - \chi_i\ \leq \chi \leq a_i - a_i $
₹5.	The state of the s
-	where $k = \sup \ x_i\ $ is fixed.
_	where k = sup xill is fixed.
	(m) 11
+	$\Rightarrow \chi - \chi \rightarrow 0$ as $m \rightarrow \infty$
	Marray Thalid
	i.e. lim x = x Noman Khalid M.Sc(PU) M.PHII(GOMSATS)
	m→∞ Contact: 0300-4285657
- (4)	Hence M is complete.
_	Question # 8(a) s- Show that a unit ball in a banach space
	unit ball in a banach space
7	is compact.
_	Sols-
	The state of the s

(ii)

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Westion # 8(b) 5- (i) State	1
Luestion # 8(b):- (i) State & prove Parallelogram Identity. (ii) State & prove Polarization	
(ii) State & prove Polarization	-
	-
C. Pasoillelogram law or Identity:-	+
gol: (i) let V be a complete inner	+
Soli- (i) let V be a complete inner product space. Then for any	-
$x, y \in V$ $\ x+y\ ^2 + \ x-y\ ^2 = 2[\ x\ ^2 + \ y\ ^2]$	+
$ x+y + x-y = \alpha [x + y]$	+
Prof: - L.H.S. = x+y + x-y	+-
10007:- 2.71.3.2 1127/11 11/2 7/11	1
= <x+y, x+y=""> + <x-y, x-y=""></x-y,></x+y,>	+
= = = = = = = = = = = = = = = = = = = =	1
25 2 2 (x, x> + < x, y> + < y, x> + < y, y>	1
838 + < x, x> - < x, y> - < y, x> + < y, y>	1
20	
= x + y + x + y	
<u>\$</u> 8 8	1
= 2 [x + y] = R.H.S.	1
Hence proved.	1
Polarization Identity:- (ii) let V be a complex inner product	1
(u) let V be a complex inner product	1
space. Then for any x, y EV	1
	1

33 _ 3
(a) Rexx, y> = 1811x+y11-11x-y113
(b) $9m < x, y > = \frac{1}{4} \left\{ x + iy ^2 - x - iy ^2 \right\}$
Porof:- (a) R.H.S. = 1 { 11 x + y 11 - 11 x - y 11 }
T
= 1 \{ < \x + y ., \x + y > - < \x - y . \}
4
$= \frac{1}{2} \left\{ \langle x, x \rangle + \langle x, y \rangle + \langle y, x \rangle + \langle y, y \rangle \right\}$
4
-4x,x>+4x,y>+4y,x>-4y,y>
3 - 7 - 7 - 7 - 7 - 7 - 7 - 7 - 7 - 7 -
= 1 22(x, y> +2/y, x> 5 3 5 5
$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} = 1$
100 S 300 S
2 Re < x, y>
Sec 3
= Re < x, y > = L. H.S.
Hence hoved.
R.H.S-13 x+iy - x-iy 3
4
$= \frac{1}{2} \left\{ \langle x + iy, x + iy \rangle - \langle x - iy, x - iy \rangle \right\}$
4
$= \frac{1}{2} \{ 2x, x + \langle x, iy \rangle + \langle iy, x \rangle + \langle iy, iy \rangle$
4 1/2
- < x, x> + < x, iy> + < iy, x> - < iy, iy> !
$= 1 \left\{ 2 < x, iy > + 2 < iy, x > \right\}$
$=\frac{1}{4} \cdot \frac{1}{4} \cdot 1$
Marie Le vonce

		35 .:				- 1
Gree	A C AUB OUB) C A	& B	C AU	1B	1 1	
omce (A	IUB) C AL	8 (A	UB)1	= B+	by lu)
						-
허	(AUB) 1 ⊆	ATOB		0.		10
					<u> </u>	7.91
Now	$let \propto$	E AT N	B			7
	$ \begin{array}{c} \text{let} \chi \\ \Rightarrow \chi \in A \end{array} $	x	EB.	special section section	* * * / . /	133
					1.3.4	
=>	< x, 4> & < x, y;	= 0	-> <u>V</u>	1168	7	.10
5 787 FE	4 < x, y	>=0	_,V	y co	-	1
Transcore and the second	< x, y >	0	¥ 4	LEAL	1B	1 4 5 1 6 1
7					1. 21	- 1
	$x \in ($	'AUB) ¹			SATS	1
					COM 0	1.6.6
=	, AINB	I C (A	UB)	(2)	3000	+ 1
	combining	0/4	(2)		200	
	(0110	77 01	NB ^L		N.S.C.	,
	(AUB	7		Proveo		- 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1
			ence		Mr.	1111
(iv) To	00000	$A^{\perp} = A$	111	THEMATIC	ACADEM	
(CTA)	A Secretary	1	MA	THE Offe	Naths	7 (1)
15080	By (i)	AGA	1	M.SCH	0305-5130	-
			N H	,		4
	=> F) ***	A^{\perp} -	- Dy	1 (ü)	-
		0			1	376
Also	by (i) 2	replacing	<u>_A</u>	by H	- 10 Style	1 -5
	717	$(A^{\perp})^{\perp \perp}$	$= A^{\perp}$	<u> </u>		1-1
	<u> </u>	<u>(n)</u>	= 7		i asil	<u> </u>
	→ A 9	- 4111		(A')	100	.6
	=7 ri =	= /1		<u> </u>	3	1
				1 - 11 - 17	ith ComScor	香草

```
duestion # 9(6) s- let A be
     complete subspace of un innur
 product space V. Then
           let A be a complete of an inner product space
   subspace
   V. Then there is a unique YEA
  such that
           \|x-y\| = \inf_{y' \in A} \|x-y'\|
Put z = x - y then z. LA so that zeA !
we know that A is closed
 subspace of V. Hence
            x = y + z - 0 where y \in A  \begin{cases} z \in A^{\perp} \end{cases}
 Also ANA' = {o} => ANA' = {o}.
To prove the uniqueness of the
expression given by O
suppose that
 x = y_1 + z_1
 so that y + Z = y, + Z,
       > y-y, = z, -z ∈ A ∩ A = {o}
     ラ 4=4 キャース
                              MATHEMATICA ACADEN
  Hence V = A \oplus A^{\perp}
```