Exercise on Page 211.

Q(1) Discuss the nature of singularities of the following fns and also calculate the residues at poles

(i) $f(z) = \operatorname{Tan} z = \dfrac{\sin z}{\cos z}$

For poles $\cos z = 0$ $\quad z = \cos^{-1}(0)$

$z = (2n+1)\pi/2 \;,\; n = 0, \pm 1, \pm 2 \cdots$

Each pt is a pole of order 1

<u>Residues at poles</u>

$$R(f, \pi/2) = \underset{z\to\pi/2}{Lt}\left[(z-\pi/2)\, f(z)\right]$$

$$= \underset{z\to\pi/2}{Lt}\left\{(z-\pi/2)\frac{\sin z}{\cos z}\right\} \qquad \frac{0}{0} \text{ form}$$

using Hosp. rule

$$= \underset{z\to\pi/2}{Lt}\left(\frac{(z-\pi/2)\cos z + \sin z}{-\sin z}\right) = -1 = b_1 .$$

(ii) $f(z) = \dfrac{z}{z^4+1}$

For poles $z^4+1=0$ $\quad z^4 = -1 \Rightarrow z = (-1)^{1/4}$

4, 4th roots of -1

let $z = -1+0i$

$|z| = 1$ $\quad r\cos\theta = -1$

$r\sin\theta = 0$

$\theta = \pm\pi, \pm 3\pi$

$z = \operatorname{cis}\theta = e^{\pm i\pi}$

$z^{1/4} = (e^{\pm i\pi})^{1/4} = e^{\pm i\pi/4},\; e^{\pm 3i\pi/4}$

are simple poles.

<u>Residues at poles</u>

$$R\left(f, e^{\pi i/4}\right) = \underset{z\to e^{i\pi/4}}{Lt}\left[\frac{(z-e^{i\pi/4})\,z}{z^4+1}\right] = \underset{z\to e^{i\pi/4}}{Lt}\left(\frac{z^2 - z e^{i\pi/4}}{z^4+1}\right) \quad \frac{0}{0}$$

using Hosp Rule

$$= \underset{z\to e^{i\pi/4}}{Lt}\left(\frac{2z - e^{i\pi/4}}{4z^3}\right) = \frac{e^{i\pi/4}}{4e^{3i\pi/4}} = \frac{1}{4}e^{-2i\pi/4}$$

$$= \frac{1}{4}\left(\frac{1}{e^{i\pi/2}}\right) = \frac{1}{4}\left(\cos\pi/2 - i\sin\pi/2\right) = \frac{-i}{4}$$

Similarly we can find other residues

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(iii) $f(z) = \frac{\sin z}{(z-\pi)^2}$

For poles $z-\pi = 0$ $z=\pi$ is a pole of ord 2

Residue

$$R(f,\pi) = \lim_{z\to\pi} \frac{d}{dz}\left[\cancel{(z-\pi)^2}\,\frac{\sin z}{\cancel{(z-\pi)^2}}\right]$$

$$= \lim_{z\to\pi}(\cos z) = -1$$

Q(2)

$$f(z) = \frac{z-2}{z^2}\sin\left(\frac{1}{z-1}\right)$$

For zeros $f(z)=0$

$\Rightarrow z=2$ & $\sin\left(\frac{1}{z-1}\right) = 0$

$$\frac{1}{z-1} = \sin^{-1}(0) = n\pi$$

$$z-1 = \frac{1}{n\pi}$$

$$z = \frac{1}{n\pi} + 1$$

Zeros of $f(z)$ are $z = 2,\ 1+\frac{1}{n\pi}$

for Poles $z=0$ is a pole of order 2

for $z = \frac{1}{n\pi}+1$ when $n\to\infty$

$z=1$ is an essential singularity limit pt. is 1

Residues at $z=0$

$$R(f,0) = \lim_{z\to0}\frac{d}{dz}\left(\cancel{(z-0)^2}\left(\frac{z-2}{\cancel{z^2}}\right)\sin\left(\frac{1}{z-1}\right)\right)$$

$$= \lim_{z\to0}\left[(z-2)\cos\left(\frac{1}{z-1}\right)(-1)(z-1)^{-2} + \sin\left(\frac{1}{z-1}\right)\right]$$

$$= 2\cos(-1) - \sin(1) = 2\cos 1 - \sin 1$$

$\cos(-\theta) = \cos\theta$

Q(3) $f(z) = e^{1/z}$

For zeros $f(z) = 0$, $e^{1/z} = 0$

$$= 1 + 1/z + \frac{1}{2!}\left(\frac{1}{z}\right)^2 + \text{——}$$

$z=0$ is an essential singularity

$$e^{1/z} = -1 = e^{\pi i} = \cos\pi + i\sin\pi$$

$\Rightarrow \frac{1}{z} = \pi i$

$\Rightarrow \frac{1}{z_k} = \pi i + 2\pi i k$, $k = 0, \pm 1, \pm 2, \pm 3$ —— There are ∞ no of zeros.

$z_k = \frac{1}{\pi i(2k+1)}$

Q(4) $f(z) = \frac{\cos z}{\sin z}$

Poles $\sin z = 0$, $z = n\pi$, $n = 0, \pm 1, \pm 2$ ——

all the poles are simple | $\because \frac{d}{dz}(\sin z) = \cos z \neq 0$ for $z = n\pi$

Residues

$$R(f, n\pi) = \lim_{z \to n\pi}\left(\frac{(z-n\pi)\cos z}{\sin z}\right) \quad \frac{0}{0} \text{ form.}$$

$$= \lim_{z \to n\pi}\left[\frac{(z-n\pi)(-\sin z) + \cos z}{\cos z}\right] = 1$$

$\therefore R(f, n\pi) = 1$

(ii) $f(z) = \frac{z^2-4}{z^5-z^3} = \frac{z^2-4}{z^3(z^2-1)}$

$z = 0$ is a pole of order 3

$z = \pm 1$ are simple pole.

$$R(f, 0) = \frac{1}{\lfloor 2}\frac{d^2}{dz^2}\left[(z-0)^3 f(z)\right]_{z=0}$$

$$= \frac{1}{\lfloor 2}\lim\frac{d^2}{dz^2}\left[\cancel{z^3}\left(\frac{z^2-4}{\cancel{z^3}(z^2-1)}\right)\right]$$

$$= \frac{1}{2}\lim\frac{d}{dz}\left[\frac{(z^2-1)2z-(z^2-4)2z}{(z^2-1)^2}\right]$$

$$= \frac{1}{2}\lim_{z\to 0}\frac{d}{dz}\left(\frac{6z}{(z^2-1)^2}\right) = 3\frac{d}{dz}\left(\frac{z}{(z^2-1)^2}\right)$$

$$= 3\lim_{z\to 0}\left\{\frac{(z^2-1)^2 \cdot 1 - z \cdot 2(z^2-1)\,2z}{(z^2-1)^4}\right\}$$

$$= 3\lim_{z\to 0}\left\{\frac{z^4 - 2z^2 + 1 - 4z^4 + 4z^2}{(z^2-1)^4}\right\} = 3$$

(iii) $f(z) = \frac{1}{z^2\sin z} = \frac{1}{z^2\left[z - \frac{z^3}{\lfloor 3} + \frac{z^5}{\lfloor 5} - + \text{——}\right]}$

$$= \frac{1}{z^3\left(1 - \frac{z^2}{\lfloor 3} + \frac{z^4}{\lfloor 5} + \text{——}\right)}$$

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$z=0$ is a pole of order 3

& $\sin z = 0 \Rightarrow z = n\pi \quad n = 0, \pm 1, \pm 2 \text{ --- }$ are simple poles

$$R(f,0) = \frac{1}{\lfloor 2} \frac{d^2}{dz^2}\left[(z-0)^3 \frac{1}{z^2 \sin z}\right]_{z=0}$$

$$\cancel{= \frac{1}{2} \lim_{z\to 0} \frac{d}{dz}\left(\frac{\sin z(1) - z\cos z}{\sin^2 z}\right)}$$

$$\cancel{= \frac{1}{2} \lim_{z\to 0}\left(\frac{\sin^2 z(\cos z - \cos z + z\sin z) - (\sin z - z\cos z)2\sin z\cos z}{(\sin z)^4}\right)}$$

$$= \frac{1}{2} \frac{d^2}{dz^2}\left[\frac{(z-0)^3}{z^2\left[z - \frac{z^3}{\lfloor 3} + \frac{z^5}{\lfloor 5} + \cdots\right]}\right] \lim_{z\to 0}$$

$$= \frac{1}{2} \frac{d^2}{dz^2}\left(\frac{1}{1 - \frac{z^2}{\lfloor 3} + \frac{z^4}{\lfloor 5} + \cdots}\right)$$

$$= \frac{1}{2} \frac{d^2}{dz^2}\left[1 - \left(\frac{z^2}{\lfloor 3} - \frac{z^4}{\lfloor 5} + \cdots\right)\right]^{-1}$$

$$= \frac{1}{2} \frac{d^2}{dz^2}\left[1 + \frac{z^2}{\lfloor 3} - \frac{z^4}{\lfloor 5} + \left(\frac{z^2}{\lfloor 3} - \frac{z^4}{\lfloor 5} \cdots\right)^2 + \cdots\right]$$

$$= \frac{1}{2}\left(\frac{2}{\lfloor 3}\right) = \frac{1}{6}$$

$$R(f, n\pi) = \lim_{z\to n\pi}\left((z-n\pi)\frac{1}{z^2 \sin z}\right) \quad \frac{0}{0} \text{ form}$$

$$= \frac{1}{2z\sin z + z^2\cos z} = \frac{1}{(n\pi)^2\cos n\pi} = \frac{(-1)^n}{n^2\pi^2}$$

$n = 0, \pm 1, \pm 2 \text{ --- }$

Q(5) $z e^{a-z} = 1 \quad a > 1 \quad |z| = 1$

Given $z e^{a-z} = 1 \Rightarrow z = \frac{1}{e^{a-z}}$ $\quad H(z) = f(z) - g(z)$

Since $a > 1$ for all z on C $\quad f(z) = z$

$\qquad |g(z)| < 1 \qquad g(z) = \frac{1}{e^{a-z}}$

& $f(z) = |z| = 1$

$|g(z)| < |f(z)|$

by Rouches Th $H(z)$ has as many zeros interior of $|z|=1$ as $f(z) = z$ one zero in interior of ~~|z|=1~~ C

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Q(6) Find the Poles Of the fn $f(z) = e^{az}(\cosh z + \cos\beta)^{-1}$
$0<\alpha<1$ & $0<\beta<\pi$ Find the Residues:

Sol

$$f(z) = e^{az}/\cosh z + \cos\beta$$

for poles: $\cosh z + \cos\beta = 0$

$$\Rightarrow \quad \frac{\cos\beta}{\cosh z} = -1 = e^{\pm i\pi + 2n\pi i}$$

$$\cos\beta = e^{\pm i\pi + 2n\pi i}(\cosh z)$$

$$\frac{e^{i\beta} + e^{-i\beta}}{2} = e^{i\pi \pm 2n\pi i}\left(\frac{e^{z} + e^{-z}}{2}\right)$$

$$e^{i\beta} + e^{-i\beta} = e^{z \pm i\pi + 2n\pi i} + e^{-z \pm i\pi + 2n\pi i}$$

Suppose $e^{i\beta} = e^{z \pm i\pi + 2n\pi i}$ & $e^{-i\beta} = e^{-z \pm i\pi + 2n\pi i}$

$$z = -i\,[\pi + 2n\pi - \beta]$$

at $p_1 = -i\,[\pi + 2n\pi - \beta]$

Residue at p_1

$$R(f, p_1) = \lim_{z\to p_1}\left[(z - p_1)\, f(z)\right]$$

$$= \lim_{z\to p_1}\left[(z - p_1)\,\frac{e^{az}}{\cosh z + \cos\beta}\right]$$

?

$\cosh i\beta$
$\cosh\beta$

$\cosh(2n\pi i - \pi + \beta)$
$\cos h(i\beta + i\pi - 2n\pi i)$
$\cosh z\,(\beta - \pi(2n+1))$
$\cos(\pi(2n+1) - \beta)$

Q.(7) Prove that residue of the fn

$$f(z) = z^{-7}\cot\pi z\,\coth\pi z \quad \text{at } z = 0$$

$$= \frac{1}{z^7}\left\{\frac{\cos\pi z\ \cosh\pi z}{\sin\pi z\ \sinh\pi z}\right\}$$

$$= \frac{1}{z^7}\frac{\left\{\left(1 - \frac{\pi^2 z^2}{2} + \frac{\pi^4 z^4}{\lfloor 4} - \frac{\pi^6 z^6}{\lfloor 6} + \frac{\pi^8 z^8}{\lfloor 8} + \cdots\right)\left(1 + \frac{\pi^2 z^2}{\lfloor 2} + \frac{\pi^4 z^4}{\lfloor 4} + \frac{\pi^6 z^6}{\lfloor 6} + \frac{\pi^8 z^8}{\lfloor 8}\right.\right.}{\left\{\left(\pi z - \frac{\pi^3 z^3}{\lfloor 3} + \frac{\pi^5 z^5}{\lfloor 5} - \frac{\pi^7 z^7}{\lfloor 7} + \frac{\pi^9 z^9}{\lfloor 9} + \cdots\right)\left(\pi z + \frac{\pi^3 z^3}{\lfloor 3} + \frac{\pi^5 z^5}{\lfloor 5} + \frac{\pi^7 z^7}{\lfloor 7} + \frac{\pi^9 z^9}{\lfloor 9}\right)\right.}$$

Then coeffs of $\frac{1}{z}$ will be the Answer

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Q(8)

$$f(z) = \frac{1}{(z^2-1)^2}\sin(1/z)$$

$$= \frac{1}{(z^2-1)^2} \cdot \left\{ \frac{1}{z} - \frac{1}{3!}\left(\frac{1}{z^3}\right) + \frac{1}{5!}\left(\frac{1}{z^5}\right) - + \text{---} \right)$$

$$= \frac{1}{(z^2-1)^2} \cdot \frac{1}{z}\left\{ 1 - \frac{1}{6z^2} + \frac{1}{120 z^4} + \text{---} \right\}$$

the pts $z = \pm 1$ are poles of order 2 and $z = 0$ is an essential singularity.

For ~~pol~~ zeros $\sin(1/z) = 0$

$$1/z = \sin^{-1}(0) = n\pi.$$

$$z = \frac{1}{n\pi}$$

$$R(f,1) = \underset{z\to 1}{Lt} \frac{d}{dz}\left(\frac{(z-1)^2 \sin(1/z)}{(z^2-1)^2}\right)$$

$$= \underset{z\to 1}{Lt} \frac{d}{dz}\left(\frac{\sin(1/z)}{(z+1)^2}\right)$$

$$= \underset{z\to 1}{Lt}\left\{ \frac{(z+1)^2 \cos 1/z \,(-1/z^2) - \sin(1/z)\, 2(z+1)}{(z+1)^4} \right\}$$

$$= -\tfrac{1}{4}(\cos 1 + \sin 1) = R f(-1)$$

Q(9) Prove that at $z = ai$ of $f(z) = \dfrac{e^{i\pi z}}{z(z^2+a^2)^2}$

Poles are $z = \pm ai$ of order 2 & $z = 0$ is simple Pole

$$R(f, ai) = \underset{z\to ai}{Lt} \frac{d}{dz}\left((z-ai)^2 f(z)\right)$$

$$= \underset{z\to ai}{Lt} \frac{d}{dz}\left(\cancel{(z-ai)^2} \cdot \frac{e^{i\pi z}}{z(z+ai)^2 \cancel{(z-ai)^2}}\right)$$

$$= \underset{z\to ai}{Lt}\left\{ \frac{z(z+ai)^2 \pi i\, e^{i\pi z} - e^{i\pi z}\left[(z+ai)^2 + z\, 2(z+ai)\right]}{\left[z(z+ai)^2\right]^2} \right\}$$

$$= \underset{z\to ai}{Lt}\left\{ \frac{z[z+ai]^2\, i\pi\, e^{i\pi z} - e^{i\pi z}\left((z+ai)^2 + 2z(z+ai)\right)}{z^2(z+ai)^4} \right\}$$

$$= \underset{z\to ai}{Lt} \frac{e^{i\pi z}\left[z(z+ai)\, i\pi - (z+ai) - 2z\right]}{z^2(z+ai)^3} =$$

$$= e^{-na}\left\{\frac{ai(in)\,2ai\;-4ai}{-a^2(8a^3i^3)}\right\}$$

$$= e^{-na}\left[\frac{2a^2n\,i^3-4ai}{i\,8a^5}\right] = e^{-na}\left\{\frac{-2ina^2-4ai}{8i\,a^5}\right\}$$

$$= \frac{-2ia\,e^{-na}(an+2)}{8i\,a^5}$$

$$= -\frac{e^{-na}(na+2)}{4a^4}$$

Q(10) Find Residue of the fn at $z=0$

$$f(z) = \frac{\operatorname{cosec} z\ \operatorname{cosech} z}{z^3}$$

$$= \frac{1}{z^3}\left(\frac{1}{\left(z-\frac{z^3}{\lfloor 3}+\frac{z^5}{\lfloor 5}-\frac{z^7}{\lfloor 7}\cdots\right)\left(z+\frac{z^3}{\lfloor 3}+\frac{z^5}{\lfloor 5}\cdots\right)}\right)$$

$$= \frac{1}{z^5}\left[\left(1-\left(\frac{z^2}{6}-\frac{z^4}{120}+\frac{z^6}{\lfloor 7}-\cdots\right)\right)^{-1}\left(1+\left(\frac{z^2}{6}+\frac{z^4}{\lfloor 5}\cdots\right)\right)^{-1}\right]$$

$$= \frac{1}{z^5}\left[\left[1+\left(\frac{z^2}{6}-\frac{z^4}{\lfloor 5}+\frac{z^6}{\lfloor 7}\cdots\right)+\left(\frac{z^2}{\lfloor 3}-\frac{z^4}{\lfloor 5}\cdots\right)^2+\cdots\right]\cdot\right.$$
$$\left.\left[1-\left(\frac{z^2}{\lfloor 3}+\frac{z^4}{\lfloor 5}\right)+\left(\frac{z^2}{\lfloor 3}+\frac{z^4}{\lfloor 5}\cdots\right)^2\cdots\right]\right]$$

For coeff of $\frac{1}{z}$ (Taking upto z^4 ..)

$$= \frac{1}{z^5}\left[-\frac{z^4}{\lfloor 5}+\frac{z^4}{\lfloor 3\lfloor 3}-\frac{z^4}{\lfloor 5}+\frac{z^2}{(\lfloor 3)^2}\cdots\right]$$

one product term $\frac{z^2}{\lfloor 3}\cdot\frac{z^2}{\lfloor 3}$

~~6 | 60 , 36~~
2 | 10 6
5 . 3

$$= \frac{z^4}{z^5}\left[-\frac{2}{\lfloor 5}+\frac{1}{\lfloor 3\lfloor 3}\right]$$

$$= \frac{1}{z}\left[-\frac{1}{60}+\frac{1}{36}\right] = \frac{1}{z}\left[\frac{-3+5}{180}\right]$$

$$= \frac{1}{z}\left(\frac{1}{90}\right)$$

Coeff of $1/z$ is $\boxed{1/90}$

$\therefore\ R(f,0) = 1/90$

Q(11) $f(z) = z^5 + z^3 + 2z + 3$ has just one zero in the first quadrant of the plane.

Sol.

$f(z) = z^5 + z^3 + 2z + 3$ (Note that odd degree & with real coeff has at least one real root.)

Since all the coeffs of terms are +ve real nos ∴ No root of the eq is +ve

Put $z = -x$

$f(-x) = -x^5 - x^3 - 2x + 3 = -[x^5 + x^3 + 2x - 3]$

$= -x^3(x^2+1) - (2x-3) \Rightarrow f(-x) > 0, \; x > -1$

By Argument Principle,

(a) On x axis $\Delta \arg f(z) = 0$

(b) For the circle $|z| = R, \quad z = Re^{i\theta}$

$f(z) = z^5\left[1 + \frac{1}{z^2} + \frac{2}{z^4} + 3/z^5\right]$

$= R^5 e^{5i\theta}\left[1 + F(z)\right] \qquad F(z) \to 0$
$z \to \infty$

Then when z moves on the I quad

change in arg $f(z) = 5(\pi/2) = \frac{5\pi}{2} = \pi/2$

(c) On y axis $z = iy$

$f(iy) = iy^5 - iy^3 + 2iy + 3$

$\arg f(iy) = \tan^{-1}\left(\frac{y^5 - y^3 + 2y}{3}\right) \Rightarrow$ $\quad y^4 - y^2 + 2$

$\tan^{-1} 0 = 0$, when $y = 0$, $\quad y = 0 \quad f(z) = 0$

Total change in variation $0 + \pi/2 = \pi/2$

$\Rightarrow$ One real root lie in I quadrant

Q(12)

(a) $f(z) = \dfrac{1}{z^3(1-z^2)}$

Poles are $z = 0$ of order 3

$z = \pm 1$ simple poles

$R(f,0) = \lim_{z\to 0} \frac{1}{2!} \frac{d^2}{dz^2}\left((z-0)^3 \frac{1}{z^3(1-z^2)}\right)$

$= \lim_{z\to 0} \frac{1}{2} \frac{d}{dz}\left((-1)(1-z^2)^{-2}(-2z)\right)$

$= 1/2 \left[\frac{d}{dz}\left(\frac{z}{(1-z^2)^2}\right)\right] = \lim \frac{(1-z^2)1 - z(2(1-z^2)(-2z))}{(1-z^2)^4} = 1$

$\Rightarrow R(f,0) = 1$

17/2
1 16
2 17
3 18
4 19
5
6 20
7 21
8 22
9 23
10 24
11 25
12 26
13 27
14 28
15 29
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31
32

(12) (a) Simple Pole at 1 & −1

(b) $f(z) = \dfrac{\sin 2z}{(z+1)^3}$ $z=-1$ is Pole of ord 3

$$R(f,-1) = \lim_{z\to -1} \frac{1}{\lfloor 2} \frac{d^2}{dz^2}\left(\frac{(z+1)^3 \sin 2z}{(z+1)^3}\right)$$

$$= \frac{1}{2}\lim_{z\to -1} \frac{d}{dz}(\cos 2z) = \lim_{z\to -1}(-2\sin 2z)$$

$$= -2\sin(-2) = 2\sin 2$$

(c) $f(z) = \dfrac{1}{\sin z}$

$$= \frac{1}{z - \frac{z^3}{\lfloor 3} + \frac{z^5}{\lfloor 5} - \cdots} = \frac{1}{z}\left(\frac{1}{1 - \frac{z^2}{\lfloor 3} + \frac{z^4}{\lfloor 5} - \cdots}\right)$$

$\sin z = 0$ $z = n\pi$ $n = 0, \pm 1, \pm 1$

$$R(f, n\pi) = \lim_{z\to n\pi}\left((z-n\pi)\frac{1}{\sin z}\right) \quad \frac{0}{0} \text{ form}$$

$$= \lim_{z\to n\pi} \frac{1}{\cos z} = \frac{1}{\cos n\pi} = (-1)^n$$

$n = 0, \pm 1, \pm 2 \ldots$

(d) $f(z) = \sin z \ \sin(1/z)$

$$= \sin z \left(\frac{1}{z} - \frac{1}{\lfloor 3}\left(\frac{1}{z^3}\right) + \cdots\right)$$

$$= \frac{\sin z}{z}\left(1 - \frac{1}{6}\frac{1}{z^2} + \cdots\right) \quad z = 0$$

$$R(f,0) = \frac{1}{z}\left(z - \frac{z^3}{\lfloor 3} + \frac{z^5}{\lfloor 5} - \cdots\right)\left(1 - \frac{1}{6}\frac{1}{z^2} - \cdots\right)$$

$$= \left(1 - \frac{z^2}{\lfloor 3} + \frac{z^4}{\lfloor 5} - \cdots\right)\left(1 - \frac{1}{6}\left(\frac{1}{z^2}\right)\right) \quad \text{No terms } \tfrac{1}{z}$$

$R = 0$

(13) $f(z) = z^{-3}\, e^{z^2}/2 = \frac{1}{2}\,\frac{1}{z^3}\, e^{z^2}$

$z = 0$ is a pole of ord 3

$$R(f,0) = \lim_{z\to 0} \frac{1}{\lfloor 2} \frac{d^2}{dz^2} \frac{z^3}{2}\frac{e^{z^2}}{z^3}$$

$$= \frac{1}{4}\lim_{z\to 0}\frac{d}{dz}\left(e^{z^2}(2z)\right)$$

$$= \frac{1}{4}\lim_{z\to 0}\left(e^{z^2}(2z)(2z) + e^{z^2}(2)\right) = \frac{2}{4} = \frac{1}{2}$$

13 (ii) $$f(z) = \frac{\sin z}{z^2}$$

$$= \frac{\left\{ z - z^3/\lfloor 3 + z^5/\lfloor 5 - + - - \right\}}{z^2}$$

$$= \frac{z \left[1 - z^2/\lfloor 6 + z^4/\lfloor 5 - + \text{———} \right]}{z^4}$$

$z = 0$ is a pole of order 1

$$R(f, 0) = \mathrm{Lt}_{z \to 0} \left((z - 0) f(z) \right) = \mathrm{Lt}_{z \to 0} \, z \frac{\sin z}{z^4} = \mathrm{Lt}_{z \to 0} \left(\frac{\sin z}{z} \right) = 1$$

$$R = 1$$

Q (14) $$f(z) = \frac{z^2 + z^4}{\sin \pi z}$$

Pole $\sin \pi z = 0$

$$\pi z = \sin^{-1}(0) = n\pi$$

$$\boxed{z = n} \qquad n = 0 \pm 1, \pm 2 ——$$

$$R(f, n) = \mathrm{Lt}_{z \to n} \frac{(z - n)(z^2 + z^4)}{\sin \pi z} \qquad \frac{0}{0} \text{ form}$$

$$= \mathrm{Lt}_{z \to n} \frac{(z - n)(2z + 4z^3) + z^2 + z^4}{\cos \pi z \cdot \pi}$$

$$= \frac{n^2 + n^4}{\pi \cos \pi n} = \frac{(-1)^n (n^2 + n^4)}{\pi}$$