LINEAR EQUATIONS AND INEQUALITIES

CHAPTER 4

Systems of linear equations play an important and motivating role in the subject of linear algebra. In fact, many problems in linear algebra reduce to finding the solution of a system of linear equations. Thus, the techniques introduced in this chapter will be applicable to abstract ideas introduced later. On the other hand, some of the abstract results will give us new insights into the structure and properties of systems of linear equations. All our systems of linear equations involve scalars as both coefficients and constants, and such scalars may come from any number field **F**. There is almost no loss in generality if the reader assumes that all our scalars are real numbers — that is, they come from the real field **R**.

In this chapter we will learn about;

- Linear equation and its consequences
- Analytical approach to solve simultaneous equations
- Inequalities and their application
- Linear programming
- Exercises about linear equations and inequalities

Linear Equation

It is an algebraic equation in which each term has an exponent of one and graphing of equation results in a straight line.

Or A linear equation in unknowns $x_1, x_2, \ldots, x_n$ is an equation that can be put in the standard form $a_1x_1 + a_2x_2 + \ldots + a_nx_n = b$ where $a_1, a_2, \ldots, a_n$ and b are constants. The constant a_k is called the coefficient of x_k, and b is called the constant term of the equation. e.g. $6x_1 + 7x_2 = 5, 2x + 3y + 4z = -1$

Solutions of Linear Equation

A **solution** of the linear equation $a_1x_1 + a_2x_2 + \ldots + a_nx_n = b$ is a list of values for the unknowns or, equivalently, a vector $\vec{u}$ in $\mathbf{K}^n$, say $x_1 = k_1, x_2 = k_2, \ldots, x_n = k_n$ or $\vec{u} = (k_1, k_2, \ldots, k_n)$ such that the following statement (obtained by substituting k_i for x_i in the equation) is true: $a_1k_1 + a_2k_2 + \ldots + a_nk_n = b$.

In such a case we say that $\vec{u}$ satisfies the equation. The set of all solutions of the system is called the **solution set** or the **general solution** of the system.

Examples for Linear and Non – Linear Equations

- $x+3y=7$ linear
- $5x+7y-8yz=16$ not linear
- $x+\pi y+ez=\log 5$ linear for constants π, e
- $x_1-2x_2-3x_3+x_4=0$ linear
- $x+3y^2=4$ not linear
- $3x+2y-xy=5$ not linear
- $\text{Sin}x+y=0$ not linear
- $\sqrt{x_1}+2x_2+x_3=1$ not linear
- $x_1+5x_2-\sqrt{2}x_3=1$ linear
- $x_1+3x_2+x_1x_3=2$ not linear
- $x_1^{-2}+x_2+8x_3=5$ not linear
- $x_1^{3/5}-2x_2+x_3=4$ not linear
- $\pi x_1-\sqrt{2}x_2=7^{\frac{1}{3}}$ linear
- $\cos\left(\frac{\pi}{7}\right)x-4y=\log 3$ not linear
- $xy=1$ not linear
- $y+7=x$ linear

A linear equation does not involve any products or roots of variables. All variables occur only to the first power, and do not appear as arguments of trigonometric, logarithmic or exponential functions.

System of Linear Equations **(System in which more than one linear equations involve)**

A system of linear equations is a list of linear equations with the same unknowns. In particular, a system of 'm' linear equations $L_1, L_2, \ldots, L_m$ in 'n' unknowns $x_1, x_2, \ldots, x_n$ can be put in the standard form

$$a_{11}x_1+a_{12}x_2+\ldots+a_{1n}x_n=b_1$$
$$a_{21}x_1+a_{22}x_2+\ldots+a_{2n}x_n=b_2$$
::
$$a_{m1}x_1+a_{m2}x_2+\ldots+a_{mn}x_n=b_m$$

$a_{ij}x_j=b_i$

$m=$ No. of equations
$n=$ No. of unknowns

Where the a_{ij} and b_i are constants. The number a_{ij} is the coefficient of the unknown x_j in the equation L_i, and the number b_i is the constant term of the equation L_i.

Square system of linear equations

The system of linear equations $a_{ij}x_j=b_i$ is called an $m\times n$ system. It is called a **square system** if $m=n$ that is, if the number m of equations is equal to the number of unknowns.

Homogeneous and non-homogeneous system of linear equations

The system of linear equations $a_{ij}x_j = b_i$ is said to be **homogeneous** if all the constant terms are zero that is, if $b_1 = 0, b_2 = 0, \ldots\ldots, b_n = 0$ .Otherwise the system is said to be **nonhomogeneous (inhomogeneous**.)

1. Solve the equation $3x + 7 = 1$.

Solution:

$$3x + 7 = 1$$
$$\Rightarrow 3x + 7 - 7 = 1 - 7 \ \Rightarrow 3x = -6$$
$$\Rightarrow \frac{3x}{3} = -\frac{6}{3} \Rightarrow x = -2$$

2. Solve the equation 2x − 3 − 2 = 3.

Solution:

$$2x - 3 - 2 = 3 \ \Rightarrow 2x - 5 = 3$$
$$\Rightarrow 2x - 5 + 5 = 3 + 5$$
$$\Rightarrow 2x = 8$$
$$\Rightarrow \frac{2x}{2} = \frac{8}{2} \ \Rightarrow x = 4$$

3. Solve the equation −7x + 1 + 2x = 9x − 8 + 1.

Solution:

$$-7x + 1 + 2x = 9x - 8 + 1$$
$$\Rightarrow -7x + 2x - 9x = -8 + 1 - 1$$
$$\Rightarrow -14x = -8 \Rightarrow x = \frac{-8}{-14} \ \Rightarrow x = \frac{4}{7}$$

4. Solve the equation; $2(x-3) - 17 = 13 - 3(x+2)$**.**

Solution:

$$2(x-3) - 17 = 13 - 3(x+2)$$
$$\Rightarrow 2x - 6 - 17 = 13 - 3x - 6$$
$$\Rightarrow 2x + 3x = 13 - 6 + 6 + 17$$
$$\Rightarrow 5x = 30 \Rightarrow x = 6$$

5. Solve the equation involving fraction; $\frac{x+2}{4} - \frac{x-1}{3} = 2$ **.**

Solution:

$$\frac{x+2}{4} - \frac{x-1}{3} = 2$$
$$\Rightarrow 12 \times \frac{x+2}{4} - 12 \times \frac{x-1}{3} = 12 \times 2$$
$$\Rightarrow 3(x+2) - 4(x-1) = 24 \ \Rightarrow 3x + 6 - 4x + 4 = 24$$
$$\Rightarrow 3x - 4x = 24 - 6 - 4 \ \Rightarrow -x = 14 \ \Rightarrow x = -14$$

6. Solve the equation involving fraction; $\frac{3x}{5}-\frac{x-3}{2}=\frac{x+2}{3}$ **.**

Solution:

$$\frac{3x}{5}-\frac{x-3}{2}=\frac{x+2}{3}$$
$$\Rightarrow 30\times\frac{3x}{5}-30\times\frac{x-3}{2}=30\times\frac{x+2}{3}$$
$$\Rightarrow 6(3x)-15(x-3)=10(x+2)$$
$$\Rightarrow 18x-15x+45=10x+20$$
$$\Rightarrow 18x-15x-10x=20-45$$
$$\Rightarrow -7x=-25$$
$$\Rightarrow x=\frac{25}{7}$$

7. Consider the following linear equation in three unknowns x, y, z:

$x+2y-3z=6$

Check $\vec{u}=(5,2,1)$ **is solution of equation or not.**

Solution:

We note that $x=5; y=2; z=1$, or, equivalently, the vector $\vec{u}=(5,2,1)$ is a solution of the equation. That is, $5+2(2)-3(1)=6$

8. Consider the following linear equation in three unknowns x, y, z:

$x+2y-3z=6$

Check $\vec{v}=(1,2,3)$ **is solution of equation or not.**

Solution:

$\vec{v}=(1,2,3)$ is not a solution, because on substitution, we do not get a true statement: $1+2(2)-3(3)=-4\neq 6$

9. Solve the system of equations by addition method; $2x + 3y = 48$ & $9x - 8y = -24$

Solution:

$2x + 3y = 48$............(i)

$9x - 8y = -24$ ....(ii)

Multiplying (i) with – 3

$-6x - 9y = -144$

$9x - 8y = -24$

Then adding we have y = 12 and putting in (i) we have x = 8.

Hence the solution set is $\{8,12\}$.

Radical Equations

An equation in which the unknown letter (variable) appears under a radical sign is called a radical equation. For example: $\sqrt{x+1}=7, \sqrt{x}=9, \sqrt{2x-3}=\sqrt{x+5}$

10. Solve $\sqrt{x}+3=7$

Solution:

$\sqrt{x}+3=7 \Rightarrow \sqrt{x}=4$

$\left(\sqrt{x}\right)^2=4^2 \Rightarrow x=16$

11. Solve $4+2\sqrt{3y+1}=3$

Solution:

$4+2\sqrt{3y+1}=3$

$2\sqrt{3y+1}=-1 \Rightarrow \sqrt{3y+1}=-\frac{1}{2}$

$\left(\sqrt{3y+1}\right)^2=\left(-\frac{1}{2}\right)^2 \Rightarrow 3y+1=\frac{1}{4}$

$\Rightarrow 3y=-\frac{3}{4} \Rightarrow y=-\frac{1}{4}$

12. Solve $\sqrt{x-5}=3$

Solution:

$\sqrt{x-5}=3 \Rightarrow \left(\sqrt{x-5}\right)^2=(3)^2$

$x-5=9 \Rightarrow x=14$

(i) The equations of the form: $l\left(ax^2+bx\right)+m\sqrt{ax^2+bx+c}=0$

13. Solve the equation $3x^2+15x-2\sqrt{x^2+5x+1}=2$

Solution:

Let $\sqrt{x^2+5x+1}=y$

$\Rightarrow x^2+5x+1=y^2$

$\Rightarrow x^2+5x=y^2-1$

$\Rightarrow 3x^2+15x=3y^2-3$

The given equation becomes $3y^2-3-2y=2$

$\Rightarrow 3y^2-2y-5=0$

$\Rightarrow (3y-5)(y+1)=0$

$\Rightarrow\ y=\dfrac{5}{3}$ or $y=-1$

$\Rightarrow\ \sqrt{x^2+5x+1}=\dfrac{5}{3}$ $\Rightarrow\ \sqrt{x^2+5x+1}=-1$

$\Rightarrow\ x^2+5x+1=\dfrac{25}{9}$ $\Rightarrow\ x^2+5x+1=1$

$\Rightarrow\ 9x^2+45x+9=25$ $\Rightarrow\ x^2+5x=0$

$\Rightarrow\ 9x^2+45x-16=0$ $\Rightarrow\ x(x+5x)=0$

$\Rightarrow (3x+16)(3x-1)=0$ $x=0\ or\ x=-5$

$\therefore\quad x=\dfrac{1}{3}\ \ or\ x=\dfrac{16}{13}$

On checking, it is found that 0 and -5 do not satisfy the given equation.
Therefore 0 and – 5 being extraneous roots cannot be included in solution set.
Hence solution set.

ii) The equation of the form: $\sqrt{x+a}+\sqrt{x+b}=\sqrt{x+c}$

14. Solve the equation $\sqrt{x+8}+\sqrt{x+3}=\sqrt{12x+13}$

Solution:

$\sqrt{x+8}+\sqrt{x+3}=\sqrt{12x+13}$

Squaring both sides, we get

$x+8+x+3+2\sqrt{x+8}\sqrt{x+3}=12x+13$

$\Rightarrow\quad 2\sqrt{x+8}\sqrt{x+3}=10x+2$

$\Rightarrow\quad \sqrt{(x+8)(x+3)}=5x+1$

Squaring again, we have

$x^2+11x+24=25x^2+10x+1$

$\Rightarrow\quad 24x^2-x-23=0$

$\Rightarrow\quad (24x+23)(x-1)=0$

$\Rightarrow\quad x=\dfrac{23}{24}\quad or\quad x=1$

On checking we find that $-\dfrac{23}{24}$ is an extraneous root. Hence solution set $=\{1\}$.

iii) The equations of the form: $\sqrt{ax^2+bx+c}+\sqrt{px^2+qx+r}=\sqrt{lx^2+mx+n}$

Where $ax^2+bx+c,\ px^2+qx+r\ \ and\ lx^2+mx+n$ have a common factor.

15. Solve the equation: $\sqrt{x^2+4x-21}+\sqrt{x^2-x-6}=\sqrt{6x^2-5x-39}$

Solution:

$x^2+4x-21=(x+7)(x-3)$

$x^2-x-6=(x+2)(x-3)$

$6x^2-5x-39=(6x+13)(x-3)$

The given equation can be written as

$\sqrt{(x+7)(x-3)}+\sqrt{(x+2)(x-3)}=\sqrt{(6x-13)(x-3)}$

$\Rightarrow\ \sqrt{x-3}\left[\sqrt{x+7}+\sqrt{x+2}-\sqrt{6x+13}\right]=0$

Enter $\sqrt{x-3}=0$ *or* $\sqrt{x+7}+\sqrt{x+2}-\sqrt{6x+13}=0$

$\sqrt{x-3}=0 \Rightarrow x-3=0 \Rightarrow x=3$

Now solve the equation $\sqrt{x+7}+\sqrt{x+2}-\sqrt{6x+13}=0$

$\Rightarrow\ \sqrt{x+7}+\sqrt{x+2}=\sqrt{6x+13}$

$\Rightarrow\ x+7+x+2+2\sqrt{(x+7)(x+2)}=6x+13$

$\Rightarrow\ 2\sqrt{(x+7)(x+2)}=4x+4$

$\Rightarrow\ x^2+9x+14=4x^2+8x+4$

$\Rightarrow\ 3x^2-x-10=0$

$\Rightarrow\ (3x+5)(x-2)=0 \Rightarrow\ x=-\frac{5}{3},2$

Thus possible roots are $3,2,-\frac{5}{3}$.

On verification, it is found that $-\frac{5}{3}$ is an extraneous root. Hence solution set $=\{2,3\}$

iv) The equations of the form: $\sqrt{ax^2+bx+c}+\sqrt{px^2+qx+r}=mx+n$

Where, $(mx+n)$ is a factor of $(ax^2+bx+c)-(px^2+qx+r)$

16. Solve the equation: $\sqrt{3x^2-7x-30}-\sqrt{2x^2-7x-5}=x-5$

Solution:

Let $\sqrt{3x^2-7x-30}=a$ *and* $\sqrt{2x^2-7x-5}=b$

$a^2-b^2=(3x^2-7x-30)-(2x^2-7x-5)$

$a^2-b^2=x^2-25$ (i)

The given equation can be written as:

$a-b=x-5$ (ii)

$$\frac{(a+b)(a-b)}{a-b}=\frac{(x+5)(x-5)}{x-5}$$ [From (i) and (ii)]

$\Rightarrow\ a+b=x+5$---------- (iii)

$\Rightarrow\ 2a=2x$ [From (ii) and (iii)]

$\Rightarrow\ a=x$

$\therefore\ \sqrt{3x^2-7x-30}=x$

$\Rightarrow\ 3x^2-7x-30=x^2$

$\Rightarrow\ 2x^2-7x-30=0$

$\Rightarrow\ (2x+5)(x-6)=0$

$\Rightarrow\ x=-\frac{5}{2},6$

On checking, we find that $-\frac{5}{2}$ is an extraneous root.

Hence solution set $=\{6\}$

Absolute Value

The absolute value of a real number x is defined as follows;

$$|x|=\begin{cases} x;\ x>0 \\ 0;\ x=0 \\ -x;\ x<0 \end{cases}$$

Absolute Value Equations

An equation that contains a variable inside the absolute value bars is called an absolute value equation.

For example: $|x|+1=5, |x-3|=4$

17. Solve $|x|=8$

Solution:

$|x|=8$

$x=\pm 8$

18. Solve $|x|=-6$

Solution:

$|x|=-6$ not possible for real numbers. So this equation has no solution.

19. Solve $|2x+5|=11$

Solution: $|2x+5|=11$

$2x+5=\pm 11$

$x=3,-8$

20. Solve $|a-1|=|2a-3|$

Solution:

$$|a-1|=|2a-3|$$

$$a-1=\pm(2a-3)$$

$$a=2,\frac{4}{3}$$

21. Consider the following system of linear equations:

$$x_1+x_2+4x_3+3x_4=5$$

$$2x_1+3x_2+x_3-2x_4=1$$

$$x_1+2x_2-5x_3+4x_4=3$$

It is a 3×4 **system because it has three equations in four unknowns. Determine whether (a)** $\vec{u}=(-8,6,1,1)$ **and (b)** $\vec{v}=(-10,5,1,2)$ **are solutions of the system or not.**

Solution:

(a) Substitute the values of $\vec{u}$ in each equation, obtaining

$$-8+6+4(1)+3(1)=5\Rightarrow 5=5$$

$$2(-8)+3(6)+1-2(1)=1\Rightarrow 1=1$$

$$-8+2(6)-5(1)+4(1)=3\Rightarrow 3=3$$

Yes, $\vec{u}$ is a solution of the system because it is a solution of each equation.

(b) Substitute the values of $\vec{v}$ into each successive equation, obtaining

$$-10+5+4(1)+3(2)=5\Rightarrow 5=5$$

$$2(-10)+3(5)+1-2(2)=1\Rightarrow -8\neq 1$$

No, $\vec{v}$ is not a solution of the system, because it is not a solution of the second equation. (We do not need to substitute $\vec{v}$ into the third equation.)

Consistent and Inconsistent Solutions

The system of linear equations is said to be **consistent** if it has one or more solutions, and it is said to be **inconsistent** if it has no solution.

Underdetermined

A system of linear equations is considered underdetermined if there are fewer equations than unknowns.

Over determined

A system of linear equations is considered over determined if there are more equations than unknowns.

22. Solve the system of equations by substitution; $2x + y = 5$ *and* $x - 2y = 15$

Solution:

$2x + y = 5 \quad \text{............}(i)$

$x - 2y = 15 \quad \text{............}(ii)$

$(ii) \Rightarrow x = 2y + 15$

$(i) \Rightarrow 2(2y+15) + y = 5 \Rightarrow 4y + 30 + y = 5 \Rightarrow 5y = -25 \Rightarrow \mathbf{y} = -5$

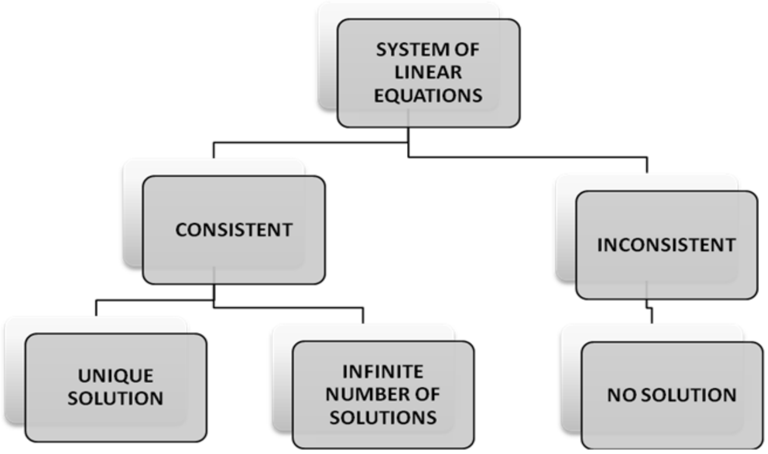
$(ii) \Rightarrow x = 2(-5) + 15 \Rightarrow x = -10 + 15 \Rightarrow \mathbf{x} = 5$

So our final answer is $x = 5$, $y = -5$

Remember

If the field **F** of scalars is infinite, such as when **F** is the real field **R** or the complex field **C**, then we have the following important result.

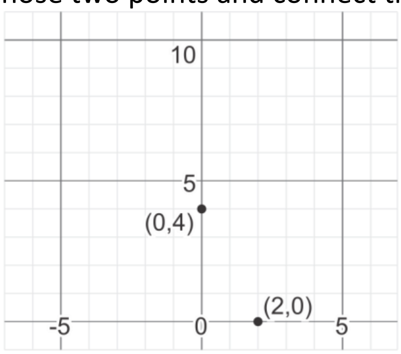
Result: Suppose the field **F** is infinite. Then any system of linear equations has
(i) a unique solution, (ii) no solution, or (iii) an infinite number of solutions.

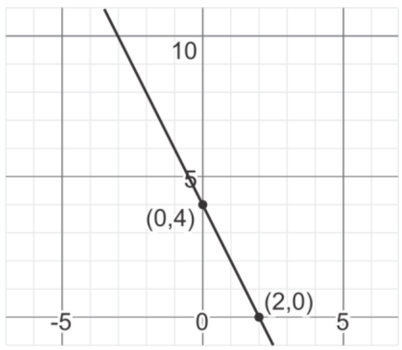


Graphing Linear Equations

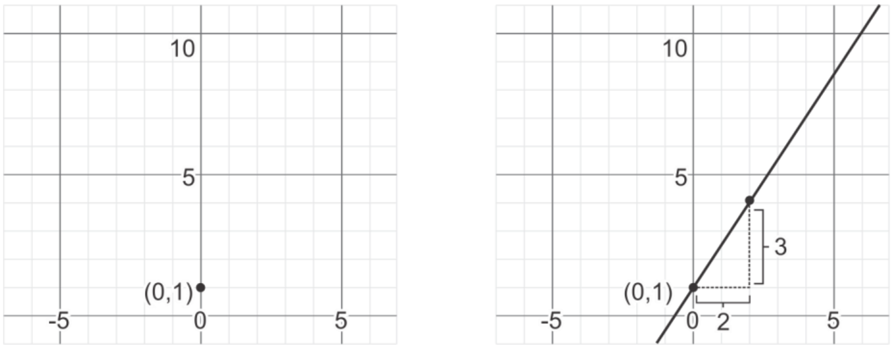
There are two methods to graph a line:

1) Use the x and y intercepts: If you have the coordinates of the x and y intercepts, plot those two points and connect them with a line.





2) Use slope-intercept form: If you have an equation in slope intercept form ($y = mx + b$), plot the y-intercept and then construct the line using the slope. If the equation is not in slope-intercept form, it can be rearranged to be in slope-intercept form.



$$m = \frac{3}{2}$$

23. Graph the equation 6x + 2y = 12.

Solution:

Given that 6x + 2y = 12

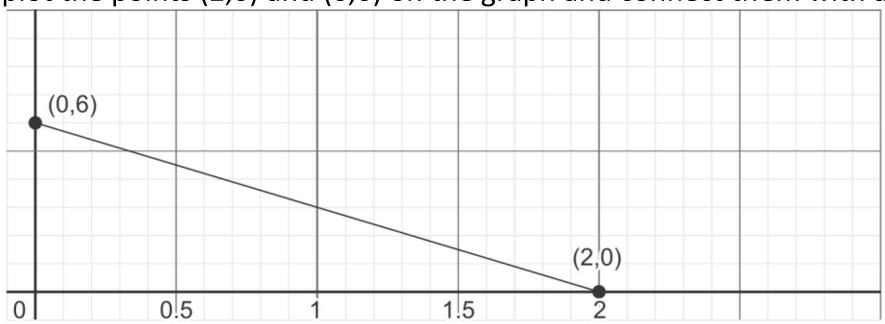
To find the x-intercept, set y equal to zero and solve for x.

$$6x + 2y = 12 \Rightarrow 6x + 2(0) = 12 \Rightarrow 6x = 12 \Rightarrow x = 2$$

To find the y-intercept, set x equal to zero and solve for y.

$$6x + 2y = 12 \Rightarrow 6(0) + 2y = 12 \Rightarrow 2y = 12 \Rightarrow y = 6$$

Now, plot the points (2,0) and (0,6) on the graph and connect them with a line:



24. Graph the equation 6x + 2y = 12.

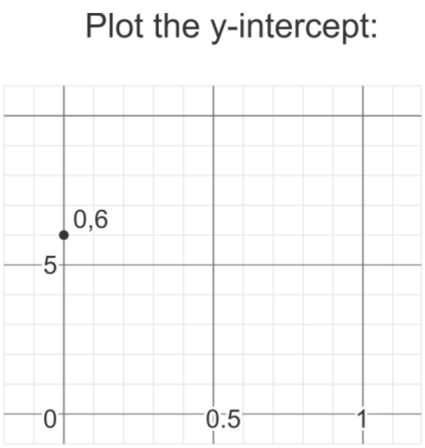
Solution:

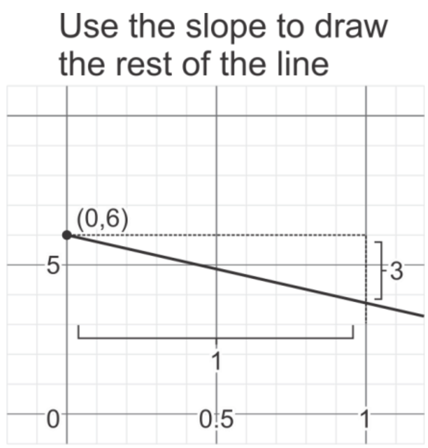
Given that 6x + 2y = 12

$$\Rightarrow y = -3x + 6$$

Comparing with $y = mx + c$ we have m = −3 and b = 6.

In other words, the slope is −3 and y – intercept is at (0,6)





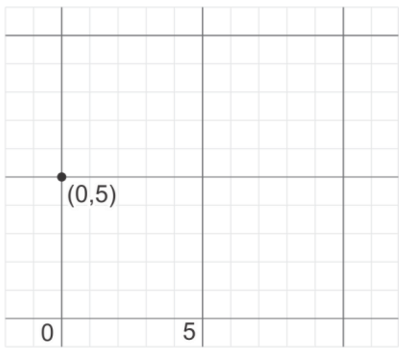
Graphs with one variable

If a graph only involves one variable, it can be graphed as a horizontal or vertical line.

25. Graph the equation y = 5.

Solution:

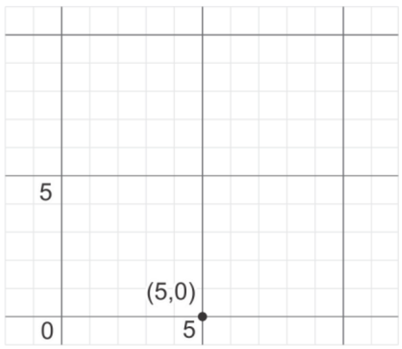
A horizontal line that crosses the y – axis at (0,5)



26. Graph the equation x = 5.

Solution:

A vertical line that crosses the x – axis at (5,0)



Geometrical Presentation / Graphics

Linear system in two unknowns arise in connection with intersection of lines.

- The lines may be parallel and distinct, in which case there is no intersection and consequently no solution.
- The lines may be intersect at only one point, in which case the system has exactly one solution.
- The lines may coincide, in which case there are infinitely many points of intersection (the points on the common line) and consequently infinitely many solutions. (in such system, all equations will be same with few common factors)

27. (A Linear System with one Solution): Solve the following system of linear equations

$x - y = 1$ **...........(i)**

$2x + y = 6$ **...........(ii)**

Solution:

$(i) \Rightarrow -2x + 2y = -2$ multiplying with -2

$\Rightarrow -2x + 2y + 2x + y = -2 + 6 \Rightarrow y = \frac{4}{3}$

$(i) \Rightarrow x - \frac{4}{3} = 1 \Rightarrow x = \frac{7}{3}$

$S.S = \left(x = \frac{7}{3}, y = \frac{4}{3} \right)$

Geometrically this means that the lines represented by the equations in the system intersect at the single point $\left(x = \frac{7}{3}, y = \frac{4}{3} \right)$

28. (A Linear System with No Solution): Solve the following system of linear equations

$x + y = 4$ **...........(i)**

$3x + 3y = 6$ **.........(ii)**

Solution:

$(i) \Rightarrow -3x - 3y = -12$ Multiplying with -3

Adding (i) with (ii) $\Rightarrow -3x - 3y + 3x + 3y = -12 + 6 \Rightarrow 0 = -6$

The result is contradictory, so the given system has no solution. Geometrically this means that the lines may be parallel and distinct, in this case there is no intersection and consequently no solution.

29. (A Linear System with Infinitely many Solutions):Solve the following system of linear equations

$4x - 2y = 1$ ....(i)

$16x - 8y = 4$ ....(ii)

Solution:

$(i) \Rightarrow -16x + 8y = -4$ multiplying with -4

Adding (i) with (ii) $\Rightarrow -16x + 8y + 16x - 8y = -4 + 4 \Rightarrow 0 = 0$

Equation $0 = 0$ does not impose any restriction on 'x' and 'y' and hence can be omitted. Thus the solution of the system are those values of 'x' and 'y' that satisfy the single equation $4x - 2y = 1$

Geometrically this means that the lines corresponding to the two equations in the original system coincide. And this system will have infinitely many solutions.

How to Find Few Solutions of Such System?

Find the value of 'x' from Common equation.

Put $y = t$ 't' being **Parameter** (arbitrary value instead of actual value)

Replace $y = t$ in given system.

Use $t = 0, 1, 2, 3, \ldots$ Upon your taste and get different answers.

We may apply same procedure by replacing 'x' and 'y'

30. Find different solutions for problem as follows using Parametric Equation (arbitrary equation using Parameter instead of actual value).

$4x - 2y = 1$ **...........(i)**

$16x - 8y = 4$ **...........(ii)**

How to find solution of more than two equations?
1st method: find x,y,z solving equations in pair (Lengthy Process)
2nd: solve two equations, find x,y and put in 3rd equation to get value of z.
3rd method: observe given equations and take common if possible and then check all equations are same or not, if same then solution will be infinite.

Solution:

$4x - 2y = 1$

$\Rightarrow x = \frac{1}{4} + \frac{1}{2}y$ and put $y = t$

$\Rightarrow x = \frac{1}{4} + \frac{1}{2}t$

$S.S = \left(x = \frac{1}{4}, y = 0\right) \quad t = 0$

$S.S = \left(x = \frac{3}{4}, y = 1\right) \quad t = 1$

$S.S = \left(x = -\frac{1}{4}, y = -1\right) \quad t = -1$

31. Find different solutions for problem as follows using Parametric Equation (arbitrary equation using Parameter instead of actual value).

$x - y + 2z = 5$ **...........(i)** $2x - y + 4z = 10$ **...........(ii)**

$3x - 3y + 6z = 15$ **...........(iii)**

Solution:

Since above all equations have same graphics or formation. Therefore will have infinitely many solutions. We will solve it using parametric equations.

In above all equations we have the parallel form $x - y + 2z = 5$

$\Rightarrow x = 5 + y - 2z$ and put $y = r, z = s \Rightarrow x = 5 + r - 2s$

$S.S = (x = 5, y = 0, z = 0) \quad r = 0, s = 0$

$S.S = (x = 6, y = 1, z = 0) \quad r = 1, s = 0$

$S.S = (x = 4, y = 1, z = 1) \quad r = 1, s = 1$

$\text{General Solution} = \{(5,0,0), (6,1,0), (4,1,1)\}$

Try Others Also!!!!!!!!

System of Simultaneous Equation

32. $2x-y=4; 2x^2-4xy-y^2=6$

Solution:

$$2x-y=4 \text{ ——— } i,\quad 2x^2-4xy-y^2=6 \text{ ——— } ii$$

$$From\ i\quad -y=4-2x \quad \Rightarrow y=2x-4 \text{ ——— } iii$$

$$(Put\ iii\ in\ ii)\quad 2x^2-4x(2x-4)-(2x-4)^2=6$$

$$2x^2-8x^2+16x-(4x^2-16x+16)-6=0$$

$$2x^2-8x^2+16x-4x^2+16x-16-6=0$$

$$-10x^2+32x-22=0 \quad \Rightarrow 10x^2-32x+22=0\ ('\times'\ by -1)$$

$$\div\ by\ 2\ both\ sides$$

$$5x^2-16x+11=0 \quad \Rightarrow 5x^2-5x-11x+11=0$$

$$5x(x-1)-11(x-1)=0 \quad \Rightarrow (x-1)(5x-11)=0$$

$$x-1=0 \quad or \quad 5x-11=0 \quad \Rightarrow x=1\ or \quad x=\frac{11}{5}$$

$$When\ \ x=1\ then\ y=2(1)-4=2-4=-2$$

$$When\ \ x=\frac{11}{5}\ then\ y=2\left(\frac{11}{5}\right)-4=\frac{22}{5}-4=\frac{22-20}{5}=\frac{2}{5}$$

$$S.S=\left\{(1,-2),(\frac{11}{5},\frac{2}{5})\right\}$$

33. $x^2+y^2+6x=1$; $x^2+y^2+2(x+y)=3$

Solution:

$$x^2+y^2+6x=1 \text{ ——} I,\quad x^2+y^2+2x+2y=3 \text{ ——} II$$

$$II-I \qquad \cancel{x^2+y^2}+2x+2y=3$$

$$\qquad\quad \cancel{x^2\pm y^2}\ \pm 6x \qquad = \underset{-}{1}$$

$$-4x+2y=2 \quad \Rightarrow \quad -2x+y=1 \quad \Rightarrow \quad y=2x+1$$

$$(Put\ value\ of\ y\ \ in\ \ I\ ) \qquad x^2+(2x+1)^2+6x=1$$

$$x^2+4x^2+4x+\cancel{1}+6x-\cancel{1}=0 \quad \Rightarrow \quad 5x^2+10x=0$$

$$5x(x+2)=0 \quad \Rightarrow \quad 5x=0 \quad or \quad x+2=0$$

$$x=0 \quad or \quad x=-2$$

$$When\ x=0 \qquad then \quad y=2\ (0)+1=0+1=1$$

$$When\ x=-2 \qquad then \quad y=2\ (-2)+1=-4+1=-3 \Rightarrow S.S=\{(0,1),(-2,-3)\}$$

Modeling with System of Linear Equations and their Solutions

34. A car rental company charges Rs.30 a day and 15 Pesa a mile for renting a car. Ali rents a car for two days and his bill comes to Rs. 108. How many miles did he drives?

Solution:

Let x be number of miles driven. Mileage cost is 0.15x and daily cost is Rs. $2(30)$.

Now according to mathematical model

$\text{Mileage cost} + \text{daily cost} = \text{total cost}$

$0.15x + 2(30) = 108$

$x = 320$ miles

35. Usman inherits Rs. 100,000 and inverts it in two certificates of deposit. One certificate pays 6% and the other pays $4\frac{1}{2}\%$ simple interest annually. If usman's total interest is Rs. 5025 per year, how much money is invested at each rate?

Solution:

Let x be amount invested at 6% , amount invested at $4\frac{1}{2}\%$ is 100,000 - x, interest earned at 6% is 0.06x and interest earned at $4\frac{1}{2}\%$ is $0.045(100,000 - x)$.

Then according to mathematical model interest at $6\% +$ interest at $4\frac{1}{2}\%$ = total interest $0.06x + 0.045(100,000 - x) = 5025$. $x = 35000$ rupees. So, Usman has invested Rs.35000 at 6% and the remaining Rs.65000 at $4\frac{1}{2}\%$

36. A square garden has a walkway 3 ft wide around its outer edge. If the area of entire garden including the walkway is 18000 ft^2 what are the dimensions of the planted area?

Solution:

Let x be the length of planted area, length of entire garden $(x+6)$ and area of entire garden $(x+6)^2$ .Then according to mathematical model

$\text{area of entire garden} = 18000\text{ft}^2$

$(x+6)^2 = 18000 \Rightarrow x + 6 = \sqrt{18000}$

$x \approx 128$ ft

The planted area of garden is about 128 ft by 128 ft.

37. A rectangular building lot is 8ft longer than its width and has an area of 2900ft^2. Find the dimension of the lot.

Solution:

Let w be the width of lot, length of lot is $(w+8)$.

Then according to mathematical model

$$\text{width of lot} \times \text{length of lot} = 2900\text{ft}^2$$

$$w(w+8) = 2900\text{ft}^2$$

$$w^2 + 8w = 2900$$

$$w^2 + 8w - 2900 = 0$$

$$w = 50 \text{ or } w = -58$$

Since the width of the lot must be a positive numbers, we conclude that $w = 50\text{ft}$.

And the length of the lot is $w + 8 = 58\text{ft}$.

38. A man who is 6ft tall wishes to find the height of a certain four story building. He measures its shadow and find it to be 28ft long, while his own shadow is $3\frac{1}{2}$ ft long. How tall is building?

Solution:

Let h be the height of building. Then according to mathematical model

$$\frac{\text{height in large triangle}}{\text{base in large triangle}} = \frac{\text{height in small triangle}}{\text{base in small triangle}}$$

$\frac{h}{28} = \frac{6}{3.5} \Rightarrow h = 48\text{ft}$ So the building is 48ft tall.

39. A manufacturer of soft drinks advertises their orange soda as "naturally flavored" although it contains only 5% orange juice. A new federal regulation stipulates that to be called 'natural' a drink must contain at least 10% fruit juice. How much pure orange juice must this manufacturer add to 900 gal of orange soda to conform to the new regulation?

Solution:

Let x be the amount of orange juice to be added. Amount of the mixture is $(900 + x)$

Amount of orange juice in the first vat $= 0.05(900) = 45$

Amount of orange juice in the second vat $= 1.x = x$

Amount of orange juice in the mixture $= 0.10(900 + x)$

Then according to mathematical model

Amount of orange juice in the first vat + Amount of orange juice in the Second vat = Amount of orange juice in the mixture

$$45 + x = 0.10(900 + x)$$

$$45 + x = 90 + 0.1x \Rightarrow x = 50$$

The manufacturer should add 50 gal of pure orange juice to the soda.

40. Because of an anticipated heavy rainstorm, the water level in a reservoir must be lowered by 1ft. Opening spillway A lowers the level by this amount in 4 hours, whereas opening the smaller spillway B does the job in 6 hours. How long will it take to lower the water level by 1ft if both spillways are opened?

Solution:

Time it takes to lower level 1ft with A and B together $= \mathrm{x}$

Distance A lowers level in 1 hour $= \frac{1}{4}\mathrm{ft}$

Distance B lowers level in 1 hour $= \frac{1}{6}\mathrm{ft}$

Distance A and B together lowers level in 1 hour $= \frac{1}{\mathrm{x}}\mathrm{ft}$

Then according to mathematical model

Fraction done by A + Fraction done by B = Fraction done by both

$$\frac{1}{4}+\frac{1}{6}=\frac{1}{\mathrm{x}}$$

$$\mathrm{x}=\frac{12}{5}$$

It will takes $2\frac{2}{5}$ hours, or 2h 24 min, to lower the water level by 1ft if both spillways are open.

Introduction to Inequalities

Inequality is a fundamental concept in mathematics that deals with the relative size or magnitude of two or more quantities. It is a statement that compares two expressions, declaring one to be greater than, less than, or equal to the other. Inequalities are ubiquitous in real-life situations, from comparing prices and temperatures to modeling complex social and economic phenomena. Understanding inequalities is essential for solving problems in various fields, including mathematics, science, economics, and engineering.

Inequality / Inequation

An expression involving the signs $<, \leq, >, \geq, \neq$ is called inequality. It is a statement in which two algebraic expression are not equal.

For example: $\mathrm{x}<3, x \leq 5, x>-2, x \geq 17, x \neq 10$

Strict Inequality

An expression involving the signs $<, >$ is called strict inequality.

For example: $\mathrm{x}<3, x>-20$

Solution of an Inequality

Variable that make the inequality true is called solution of that inequality.

41. Solve $3x + 7 \geq 1$

Solution:

$$3x+7\geq 1 \Rightarrow 3x+7-7\geq 1-7 \Rightarrow 3x\geq -6 \Rightarrow \frac{3x}{3}\geq -\frac{6}{3} \Rightarrow x\geq -2$$

42. Solve $-5x - 8 \geq 2$

Solution:

$$-5\text{x}-8\geq 2 \Rightarrow -5\text{x}-8+8\geq 2+8 \Rightarrow -5\text{x}\geq 10 \Rightarrow \frac{-5\text{x}}{-5}\leq \frac{10}{-5} \Rightarrow \text{x}\leq -2$$

43. Solve $2x+3>7$ or $4x-1<3$

Solution:

$2x+3>7$ or $4x-1<3$

$2x>4$ or $4x<4$

$x>2$ or $x<1$

Solution Set = $\{\text{x} | \text{x} \in \text{R} \wedge \text{x} > 2 \ \ or \ x<1\}$

44. Solve 3x + 5 < x - 7

Solution:

$3x + 5 < x - 7$

$3x - x < -7 - 5$

$2x < -12 \Rightarrow x < -6$

The solution set is $\{x \in R : x < -6\} =]-\infty, -6[$

Linear inequalities with absolute values

Inequalities are fundamental concepts in mathematics, governing relationships between quantities, sizes and magnitudes. They permeate various branches of mathematics, including algebra, geometry and calculus, and have far-reaching implications in fields such as economics, social sciences and engineering. Understanding inequalities is crucial for solving problems, modeling real-world phenomena and making informed decisions, enabling us to analyze, optimize and address complex challenges in diverse disciplines.

Absolute Value Inequality

An inequality involving an absolute value expression, e.g., $|2x + 3| < 5$

Types of Absolute Value Inequalities

1. Simple Absolute Value Inequality: $|x| < a$, $|x| > a$, $|x| \leq a$, $|x| \geq a$
2. Compound Absolute Value Inequality: $a < |x| < b$, $a \leq |x| \leq b$

Rules for Solving Absolute Value Inequalities

1. Simple Absolute Value Inequalities:

- $|x| < a \Rightarrow -a < x < a$
- $|x| > a \Rightarrow x < -a$ or $x > a$
- $|x| \leq a \Rightarrow -a \leq x \leq a$
- $|x| \geq a \Rightarrow x \leq -a$ or $x \geq a$

2. Compound Absolute Value Inequalities:

- $a < |x| < b \Rightarrow -b < x < -a$ or $a < x < b$
- $a \le |x| \le b \Rightarrow -b \le x \le -a$ or $a \le x \le b$

3. Linear Absolute Value Inequalities:

- $|ax + b| < c \Rightarrow -(c/b) < x < (c-b)/a$
- $|ax + b| > c \Rightarrow x < -(c+b)/a$ or $x > (c-b)/a$

Graphical Representation

1. Simple Absolute Value Inequalities: Open/closed intervals on the number line.
2. Compound Absolute Value Inequalities: Union/intersection of intervals.

45. Solve $|2x + 3| < 5$

Solution:

$-2x - 3 < 5$ and $2x + 3 < 5$

$-2x < 8$ and $2x < 2$

$x > -4$ and $x < 1$

Interval: $(-4, 1)$

46. Solve $|x - 2| > 3$

Solution:

$x - 2 < -3$ or $x - 2 > 3$

$x < -1$ or $x > 5$

Interval: $(-\infty, -1) \cup (5, \infty)$

47. Solve $|3x + 2| \le 4$

Solution:

$-4 \le 3x + 2 \le 4$

$-6 \le 3x \le 2$

$-2 \le x \le 2/3$

Interval: $[-2, 2/3]$

48. Solve $|2x - 1| \ge 5$

Solution:

$2x - 1 \ge 5$ or $2x - 1 \le -5$

$2x \ge 6$ or $2x \le -4$

$x \ge 3$ or $x \le -2$

Interval: $(-\infty, -2] \cup [3, \infty)$

49. Solve $|x + 4| < 2$

Solution:

$-2 < x + 4 < 2$

$-6 < x < -2$

Interval: $(-6, -2)$

50. Solve |5x + 6| ≥ 5

Solution:

We first write equivalent inequalities by removing: he absolute value sign.

Now, $|5x+6| \geq 5$ if and only if

$5x+6 \geq 5$ or $-(5x+6) \geq 5$

If $5x+6 \geq 5$ then $x \geq -\frac{1}{5}$

If $-(5x+6) \geq 5$ then $-5x-6 \geq 5$ *or* $-5x \geq 11$

or $5x \leq -11$ *or* $x \leq -\frac{11}{5}$

The solution set is $\left\{x \in R : x \geq -\frac{1}{5} \text{ or } x \leq \frac{-11}{5}\right\} =]-\infty, \frac{-11}{5}] \cup [\frac{-1}{5}, \infty[.$

Alternative Method

The associated equation is $|5x+6| = 5$. Therefore, $5x+6 = \pm 5$

i.e., $x = -\frac{1}{5}, \frac{-11}{5}$ are the boundary numbers. we locate them on the real line so that it is divided into regions A, B, C as shown.

Region A. test $x = -3$: $|-15+6| \geq 5$ *True*

Region B. test $x = -1$: $|-5+6| \geq 5$ *False*

Region C. test $x = 0$: $|6| \geq 5$ *True*

The solution set $= \left\{x \in R : x \leq \frac{-11}{5} \text{ or } x \geq \frac{-1}{5}\right\} =]-\infty, \frac{-11}{5}] \cup [\frac{-1}{5}, \infty[.$

51. Solve $x^2 - 5x + 6 < 0$

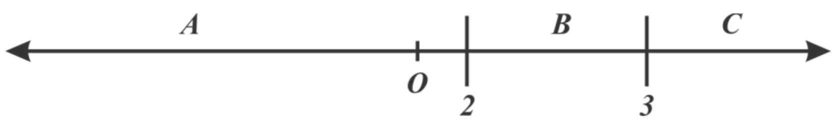
Solution:

The associated equation

$x^2 - 5x + 6 = 0$

$x = 2$ *and* 3

Which are the boundary numbers for the given inequality. The real line is divided the boundary numbers into regions as shown



Region A. test $x=0$: $6<0$ *False*

Region B. test $x=\frac{5}{2}:\left(\frac{5}{2}\right)^2-\frac{25}{2}+6<0$ *True*

Region C. test $x=4$: $16-20+6<0$ *False*

Thus only region B is in the solution set.

Therefore, the solution set is $\{x\in R:2<x<3\}=\ ]2,3[$

52. Solve $x^2-2x+2>0$

Solution:

The associated equation is $x^2-2x+2=0$

$$x=\frac{2\pm\sqrt{4-8}}{2}=1\pm i$$

Which are complex number and cannot be represented on the real line. Thus there are no boundary numbers. We have only one region, the entire real line.

Region. Entire real line, test $x=1:1^2-2+2>0$ True

Thus the set of all real numbers is the solution set.

We note that the discriminate $4-8$ *of* x^2-2x+2 is negative and the expression is positive for all real numbers. The converse is also true i.e., if a quadratic expression is to be positive for all real numbers, then its discriminate must be negative.

53. Solve $\frac{x^2-2}{1-2x}>1$

Solution: The associated equation is $\frac{x^2-2}{1-2x}=1$

i.e., $x^2-2=1-2x$

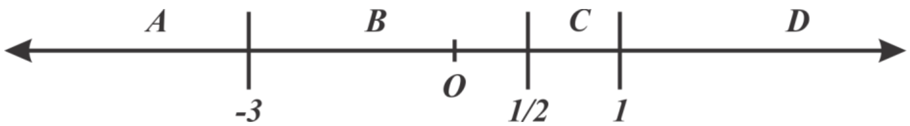
or $x^2+2x-3=0$

and so $x=-3,1$

Are the boundary numbers for the inequality

The denominator of $\frac{x^2-2}{1-2x}$ is zero for $x=\frac{1}{2}$. This is a free boundary numbers for the given inequality and cannot be in the solution set.

The real line is divided by the boundary numbers and the free boundary number into regions as shown



Region A. test $x=-4$: $\frac{16-2}{1+8}>1$ True

Region B. test $x=-1$: $\frac{1-2}{1+2}>1$ False

Region C. test $x=\frac{3}{4}$: $\frac{\frac{9}{16}-2}{1-\frac{3}{2}}>1$ True

Region D. test $x=2$: $\frac{4-2}{1-4}>1$ False

The solution set is $\{x\in R: x<-3\}\cup\left\{x\in R:\frac{1}{2}<x<1\right\}$ $=]-\infty,-3[\cup]\frac{1}{2},1[$

Linear Programming: Programming that deals with the optimization (maximization or minimization) of the function is called linear programming.

Boundary of Half Plane: $\mathrm{ax}+\mathrm{by}<c$ is called half plane region and line $\mathrm{ax}+\mathrm{by}=\mathrm{c}$ is called boundary of half plane.

Left and Right Half Plane: Vertical line divides the plane into left and right half plane.

Upper and Lower Half Plane: Non – Vertical line divides the plane into lower and upper half plane.

Vertex or Corner Point: A point of a solution region where two of its boundary lines interest is called vertex.

Non-Negative Constraints: The variable used in system of linear inequalities relating to problem of every day life is non-negative and are called non-negative constraints

Decision Variables: The non – negative constraints play an important rule for taking decision, so these variables are called decision variables.

Solution Region: We draw graph of each inequality in the system of the same coordinate axes and then take intersection of the graph. The common region so obtained is called the solution region.

Feasible Region: A region which is restricted to first quadrant is called feasible region.

Feasible Solution: Each point of feasible region is called feasible solution.

Optimal Solution: The feasible solution which maximizes or minimizes the objective function is called the optimal solution.

Objective Function: A function which is to be maximized or minimized is called an objective function.

Problem Constraints: The systems of linear inequalities involved in the problem concerned are called problem constraints.

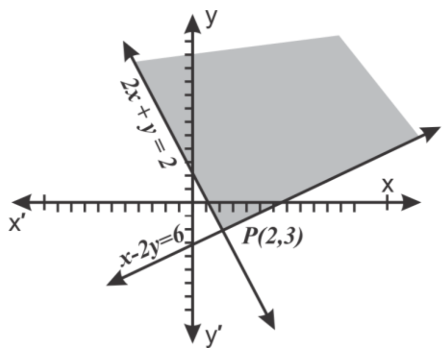
Convex: If the line segment obtained by joining any two points of a region lies entirely within the region then the region is called convex.

54. Graph the system of linear inequalities $x-y\le 6$; $2x+y\ge 2$

Solution:

$x-y\le 6$ ....(i)

$2x+y\ge 2$ ....(ii)



Associated equations

$x-y=6$ ....(iii)

$2x+y=2$ ....(iv)

To find Points

$(iii)\Rightarrow$ Put $x=0, y=-3$ then point is $(0,-3)$

$(iii)\Rightarrow$ Put $y=0, x=6$ then point is $(6,0)$

$(iv)\Rightarrow$ Put $x=0, y=2$ then point is $(0,2)$

$(iv)\Rightarrow$ Put $y=0, x=1$ then point is $(1,0)$

To check Region put $(0,0)$ **in** $(\mathbf{i})$ **and** $(\mathbf{ii})$

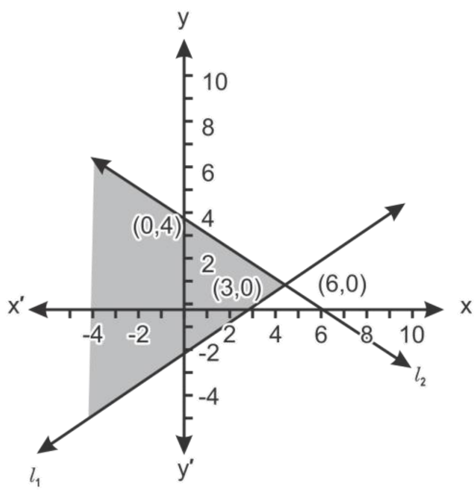
$(i)\Rightarrow 0<6$ true $(ii)\Rightarrow 0>2$ false

55. Indicate the solution of linear inequalities by shading $2x-3y\le 6$; $2x+3y\le 12$

Solution:

$2x-3y\le 6$ ....(i)

$2x+3y\le 12$ ....(ii)



Associated equations

$2x-3y=6$ ....(iii)

$2x+3y=12$ ....(iv)

To find Points

$(iii)\Rightarrow$Put $x=0, y=-2$ then point is $(0,-2)$

$(iii)\Rightarrow$Put $y=0, x=3$ then point is $(3,0)$

$(iv)\Rightarrow$Put $x=0, y=4$ then point is $(0,4)$

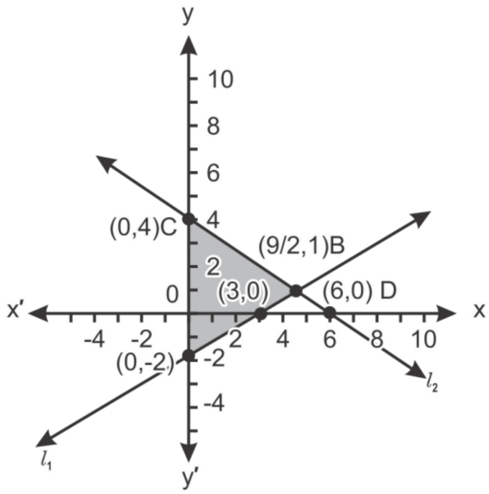
$(iv)\Rightarrow$ Put $y=0, x=6$ then point is $(6,0)$

To check Region put $(0,0)$ **in** $(\mathbf{i})$ **and** $(\mathbf{ii})$

$(i)\Rightarrow 0<6$ true $(ii)\Rightarrow 0<12$ true

56. Graph the solution region also find the corner points in each case

$2x-3y\le 6$; $2x+3y\le 12$



Solution:

$2x-3y\le 6$ ....(i)

$2x+3y\le 12$ ....(ii)

Associated equations

$2x-3y=6$ ....(iii)

$2x+3y=12$ ....(iv)

To find Points

$(iii)\Rightarrow$ Put $x=0, y=-2$ then point is $(0,-2)$

$(iii)\Rightarrow$ Put $y=0, x=3$ then point is $(3,0)$

$(iv)\Rightarrow$ Put $x=0, y=4$ then point is $(0,4)$

$(iv)\Rightarrow$ Put $y=0, x=6$ then point is $(6,0)$

To check Region put $(0,0)$ in $(\mathbf{i})$ and $(\mathbf{ii})$

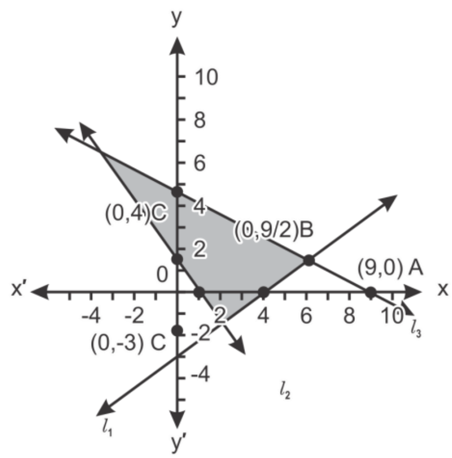
$(i)\Rightarrow 0<6$ true and $(ii)\Rightarrow 0<12$ true

Adding (iii) and (iv) we have $x=\dfrac{9}{2}$ and putting $x=\dfrac{9}{2}$ in (iii) we have $y=1$

Corner Points: $A(0,-2), B\left(\dfrac{9}{2},1\right), C(0,4)$

57. Graph the solution region of the following system of linear inequalities by shading

$3x-4y\le 12$; $3x+2y\ge 3$; $x+2y\le 9$



Solution:

$3x-4y\le 12$ ....(i)

$3x+2y\ge 3$ ....(ii)

$x+2y\le 9$ ....(iii)

Associated equations

$3x-4y=12$ ....(iv)

$3x+2y=3$ ....(v)

$x+2y=9$ ....(vi)

To find Points

$(iv)\Rightarrow$ Put $x=0, y=-3$ then point is $(0,-3)$

$(iv)\Rightarrow$ Put $y=0, x=4$ then point is $(4,0)$

$(v) \Rightarrow$ Put $x=0, y=\frac{3}{2}$ then point is $\left(0,\frac{3}{2}\right)$

$(v) \Rightarrow$ Put $y=0, x=1$ then point is $(1,0)$

$(vi) \Rightarrow$ Put $x=0, y=\frac{9}{2}$ then point is $\left(0,\frac{9}{2}\right)$

$(vi) \Rightarrow$ Put $y=0, x=9$ then point is $(9,0)$

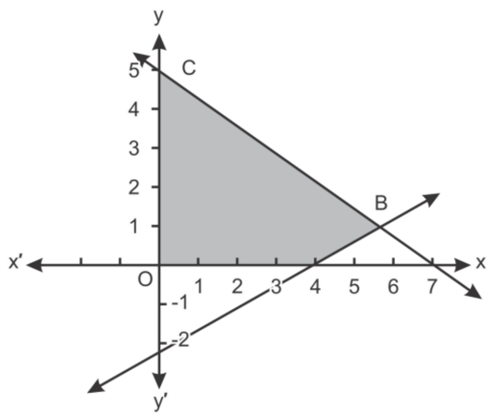
To check Region put $(0,0)$ in $(\mathbf{i}),(\mathbf{ii})$ and $(\mathbf{iii})$

$(i) \Rightarrow 0<12$ true, $(ii) \Rightarrow 0>3$ false and $(iii) \Rightarrow 0<9$ true

58. Graph the feasible region also find corner points

$5x+7y \le 35$; $x-2y \le 4$; $x \ge 0, y \ge 0$

Solution:



$5x+7y \le 35$ ....(i)

$x-2y \le 4$ ....(ii)

Associated equations

$5x+7y=35$ ....(iii)

$x-2y=4$ ....(iv)

To find Points

$(iii) \Rightarrow$ Put $x=0, y=5$ then point is $(0,5)$

$(iii) \Rightarrow$ Put $y=0, x=7$ then point is $(7,0)$

$(iv) \Rightarrow$ Put $x=0, y=-2$ then point is $(0,-2)$

$(iv) \Rightarrow$ Put $y=0, x=4$ then point is $(4,0)$

To check Region put $(0,0)$ in $(\mathbf{i})$ and $(\mathbf{ii})$

$(i) \Rightarrow 0<35$ true and $(ii) \Rightarrow 0<4$ true

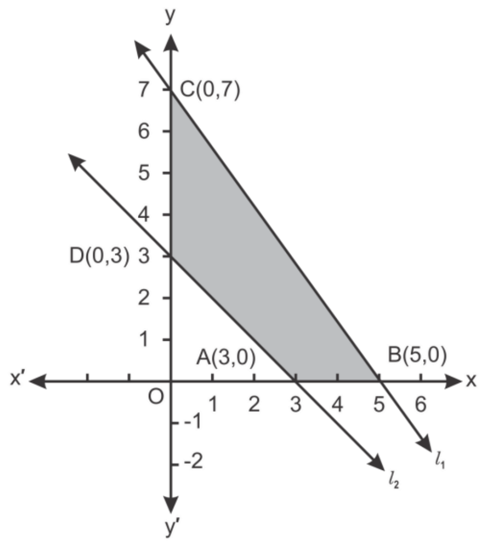
Multiplying (iv) with 5 and Subtracting from (iii)

We have $y=\frac{15}{17}$ and putting $y=\frac{15}{17}$ in (iv) we have $x=\frac{98}{17}$

Corner Points: $O(0,0), A(4,0), B\left(\frac{98}{17},\frac{15}{17}\right), C(0,5)$

59. Maximize as well as minimize $z = 2x + y$ for given linear inequalities

$x + y \geq 3$; $7x + 5y \leq 35$; $x \geq 0, y \geq 0$



Solution:

$x + y \geq 3$ ....(i)

$7x + 5y \leq 35$ ....(ii)

Associated equations

$x + y = 3$ ....(iii)

$7x + 5y = 35$ ....(iv)

To find Points

$(iii) \Rightarrow$ Put $x = 0, y = 3$ then point is $(0,3)$

$(iii) \Rightarrow$ Put $y = 0, x = 3$ then point is $(3,0)$

$(iv) \Rightarrow$ Put $x = 0, y = 7$ then point is $(0,7)$

$(iv) \Rightarrow$ Put $y = 0, x = 5$ then point is $(5,0)$

To check Region put $(0,0)$ in $(\mathbf{i})$ and $(\mathbf{ii})$

$(i) \Rightarrow 0 > 3$ false and $(ii) \Rightarrow 0 < 35$ true

Corner Points: $A(3,0), B(5,0), C(0,7), D(0,3)$

At A: $z = f(3,0) = 2(3) + 0 = 6$

At B: $z = f(5,0) = 2(5) + 0 = 10$

At C: $z = f(0,7) = 2(0) + 7 = 7$

At D: $z = f(0,3) = 2(0) + 3 = 3$

$z = 2x + y$ is maximum at $(5,0)$

$z = 2x + y$ is minimum at $(0,3)$

60. A machine can produce product A by using 2 units of chemical and 1 unit of a compound or can produce product B by using 1 unit of chemical and 2 units of the compound. Only 800 units of chemical and 1000 units of the compound are available. The profits per unit of A and B are Rs. 30 and Rs. 20 respectively, maximize the profit function.

Solution:

Suppose units of A are x and units of B are y then $f(x, y) = 2x + y$ then by condition

$2x + y \leq 800$ ....(i)

$x + 2y \leq 1000$ ....(ii)

Associated equations

$2x + y = 800 \quad ....(iii)$

$x + 2y = 1000 \quad ....(iv)$

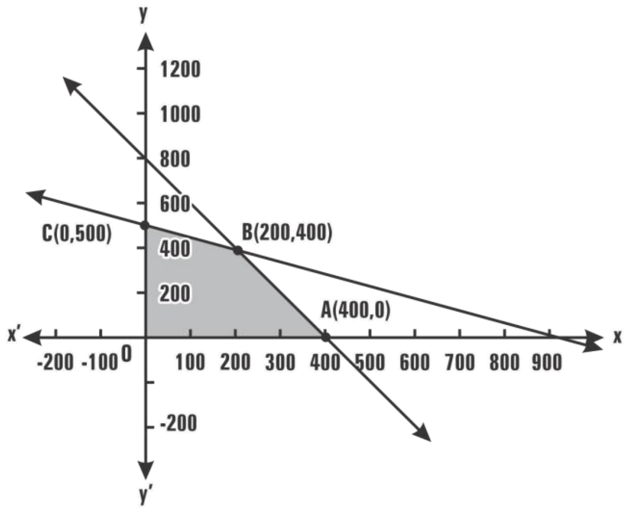
To find Points

$(iii) \Rightarrow$ Put $x = 0, y = 800$ then point is $(0,800)$

$(iii) \Rightarrow$ Put $y = 0, x = 400$ then point is $(400,0)$

$(iv) \Rightarrow$ Put $x = 0, y = 500$ then point is $(0,500)$

$(iv) \Rightarrow$ Put $y = 0, x = 1000$ then point is $(1000,0)$



To check Region put $(0,0)$ **in** $(\mathbf{i})$ **and** $(\mathbf{ii})$

$(i) \Rightarrow 0 < 800$ true and $(ii) \Rightarrow 0 < 1000$ true

Multiplying (iii) with 2 and subtracting (iii) from (iv)

We have $x = 200$ and putting $x = 200$ in (iv) we have $y = 400$

Corner Points: $O(0,0), A(400,0), B(200,400), C(0,500)$

At O: $f(0,0) = 30(0) + 20(0) = 0$

At A: $f(400,0) = 30(400) + 20(0) = 12000$

At B: $f(200,400) = 30(200) + 20(400) = 14000$

At C: $f(0,500) = 30(0) + 20(500) = 1000$

So profit is maximum at corner $B(200,400)$.

Exercise

1) Solve the following linear equations in one variable.

i. $5x-2-x=4-3x-27$

ii. $7(2-5x)+27=18x-3(8-4x)$

iii. $\frac{5x}{4}+\frac{1}{2}=0$

iii. $\frac{x}{2}+\frac{x}{3}-\frac{x}{4}+\frac{x}{5}=7\frac{5}{6}$

v. $\frac{y+1}{3}+\frac{y+1}{2}=2-\frac{y+3}{2}$

vi. $0.5x=6.3-0.2x$

2) Solve the following system of equations

i. $x+y=5;\ x^2+2y^2=17$

ii. $3x+2y=7;\ 3x^2=25+2y^2$

iii. $x+y=5;\ \frac{2}{x}+\frac{3}{y}=2,\ x\neq 0, y\neq 0$

iv. $x+y=a+b;\ \frac{a}{x}+\frac{b}{y}=2$

v. $3x+4y=25;\ \frac{3}{x}+\frac{4}{y}=2$

vi. $(x-3)^2+y^2=5; 2x=y+6$

vii. $(x+3)^2+(y-1)^2=5\ ;\ x^2+y^2+2x=9$

viii. $x^2+(y+1)^2=18\ ;\ (x+2)^2+y^2=21$

3) Solve the following radical equations.

i. $\sqrt{2x}=4$

ii. $\sqrt{x-3}=2$

iii. $\sqrt{5x-4}=14$

iv. $5-\sqrt{2x-1}=0$

v. $\sqrt{9-2x}=\sqrt{5x-12}$

vi. $3x^2+2x-\sqrt{3x^2+2x-1}=3$

vii. $x^2-\frac{x}{2}-7=x-3\sqrt{2x^2-3x+2}$

viii. $\sqrt{2x+8}+\sqrt{x+5}=7$

ix. $\sqrt{3x+4}=2+\sqrt{2x-4}$

x. $\sqrt{x+7}+\sqrt{x+2}=\sqrt{6x+13}$

xi. $\sqrt{x^2+x+1}-\sqrt{x^2+x-1}=1$

xii. $\sqrt{x^2+2x-3}+\sqrt{x^2+7x-8}=\sqrt{5(x^2+3x-4)}$

xiii. $\sqrt{2x^2-5x-3}+3\sqrt{2x+1}=\sqrt{2x^2+25x+12}$

xiv. $\sqrt{3x^2-5x+2}+\sqrt{6x^2-11x+5}=\sqrt{5x^2-9x+4}$

xv. $(x+4)(x+1)=\sqrt{x^2+2x-15}+3x+31$

xvi. $\sqrt{3x^2-2x+9}+\sqrt{3x^2-2x-4}=13$

xvii. $\sqrt{5x^2+7x+2}-\sqrt{4x^2+7x+18}=x-4$

4) Solve the following absolute value equations.

i. $\frac{|10-x|}{5}=\frac{|2x-5|}{2}; x\in R$ ii. $|x+2|=6$

iii. $|5x|+10=5$ iv. $\frac{|1-2y|}{4}=3$

v. $|z+3|-3=5-|z+3|$

5) Solve the following inequalities.

i. $2\le x\le 5$ ii. $-4<x<-\frac{3}{2}$ iii. $x\le 4; x\in W$

iv. $3x+21<1-x$ or $3x+8<3-2x$ v. $1-5x>16$ and $3-\frac{3x}{2}\le 9$

6) Consider the following system of linear equations:

$2x_1-4x_2-x_3=1$

$x_1-3x_2+x_3=1$

$3x_1-5x_2-3x_3=1$

Determine whether given 3 – tuples are solutions of the system?

(a) $(3,1,1)$

(b) $(3,-1,1)$

(c) $(13,5,2)$

(d) $\left(\frac{13}{2},\frac{5}{2},2\right)$

(e) $(17,7,5)$

7) Consider the following system of linear equations:

$x+2y-2z=3$, $3x-y+z=1$, $-x+5y-5z=5$

Determine whether given 3 – tuples are solutions of the system?

a) $\left(\frac{5}{7},\frac{8}{7},1\right)$

b) $\left(\frac{5}{7},\frac{8}{7},0\right)$

c) $(5,8,1)$

d) $\left(\frac{5}{7},\frac{10}{7},\frac{2}{7}\right)$

e) $\left(\frac{5}{7},\frac{22}{7},2\right)$

8) In each part, solve the linear system, if possible, and use the result to determine whether the lines represented by the equations in the system have zero, one, or infinitely many points of intersection. If there is a single point of intersection, give its coordinates, and if there are infinitely many, find parametric equations for them.

a) $3x-2y=4$ and $6x-4y=9$

b) $2x-4y=1$ and $4x-8y=2$

c) $x-2y=0$ and $x-4y=8$

9) In each part use parametric equations to describe solution set of linear equations

a) $7x-5y=3$

b) $x+10y=2$

c) $3x_1-5x_2+4x_3=7$

d) $-8x_1+2x_2-5x_3+6x_4=1$

e) $3v-8w+2x-y+4z=0$

f) $4x_1+2x_2+3x_3+x_4=20$

g) $x_1+3x_2-12x_3=3$

h) $v+w+x-5y+7z=0$

10) In each part use parametric equations to describe the infinitely many solutions of linear equations.

a) $2x-3y=1$ and $6x-9y=3$

b) $6x_1+2x_2=-8$ and $3x_1+x_2=-4$

c) $x_1+3x_2-x_3=-4$, $3x_1+9x_2-3x_3=-12$ and $-x_1-3x_2+x_3=4$

d) $2x-y+2z=-4$, $6x-3y+6z=-12$ and $-4x+2y-4z=8$ 3

11) Solve |3x - 2| > 7

12) Solve |2x + 5| ≤ 3

13) Solve |x - 3| ≥ 2

14) Solve |4x - 1| < 9

15) Solve |2x - 3| ≤ 6

16) Let $\delta > 0$ and $a \in R$. Show that $a-\delta < x < a+\delta$ if and only if $|x-a| < \delta$

17) Solve each of the following inequalities:

i. $|2x+5| > |2-5x|$ ii. $\left|\frac{x+8}{12}\right| < \frac{x-1}{10}$

iii. $|x|+|x-1| > 1$ iv. $12x^2-25x+12 > 0$

v. $\frac{x-1}{2}-\frac{1}{x} > \frac{4}{x}+5$ vi. $|x^2-x+1| > 1$

vii. $x^{-2}-4x^{-1}+4 > 0$ viii. $\frac{2x}{x+2} \geq \frac{x}{x-2}$

ix. $x^4-5x^3-4x^2+20x \leq 0.$

18) Graph the solution region of the following system of linear inequalities by shading also find the corner points in each case.

a) $x+y\le 5$; $-2x+y\le 2$; $y\ge 0$

b) $5x+7y\le 35$; $x-2y\le 4$; $x\ge 0$

c) $3x+2y\ge 6$; $x+3y\le 6$; $y\ge 0$

d) $2x+y\le 4$; $2x-3y\ge 12$; $x+2y\le 6$

e) $3x-2y\ge 3$; $x+4y\le 12$; $3x+y\le 12$

19) Graph the feasible region of the following system of linear inequalities by shading also find the corner points in each case.

a) $x+y\le 5$; $-2x+y\le 2$; $x\ge 0, y\ge 0$

b) $2x-3y\le 6$; $2x+3y\le 12$; $x\ge 0, y\ge 0$

c) $3x+2y\ge 6$; $x+y\le 4$; $x\ge 0, y\ge 0$

d) $2x+y\le 20$; $8x+15y\ge 120$; $x+y\le 11$; $x\ge 0, y\ge 0$

e) $x+2y\le 14$; $3x+4y\le 36$; $2x+y\le 10$; $x\ge 0, y\ge 0$

20) Maximize $z=2x+5y$ for given linear inequalities

$2y-x\le 8$; $x-y\le 4$; $x\ge 0, y\ge 0$

21) Maximize $z=2x+3y$ for given linear inequalities

$2x+y\le 8$; $x+2y\le 14$; $x\ge 0, y\ge 0$

22) Each unit of food X costs Rs. 25 and contains 2 units of protein and 4 units of iron while each unit of food Y costs Rs. 30 and contains 3 units of protein and 2 unit of iron. Each animal must receive at least 12 units of protein and 16 units of iron each day. How many units of each food should be fed to each animal at the smallest possible cost?

23) A dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5760 to invest and has space at most for 20 items. A fan costs him Rs. 360 and a sewing machine costs Rs. 240. His expectation is that the can sell a fan at a profit of Rs. 22 and a sewing machine at a profit of Rs. 18. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximize his profit?