

# LOGIC AND ITS APPLICATIONS

## CHAPTER

# 2

In today's complex world, it is not so easy to summarize the topic of logic. For lawyers and business people, logic is the science of correct reasoning. They often use logic to construct valid arguments, analyze legal contracts, and solve complicated problems. The principles of logic can also be used as a production tool. For example, programmers use logic to design computer software, engineers use logic to design the electronic circuits in computers, and mathematicians use logic to solve problems and construct mathematical proofs. The rules of logic give precise meaning to mathematical statements. These rules are used to distinguish between valid and invalid mathematical arguments. Because a major goal of this chapter is to teach the reader how to understand and how to construct correct mathematical arguments, we begin our study of discrete mathematics with an introduction to logic. Besides the importance of logic in understanding mathematical reasoning, logic has numerous applications to computer science. These rules are used in the design of computer circuits, the construction of computer programs, the verification of the correctness of programs, and in many other ways. Furthermore, software systems have been developed for constructing some, but not all, types of proofs automatically. We will discuss these applications of logic in this and later chapters.

In this chapter we will learn about;

- Logical reasoning, importance and their application in modern age
- Propositions, prepositional variables
- Induction, deduction, logical connectives
- Compound statements, truth values, truth tables and applications
- Negation, conjunction, disjunction
- Tautologies, contradiction and contingencies, equivalent statements
- The conditional and related statements
- Equivalent statements and logical equivalence
- Truth sets, a link between set theory and logic
- Arguments and their validity
- Categorical prepositions and Venn diagram
- Categorical propositions and Euler diagrams
- Quantifiers and predicates
- Logical fallacies and its types
- Exercises about logic and its applications

## Logic

Logic is the discipline that deals with the method of reasoning. In this chapter, we will encounter several facets of logic. Specifically, we will use logic to

- Analyze information and relationships between statements.
- Determine the validity of arguments and theorems.
- Determine valid conclusions based on given assumptions, and
- Analyze electronic circuits.
- In computer science to verify the correctness of programs.
- In physical science to draw conclusion from experiments.

**Propositions/Statements:** A proposition/statement is a declarative sentence that is either true or false, but not both simultaneously.

It may not be necessary to determine whether a sentence is true or false to determine whether it is a statement. For instance, the following sentence is either true or false:

*Every even number greater than 2 can be written as the sum of two prime numbers.*

At this time mathematicians have not determined whether the sentence is true or false, but they do know that it is either true or false and that it is not both true and false. Thus the sentence is a statement.

**Arguments:** Arguments are a set of statements.

**Premises:** The premises are the statements being offered in support for the conclusion.

**Conclusion:** The statements being argued for.

**Proportional Variables:** The letters ( $p, q, r, \dots$ ) that can be replaced by statements are called proportional variables. e.g. p: Today is Friday, q: It is raining, r: I am going to a movie.

### 1. Determine whether each sentence is a statement.

- a. Florida is a state in the United States.    b. The word dog has four letters.
- c. How are you?    d.  $9^{(9^9)} + 2$  is a prime number.
- e.  $x + 1 = 5$

#### Solution:

- a. Florida is one of the 50 states in the United States, so this sentence is true and it is **a statement**.
- b. The word dog consists of exactly three letters, so this sentence is false and it is **a statement**.
- c. The sentence "How are you?" is a question; it is not a declarative sentence. Thus it is **not a statement**.
- d. You may not know whether  $9^{(9^9)} + 2$  is a prime number; however, you do know that it is a whole number larger than 1, so it is either a prime number or it is not a prime number. The sentence is either true or false, and it is not both true and false simultaneously, so it is **a statement**.

- e.  $x + 1 = 5$  is a statement. It is known as **an open statement**. It is true for  $x = 4$  and it is false for any other value of  $x$ . For any given value of  $x$ , it is true or false but not both.

**2. Determine whether each sentence is a statement.**

- Open the door.**
- 7055 is a large number.**
- $4 + 5 = 8$
- In the year 2019, the president of the United States will be a woman.**
- $x > 3$

**Solution:**

- The sentence "Open the door" is a command. It is not a statement.
- The word large is not a precise term. It is not possible to determine whether the sentence "7055 is a large number" is true or false and thus the sentence is not a statement.
- The sentence  $4 + 5 = 8$  is a false statement.
- At this time we do not know whether the given sentence is true or false, but we know that the sentence is either true or false and that it is not both true and false. Thus the sentence is a statement.
- The sentence  $x > 3$  is a statement because for any given value of  $x$ , the inequality  $x > 3$  is true or false, but not both.

**Induction**

To draw general conclusion from limited number of observations or experiences is called induction.

e.g. A person gets penicillin injection once or twice and experiences reaction soon afterwards. He generalizes that he is allergic to penicillin.

**Deduction**

To draw general conclusion from well known facts is called deduction.

e.g. All men are mortal. We are men. Therefore, we are all mortal.

**Logical Connectives**

Symbols that are used to combine statements or propositional variables are called logical connectives. George Boole used symbols such as  $p$ ,  $q$ ,  $r$ , and  $s$  to represent statements and the symbols  $\wedge$ ,  $\vee$ ,  $\sim$ ,  $\rightarrow$  and  $\leftrightarrow$  to represent connectives. See Table,

Original Statement	Connective	Statement in Symbolic Form	Type of Compound Statement
not $p$	not	$\sim p$	negation
$p$ and $q$	and	$p \wedge q$	conjunction
$p$ or $q$	or	$p \vee q$	disjunction
if $p$ , then $q$	If...then	$p \rightarrow q$	conditional
$p$ if and only if $q$	if and only if	$p \leftrightarrow q$	biconditional

### Compound Statements

Two or more sentences are connected to form a compound statement. Connecting statements with words and phrases such as and, or, not, if ... then, and if and only if creates a **compound statement**. For instance, "I will attend the meeting or I will go to school" is a **compound statement**. It is composed of the two component statements "I will attend the meeting" and "I will go to school." The word **or** is a **connective** for the two component statements.

### Truth Value and Truth Tables

In logic and mathematics, a **truth value** is a value that indicates whether a statement is true or false. The **truth value** of a statement is true (T) if the statement is true and false (F) if the statement is false. A **truth table** is a table that shows the truth values of a statement for all possible truth values of its components.

### Negation

The negation of the statement "Today is Friday" is the statement "Today is not Friday." In symbolic logic, the tilde symbol  $\sim$  is used to denote the negation of a statement. If a statement  $p$  is true, its negation  $\sim p$  is false, and if a statement  $p$  is false, its negation  $\sim p$  is true. See the table. The negation of the negation of a statement is the original statement. Thus,  $\sim p$  can be replaced by  $p$  in any statement.

**The Truth Table for  $\sim p$**

$p$	$\sim p$
T	F
F	T

### 3. Write the negation of each statement.

- Bill Gates has a yacht.
- The number 10 is a prime number.
- The Dolphins lost the game.

**Solution:**

- Bill Gates does not have a yacht.
- The number 10 is not a prime number.
- The Dolphins did not lose the game.

### 4. Write the negation of each statement.

- 1001 is divisible by 7.
- 5 is an even number.
- The fire engine is not red.

**Solution:**

- 1001 is not divisible by 7.
- 5 is not an even number.
- The fire engine is red.

5. Write the negation of each of the following statements.
- Some baseball players are worth a million dollars.
  - All movies are worth the price of admission.
  - No odd numbers are divisible by 2.

**Solution:**

- No baseball player is worth a million dollars.
  - Some movies are not worth the price of admission.
  - Some odd numbers are divisible by 2.
6. Write the negations of the following statements.
- All bears are brown.
  - No math class is fun.
  - Some vegetables are not green.

**Solution:**

- Some bears are not brown.
- Some math classes are fun.
- All vegetables are green.

**Remark**

The notation for the negation operator is not standardized. Although  $\sim p$  and  $\bar{p}$  are the most common notations used in mathematics to express the negation of  $p$ , other notations you might see are  $\neg p$ ,  $-p$ ,  $p'$ ,  $\bar{N}p$ , and  $!p$ .

7. Find the negation of the proposition "Michael's PC runs Linux" and express this in simple English.

**Solution:**

The negation is "It is not the case that Michael's PC runs Linux."  
This negation can be more simply expressed as  
"Michael's PC does not run Linux."

8. Find the negation of the proposition "Vandana's smartphone has at least 32 GB of memory" and express this in simple English.

**Solution:**

The negation is "It is not the case that Vandana's smartphone has at least 32 GB of memory." This negation can also be expressed as "Vandana's smartphone does not have at least 32 GB of memory" or even more simply as "Vandana's smartphone has less than 32 GB of memory."

**9. Consider the following statements.**

- p:** Today is Friday.  
**q:** It is raining.  
**r:** I am going to a movie.  
**s:** I am not going to the basketball game.

Write the following compound statements in symbolic form.

- a.** Today is Friday and it is raining.  
**b.** It is not raining and I am going to a movie.  
**c.** I am going to the basketball game or I am going to a movie.  
**d.** If it is raining, then I am not going to the basketball game.

**Solution:**

- a.**  $p \wedge q$                       **b.**  $\sim q \wedge r$                       **c.**  $\sim s \vee r$                       **d.**  $q \rightarrow s$

**10. Consider the following statements.**

- p:** Today is Friday.  
**q:** It is raining.  
**r:** I am going to a movie.  
**s:** I am not going to the basketball game.

Write the following compound statements in symbolic form.

- a.** Today is not Friday and I am going to a movie.  
**b.** I am going to the basketball game and I am not going to a movie.  
**c.** I am going to a movie if and only if it is raining.  
**d.** If today is Friday, then I am not going to a movie.

**Solution:**

- a.**  $\sim p \wedge r$       **b.**  $\sim s \wedge \sim r$       **c.**  $r \leftrightarrow q$       **d.**  $p \rightarrow r$

**11. Consider the following statements.**

- p:** The game will be played in Atlanta.  
**q:** The game will be shown on CBS.  
**r:** The game will not be shown on ESPN.  
**s:** The Dodgers are favored to win.

Write each of the following symbolic statements in words.

- a.**  $p \wedge q$       **b.**  $\sim r \wedge s$       **c.**  $s \leftrightarrow p$

**Solution:**

- a.** The game will be shown on CBS and the game will be played in Atlanta.  
**b.** The game will be shown on ESPN and the Dodgers are favored to win.  
**c.** The Dodgers are favored to win if and only if the game will not be played in Atlanta.

**Conjunction/ The logic of AND**

The conjunction of two statements  $p$  and  $q$  is true if and only if both  $p$  and  $q$  are true otherwise false. It is denoted by  $p \wedge q$  and read as  $p$  and  $q$ .

$p$	$q$	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

**Disjunction/ The logic of OR**

The disjunction of two statements  $p$  and  $q$  is false if and only if both  $p$  and  $q$  are false otherwise true. It is denoted by  $p \vee q$  and read as  $p$  or  $q$ . The disjunction  $p \vee q$  is true if  $p$  is true, if  $q$  is true, or if both  $p$  and  $q$  are true.

$p$	$q$	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

**12. Evaluate the truth value of the following two statements.**

- a. The capital of France is Paris and Antarctica is cold.
- b. The capital of France is Paris and the capital of America is Madrid.

**Solution:**

- a. The statement contains two distinct propositions: The capital of France is Paris and Antarctica is cold. Because both propositions are true, their conjunction is also true.
- b. The statement contains two distinct propositions: The capital of France is Paris and the capital of America is Madrid. Although the first proposition is true, the second is false. Therefore, their conjunction is false.

**13. Consider the statement airplanes can fly or cows can read. Is it true?**

**Solution:**

The statement is a disjunction of two propositions: (1) airplanes can fly; (2) cows can read. The first proposition is clearly true, while the second is clearly false, which makes the disjunction  $p$  or  $q$  true. That is, the statement airplanes can fly or cows can read is true.

**14. Determine whether each statement is true or false.**

- a.  $7 \geq 5$
- b. 5 is a whole number and 5 is an even number.
- c. 2 is a prime number and 2 is an even number.

**Solution:**

- a.  $7 \geq 5$  means  $7 > 5$  or  $7 = 5$ . Because  $7 > 5$  is true, the statement  $7 \geq 5$  is a true statement.
- b. This is a false statement because 5 is not an even number.
- c. This is a true statement because each component statement is true.

**15. Determine whether each statement is true or false.**

- a. **21 is a rational number and 21 is a natural number.**
- b.  $4 \leq 9$
- c.  $-7 \geq -3$

**Solution:**

- a. True. A conjunction is true provided both components are true.
- b. True. A disjunction is true provided at least one component is true.
- c. False. If both components of a disjunction are false, then the disjunction is false.

**16. Find the conjunction of the propositions p and q where p is the proposition "Rebecca's PC has more than 16 GB free hard disk space" and q is the proposition "The processor in Rebecca's PC runs faster than 1 GHz."**

**Solution:**

The conjunction of these propositions,  $p \wedge q$ , is the proposition "Rebecca's PC has more than 16 GB free hard disk space, and the processor in Rebecca's PC runs faster than 1 GHz." This conjunction can be expressed more simply as "Rebecca's PC has more than 16 GB free hard disk space, and its processor runs faster than 1 GHz." For this conjunction to be true, both conditions given must be true. It is false when one or both of these conditions are false.

**17. Translate the statement "Students who have taken calculus or introductory computer science can take this class" in a statement in propositional logic using the propositions p: "A student who has taken calculus can take this class" and q: "A student who has taken introductory computer science can take this class."**

**Solution:**

We assume that this statement means that students who have taken both calculus and introductory computer science can take the class, as well as the students who have taken only one of the two subjects. Hence, this statement can be expressed as  $p \vee q$ , the inclusive or, or disjunction, of p and q.

**18. What is the disjunction of the propositions p and q, where p is the proposition "Rebecca's PC has more than 16 GB free hard disk space" and q is the proposition "The processor in Rebecca's PC runs faster than 1 GHz."**

**Solution:**

The disjunction of p and q,  $p \vee q$ , is the proposition "Rebecca's PC has at least 16 GB free hard disk space, or the processor in Rebecca's PC runs faster than 1 GHz." This proposition is true when Rebecca's PC has at least 16 GB free hard disk space, when the PC's processor runs faster than 1 GHz, and when both conditions are true. It is false when both of these conditions are false, that is, when Rebecca's PC has less than 16 GB free hard disk space and the processor in her PC runs at 1 GHz or slower.

**Tautologies**

A tautology is a statement that is always true. For example  $p \rightarrow q \leftrightarrow \sim q \rightarrow \sim p$

**Contradiction / Self-Contradiction / Absurdity**

A self-contradiction is a statement that is always false. For example  $p \rightarrow \sim p$

**Contingency**

A statement which can be true or false depending upon the truth values of the variables involved in it is called a contingency e.g.,  $(p \rightarrow q) \wedge (p \vee q)$  is a contingency.

**19. Is the statement  $x + 2 = 5$  a tautology or a self-contradiction?**

**Solution:** Neither.

The statement is not true for all values of  $x$ , and it is not false for all values of  $x$ .

**20. Show that  $p \vee (\sim p \vee q)$  is a tautology.**

**Solution:**

The table shows that  $p \vee (\sim p \vee q)$  is always true. Thus  $p \vee (\sim p \vee q)$  is a tautology.

$p$	$q$	$\sim p$	$\sim p \vee q$	$p \vee (\sim p \vee q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

**21. Show that  $p \wedge (\sim p \wedge q)$  is a self-contradiction.**

**Solution:**

$p$	$q$	$\sim p$	$\sim p \wedge q$	$p \wedge (\sim p \wedge q)$
T	T	F	F	F
T	F	F	F	F
F	T	T	T	F
F	F	T	F	F

Table shows that  $p \wedge (\sim p \wedge q)$  is always false. Thus  $p \wedge (\sim p \wedge q)$  is a self-contradiction

**22. Construct a table for  $\sim(\sim p \vee q) \vee q$** 

**Solution:**

$p$	$q$	$\sim p$	$\sim p \vee q$	$\sim(\sim p \vee q)$	$\sim(\sim p \vee q) \vee q$
T	T	F	T	F	T
T	F	F	F	T	T
F	T	T	T	F	T
F	F	T	T	F	F

23. Construct a truth table for  $(p \wedge \sim q) \vee (\sim p \vee q)$

Solution:

$p$	$q$	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \vee q$	$(p \wedge \sim q) \vee (\sim p \vee q)$
T	T	F	F	F	T	T
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

24. Construct a table for  $(p \wedge q) \wedge (\sim r \vee q)$

Solution:

$p$	$q$	$r$	$p \wedge q$	$\sim r$	$\sim r \vee q$	$(p \wedge q) \wedge (\sim r \vee q)$
T	T	T	T	F	T	T
T	T	F	T	T	T	T
T	F	T	F	F	F	F
T	F	F	F	T	T	F
F	T	T	F	F	T	F
F	T	F	F	T	T	F
F	F	T	F	F	F	F
F	F	F	F	T	T	F

25. Construct a table for  $(\sim p \wedge r) \vee (q \wedge \sim r)$

Solution:

$p$	$q$	$r$	$\sim p$	$\sim r$	$\sim p \wedge r$	$q \wedge \sim r$	$(\sim p \wedge r) \vee (q \wedge \sim r)$
T	T	T	F	F	F	F	F
T	T	F	F	T	F	T	T
T	F	T	F	F	F	F	F
T	F	F	F	T	F	F	F
F	T	T	T	F	T	F	T
F	T	F	T	T	F	T	T
F	F	T	T	F	T	F	T
F	F	F	T	T	F	F	F

26. Construct a truth table for  $p \vee [\sim(p \wedge \sim q)]$

Solution:

$p$	$q$	$\sim q$	$p \wedge \sim q$	$\sim(p \wedge \sim q)$	$p \vee [\sim(p \wedge \sim q)]$
T	T	F	F	T	T
T	F	T	T	F	T
F	T	F	F	T	T
F	F	T	F	T	T

**27. Construct a truth table for  $\sim p \vee (p \wedge q)$** **Solution:**

$p$	$q$	$\sim p$	$p \wedge q$	$\sim p \vee (p \wedge q)$
T	T	F	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

**Implication or Conditional Statement**

A compound statement of the form if  $p$  then  $q$ , also written  **$p$  implies  $q$** , is called a **conditional** or an **implication**, denoted by  $p \rightarrow q$ ,  $p$  is called the **antecedent** or **hypothesis** and  $q$  is called the **consequent** or the **conclusion**.

For instance, in the conditional statement, "If our school was this nice, I would go there more than once a week," the antecedent is "our school was this nice" and the consequent is "I would go there more than once a week."

Conditional statements can be written in if  $p$ , then  $q$  form or in if  $p$ ,  $q$  form. For instance, all of the following are conditional statements.

- If we order pizza, then we can have it delivered.
- If you go to the movie, you will not be able to meet us for dinner.
- If  $n$  is a prime number greater than 2, then  $n$  is an odd number.

**28. Recast each of the following statements in the form if  $p$ , then  $q$ .**

- a. A rise in sea level will devastate Florida.
- b. A red tag on an item is sufficient to mean it's on sale.
- c. Eating vegetables is necessary for good health.

**Solution:**

- a. This statement is equivalent to if sea level rises, then Florida will be devastated.
- b. This statement can be written as if an item is marked with a red tag, then it is on sale.
- c. This statement can be expressed as if a person is in good health, then the person eats vegetables.

**Truth Value of the Conditional**

The conditional is false if  $p$  is true and  $q$  is false. It is true in all other cases.

A conditional is regarded as false only when the antecedent is true and consequent is false. In all other cases it is considered to be true. Its truth table is, therefore, of the adjoining form;

$p$	$q$	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

29. Determine the truth value of each of the following.

- If 2 is an integer, then 2 is a rational number.
- If 3 is a negative number, then  $5 > 7$ .
- If  $5 > 7$  then  $2 + 7 = 4$ .

**Solution:**

- Because the consequent is true, this is a true statement.
- Because the antecedent is false, this is a true statement.
- Because the antecedent is true and the consequent is false, this is a false statement.

30. Evaluate the truth of the statement if  $2 + 2 = 5$ , then  $3 + 3 = 4$ .

**Solution:**

The statement has the form if  $p$ , then  $q$ , where  $p$  is  $2 + 2 = 5$  and  $q$  is  $3 + 3 = 4$ . Both  $p$  and  $q$  are clearly false. However, according to the rules of logic, the conditional if  $p$ , then  $q$  is true any time  $p$  is false, regardless of what  $q$  says. Therefore, the statement if  $2 + 2 = 5$ , then  $3 + 3 = 4$  is true.

31. Determine the truth value of each of the following.

- If  $4 \geq 3$  then  $2 + 5 = 6$ .
- If  $5 > 9$  then  $4 > 9$ .
- If Tuesday follows Monday, then April follows March.

**Solution:**

- Because the antecedent is true and the consequent is false, the statement is a false statement.
- Because the antecedent is false, the statement is a true statement.
- Because the consequent is true, the statement is a true statement.

32. Construct a truth table for a statement involving a conditional

$$[p \wedge (q \vee \sim p)] \rightarrow \sim p$$

**Solution:**

$p$	$q$	$\sim p$	$q \vee \sim p$	$p \wedge (q \vee \sim p)$	$[p \wedge (q \vee \sim p)] \rightarrow \sim p$
T	T	F	T	T	F
T	F	F	F	F	T
F	T	T	T	F	T
F	F	T	T	F	T

33. Construct a truth table for a statement involving a conditional  $[p \wedge (p \rightarrow q)] \rightarrow q$

**Solution:**

$p$	$q$	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$[p \wedge (p \rightarrow q)] \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

- 34. Let  $p$  be the statement “Maria learns discrete mathematics” and  $q$  the statement “Maria will find a good job.” Express the statement  $p \rightarrow q$  as a statement in English.**

**Solution:**

From the definition of conditional statements, we see that when  $p$  is the statement “Maria learns discrete mathematics” and  $q$  is the statement “Maria will find a good job,”  $p \rightarrow q$  represents the statement “If Maria learns discrete mathematics, then she will find a good job.”

- 35. What is the value of the variable  $x$  after the statement  
if  $2 + 2 = 4$  then  $x := x + 1$   
if  $x = 0$  before this statement is encountered?  
(The symbol  $:=$  stands for assignment.**

**The statement  $x := x + 1$  means the assignment of the value of  $x + 1$  to  $x$ .)**

**Solution:**

Because  $2 + 2 = 4$  is true, the assignment statement  $x := x + 1$  is executed.  
Hence,  $x$  has the value  $0 + 1 = 1$  after this statement is encountered.

### **Equivalent Forms of the Conditional**

Every conditional statement can be stated in many equivalent forms. It is not even necessary to state the antecedent before the consequent. For instance, the conditional “If I live in Boston, then I must live in Massachusetts” can also be stated as I must live in Massachusetts, if I live in Boston.

Table lists some of the various forms that may be used to write a conditional statement.

#### **Common Forms of $p \rightarrow q$**

<b>Every conditional statement <math>p \rightarrow q</math> can be written in the following equivalent forms</b>	
If $p$ , then $q$ .	Every $p$ is a $q$ .
If $p$ , $q$ .	$q$ , if $p$ .
$p$ only if $q$ .	$q$ provided $p$ .
$p$ implies $q$ .	$q$ is a necessary condition for $p$ .
Not $p$ or $q$ .	$p$ is a sufficient condition for $q$ .

- 36. Write each of the following in “If  $p$ , then  $q$ ” form.**
- The number is an even number provided it is divisible by 2.**
  - Today is Friday, only if yesterday was Thursday.**

**Solution:**

- If it is divisible by 2, then the number is an even number.
- If today is Friday, then yesterday was Thursday

- 37. Write each of the following in “If  $p$ , then  $q$ ” form.**
- Every square is a rectangle.**
  - Being older than 30 is sufficient to show I am at least 21.**

**Solution:**

- If it is a square, then it is a rectangle.
- If I am older than 30, then I am at least 21.

### The Converse, the Inverse, and the Contrapositive

Every conditional statement has three related statements. They are called the converse, the inverse, and the contrapositive.

The converse of  $p \rightarrow q$  is  $q \rightarrow p$ , The inverse of  $p \rightarrow q$  is  $\sim p \rightarrow \sim q$

The contrapositive of  $p \rightarrow q$  is  $\sim q \rightarrow \sim p$

The above definitions show the following:

- The converse of  $p \rightarrow q$  is formed by interchanging the antecedent  $p$  with the consequent  $q$ .
- The inverse of  $p \rightarrow q$  is formed by negating the antecedent  $p$  and negating the consequent  $q$ .
- The contrapositive of  $p \rightarrow q$  is formed by negating both the antecedent  $p$  and the consequent  $q$  and interchanging these negated statements.

$p$	$q$	Conditional $p \rightarrow q$	Converse $q \rightarrow p$	Inverse $\sim p \rightarrow \sim q$	Contrapositive $\sim q \rightarrow \sim p$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

**38. Write the converse, inverse, and contrapositive of**

**If I get the job, then I will rent the apartment.**

**Solution:**

**Converse:** If I rent the apartment, then I get the job.

**Inverse:** If I do not get the job, then I will not rent the apartment.

**Contrapositive:** If I do not rent the apartment, then I did not get the job.

**39. Write the converse, inverse, and contrapositive of**

**If we have a quiz today, then we will not have a quiz tomorrow.**

**Solution:**

**Converse:** If we are not going to have a quiz tomorrow, then we will have a quiz today.

**Inverse:** If we don't have a quiz today, then we will have a quiz tomorrow.

**Contrapositive:** If we have a quiz tomorrow, then we will not have a quiz today.

**40. Determine whether the given statements are equivalent.**

a. **If a number ends with a 5, then the number is divisible by 5.**

**If a number is divisible by 5, then the number ends with a 5.**

b. **If two lines in a plane do not intersect, then the lines are parallel.**

**If two lines in a plane are not parallel, then the lines intersect**

**Solution:**

a. The second statement is the converse of the first. The statements are not equivalent.

b. The second statement is the contrapositive of the first. The statements are equivalent.

**41. Determine whether the given statements are equivalent.**

- a. If  $a=b$  then  $a.c=b.c$   
 If  $a \neq b$  then  $a.c \neq b.c$
- b. If I live in Nashville, then I live in Tennessee.  
 If I do not live in Tennessee, then I do not live in Nashville.

**Solution:**

- a. The second statement is the inverse of the first statement. Thus the statements are not equivalent. This can also be demonstrated by the fact that the first statement is true for  $c = 0$  and the second statement is false for  $c = 0$ .
- b. The second statement is the contrapositive of the first statement. Thus the statements are equivalent.

**Remark**

In mathematics, it is often necessary to prove statements that are in “If  $p$ , then  $q$ ” form. If a proof cannot readily be produced, mathematicians often try to prove the contrapositive, “If  $\sim q$  then  $\sim p$ ” Because a conditional and its contrapositive are equivalent statements, a proof of either statement also establishes the proof of the other statement.

**42. A mathematician wishes to prove the following statement about the integer  $x$ .**

If  $x^2$  is an odd integer, then  $x$  is an odd integer. (I)

If the mathematician is able to prove the statement, “If  $x$  is an even integer, then  $x^2$  is an even integer,” does this also prove statement (I)?

**Solution:**

Yes, because the second statement is the contrapositive of (I).

**43. Write the contrapositive of each statement and use the contrapositive to determine whether the original statement is true or false.**

- a. If  $a+b$  is not divisible by 5, then  $a$  and  $b$  are not both divisible by 5.
- b. If  $x^3$  is an odd integer, then  $x$  is an odd integer. (Assume  $x$  is an integer.)
- c. If a geometric figure is not a rectangle, then it is not a square.

**Solution:**

- a. If  $a$  and  $b$  are both divisible by 5, then  $a+b$  is divisible by 5. This is a true statement, so the original statement is also true.
- b. If  $x$  is an even integer, then  $x^3$  is an even integer. This is a true statement, so the original statement is also true.
- c. If a geometric figure is a square, then it is a rectangle. This is a true statement, so the original statement is also true.

- 44. Write the contrapositive of each statement and use the contrapositive to determine whether the original statement is true or false.**
- If  $3 + x$  is an odd integer, then  $x$  is an even integer. (Assume  $x$  is an integer.)
  - If two triangles are not similar triangles, then they are not congruent triangles.  
**Note: Similar triangles have the same shape. Congruent triangles have the same size and shape.**
  - If today is not Wednesday, then tomorrow is not Thursday.

**Solution:**

- Contrapositive: If  $x$  is an odd integer, then  $3 + x$  is an even integer. The contrapositive is true and so the original statement is also true.
- Contrapositive: If two triangles are congruent triangles, then the two triangles are similar triangles. The contrapositive is true and so the original statement is also true.
- Contrapositive: If tomorrow is Thursday, then today is Wednesday. The contrapositive is true and so the original statement is also true.

- 45. Find the contrapositive, the converse, and the inverse of the conditional statement "The home team wins whenever it is raining."**

**Solution:**

Because "q whenever p" is one of the ways to express the conditional statement  $p \rightarrow q$ , the original statement can be rewritten as "If it is raining, then the home team wins."

Consequently, the contrapositive of this conditional statement is "If the home team does not win, then it is not raining."

The converse is "If the home team wins, then it is raining."

The inverse is "If it is not raining, then the home team does not win."

Only the contrapositive is equivalent to the original statement.

### **Biconditional**

The statement  $p \leftrightarrow q$  is called a biconditional and is read as "p if and only if q." It is equivalent to the following form;  $p \leftrightarrow q \equiv [(p \rightarrow q) \wedge (q \rightarrow p)]$

$p$	$q$	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

**46. State whether each biconditional is true or false.**

- a.  $x + 4 = 7$  if and only if  $x = 3$ .      b.  $x^2 = 6x^2$  if and only if  $x = 6$ .

**Solution:**

- a. Both components are true when  $x = 3$  and both are false when  $x \neq 3$ . Both components have the same truth value for any value of  $x$ , so this is a true statement.
- b. If  $x = -6$  the first component is true and the second component is false. Thus, this is a false statement.

**47. State whether each biconditional is true or false.**

- a.  $x > 7$  if and only if  $x > 6$ .      b.  $x + 5 > 7$  if and only if  $x > 2$ .

**Solution:**

- a. Let  $x = 6.5$ . Then the first component of the biconditional is false and the second component of the biconditional is true. Thus the given biconditional statement is false.
- b. Both components of the biconditional are true for  $x > 2$ , and both components are false for  $x \leq 2$ . Because both components have the same truth value for any real number  $x$ , the given biconditional is true.

**48. Let  $p$  be the statement "You can take the flight," and let  $q$  be the statement "You buy a ticket." Then find  $p \leftrightarrow q$ .****Solution:**

Then  $p \leftrightarrow q$  is the statement "You can take the flight if and only if you buy a ticket." This statement is true if  $p$  and  $q$  are either both true or both false, that is, if you buy a ticket and can take the flight or if you do not buy a ticket and you cannot take the flight. It is false when  $p$  and  $q$  have opposite truth values, that is, when you do not buy a ticket, but you can take the flight (such as when you get a free trip) and when you buy a ticket but you cannot take the flight (such as when the airline bumps you).

**Equivalent Statements**

Two statements are equivalent if they both have the same truth values for all possible truth values of their component statements. Equivalent statements have identical truth values in the final columns of their truth. The notation  $p \equiv q$  is used to indicate that the statements  $p$  and  $q$  are equivalent.

**49. Show that  $\sim(p \vee \sim q)$  and  $\sim p \wedge q$  are equivalent statements.****Solution:**

Construct two truth tables and compare the results. The truth tables below show that  $\sim(p \vee \sim q)$  and  $\sim p \wedge q$  have the same truth values for all possible truth values of their component statements. Thus the statements are equivalent.

$p$	$q$	$\sim(p \vee \sim q)$	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	F	F

50. Show that  $p$  and  $p \vee (p \wedge \sim q)$  are equivalent statements.

Solution:

$p$	$q$	$\sim q$	$p \wedge \sim q$	$p \vee (p \wedge \sim q)$
T	T	F	F	T
T	F	T	T	T
F	T	F	F	F
F	F	T	F	F

This shows that  $p \equiv p \vee (p \wedge \sim q)$

### De Morgan's Laws for Statements

For any statements  $p$  and  $q$ ,

$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$

$$\& \quad \sim(p \wedge q) \equiv \sim p \vee \sim q$$

$p$	$q$	$\sim(p \vee q)$	$\sim p \wedge \sim q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

$p$	$q$	$\sim(p \wedge q)$	$\sim p \vee \sim q$
T	T	F	F
T	F	F	T
F	T	F	T
F	F	F	T

51. Use one of De Morgan's laws to restate the following sentence in an equivalent form. *It is not the case that I graduated or I got a job.*

Solution:

Let  $p$  represent the statement "I graduated." Let  $q$  represent the statement "I got a job." In symbolic form, the original sentence is  $\sim(p \vee q)$ . One of De Morgan's laws states that this is equivalent to  $\sim p \wedge \sim q$ . Thus a sentence that is equivalent to the original sentence is "I did not graduate and I did not get a job."

52. Use one of De Morgan's laws to restate the following sentence in an equivalent form. *It is not true that I am going to the dance and I am going to the game.*

Solution:

Let  $d$  represent "I am going to the dance." Let  $g$  represent "I am going to the game." The original sentence in symbolic form is  $\sim(d \wedge g)$ . Applying one of De Morgan's laws, we find that  $\sim(d \wedge g) \equiv \sim(d \vee \sim g)$ . Thus an equivalent form of "It is not true that I am going to the dance and I am going to the game" is "I am not going to the dance or I am not going to the game."

**Logically Equivalent**

The compound propositions  $p$  and  $q$  are called logically equivalent if  $p \leftrightarrow q$  is a tautology. The notation  $p \equiv q$  denotes that  $p$  and  $q$  are logically equivalent.

- 53. Consider the true statement if a creature is a whale, then it is a mammal. Write its converse, inverse, and contrapositive. Evaluate the truth of each statement. Which statements are logically equivalent?**

**Solution:**

The statement has the form if  $p$ , then  $q$ , where  $p$  = a creature is a whale and  $q$  = a creature is a mammal. Therefore, we find

**Converse** (if  $q$ , then  $p$ ): if a creature is a mammal, then it is a whale. This statement is false, because most mammals are not whales.

**Inverse** (if not  $p$ , then not  $q$ ): if a creature is not a whale, then it is not a mammal. This statement is also false; for example, dogs are not whales, but they are mammals.

**Contrapositive** (if not  $q$ , then not  $p$ ): if a creature is not a mammal, then it is not a whale. Like the original statement, this statement is true, because all whales are mammals.

Note that the original proposition and its contrapositive have the same truth value and are logically equivalent. Similarly, the converse and inverse have the same truth value and are logically equivalent.

- 54. Show that  $p \rightarrow q$  and  $\sim p \vee q$  are logically equivalent. (This is known as the conditional- disjunction equivalence.)**

**Solution:**

We construct the truth table for these compound propositions in Table. Because the truth values of  $\sim p \vee q$  and  $p \rightarrow q$  agree, they are logically equivalent.

$p$	$q$	$\sim p$	$\sim p \vee q$	$p \rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

- 55. Show that  $p \vee (q \wedge r)$  and  $(p \vee q) \wedge (p \vee r)$  are logically equivalent. This is the distributive law of disjunction over conjunction.**

**Solution:**

We construct the truth table for these compound propositions in Table. Because the truth values of  $p \vee (q \wedge r)$  and  $(p \vee q) \wedge (p \vee r)$  agree, these compound propositions are logically equivalent.

$p$	$q$	$r$	$q \wedge r$	$p \vee (q \wedge r)$	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	F	F	F
F	F	T	F	F	T	T	F
F	F	F	F	F	F	F	F

Logical Equivalences Involving Conditional Statements.	Logical Equivalences Involving Biconditional Statements
$p \rightarrow q \equiv \sim p \vee q$ $p \rightarrow q \equiv \sim q \rightarrow \sim p$ $p \vee q \equiv \sim p \rightarrow q$ $p \wedge q \equiv \sim (p \rightarrow \sim q)$ $\sim (p \rightarrow q) \equiv p \wedge \sim q$ $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$ $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$ $(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$ $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$ $p \leftrightarrow q \equiv \sim p \leftrightarrow \sim q$ $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$ $\sim (p \leftrightarrow q) \equiv p \leftrightarrow \sim q$

## Truth Sets, A link between Set Theory and Logic

Logical propositions  $p, q$  etc., are formulae expressed in terms of some variables. For the sake of simplicity and convenience we may assume that they are all expressed in terms of a single variable  $x$  where  $x$  is a real variable. Thus  $p = p(x)$  where,  $x \in R$ . All those values of  $x$  which make the formula  $p(x)$  true form a set, say  $P$ . Then  $P$  is the truth set of  $p$ . Similarly, the truth set,  $Q$ , of  $q$  may be defined. We can extend this notion and apply it in other cases.

- Truth set of  $\sim p$ :** Truth set of  $\sim p$  will evidently consist of those values of the variable for which  $p$  is false i.e., they will be members of  $P'$ , the complement of  $P$ .
- $p \vee q$ :** Truth set of  $p \vee q = p(x) \vee q(x)$  consists of those values of the variable for which  $p(x)$  is true or  $q(x)$  is true or both  $p(x)$  and  $q(x)$  are true. Therefore, truth set of  $p \vee q$  will be:  $p \vee q = P \cup Q = \{x | p(x) \text{ is true or } q(x) \text{ is true}\}$

- iii)  $p \wedge q$ : Truth set of  $p(x) \wedge q(x)$  will consist of those values of the variable for which both  $p(x)$  and  $q(x)$  are true. Evidently truth set is  
 $p \wedge q = P \cap Q = \{x | p(x) \text{ is true and } q(x) \text{ is true}\}$
- iv)  $p \rightarrow q$ : We know that  $p \rightarrow q$  is equivalent to  $\sim p \vee q$  therefore truth set of  $p \rightarrow q$  will be  $P' \cup Q$
- v)  $p \leftrightarrow q$ : We know that  $p \leftrightarrow q$  means that  $p$  and  $q$  are simultaneously true or false. Therefore, in this case truth sets of  $p$  and  $q$  will be the same i.e.  $P = Q$

**Note**

- Evidently truth set of a tautology is the relevant universal set and that of an absurdity is the empty set.
- With the help of the above results we can express any logical formula in set-theoretic form and vice versa.

**56. Give logical proofs of the following theorems:**  $(A \cup B)' = A' \cap B'$

**Solution:** The corresponding formula of logic is  $\sim(p \vee q) \equiv \sim p \wedge \sim q$

We construct truth table of the two sides.

$p$	$q$	$\sim p$	$\sim q$	$p \vee q$	$\sim(p \vee q)$	$\sim p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

The last two columns of the table establish the equality of the two sides of equation.

**57. Give logical proofs of the following theorems:**  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

**Solution:** Logical form of the theorem is  $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

We construct the table for the two sides of this equation

$p$	$q$	$r$	$q \vee r$	$p \wedge (q \vee r)$	$(p \wedge q)$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

Comparison of the entries of columns 5 and 8 is sufficient to establish the desired result.

## Argument and Valid Argument

An **argument** consists of a set of statements called **premises** and another statement called the **conclusion**. An argument is **valid** if the conclusion is true whenever all the premises are assumed to be true. An argument is **invalid** if it is not a valid argument.

### Remember

In the argument about Aristotle, the two premises and the conclusion are shown below. It is customary to place a horizontal line between the premises and the conclusion.

First Premise: If Aristotle was human, then Aristotle was mortal.

Second Premise: Aristotle was human.

Conclusion: Therefore, Aristotle was mortal.

Arguments can be written in symbolic form. For instance, if we let  $h$  represent the statement "Aristotle was human" and  $m$  represent the statement "Aristotle was mortal," then the argument can be expressed as

$$h \rightarrow m$$

$$\frac{h}{\quad}$$

$$\therefore m$$

The three dots  $\therefore$  are a symbol for "therefore."

### 58. Write the following argument in symbolic form.

**The fish is fresh or I will not order it. The fish is fresh. Therefore I will order it.**

#### Solution:

Let  $f$  represent the statement "The fish is fresh." Let  $o$  represent the statement "I will order it." The symbolic form of the argument is

$$f \vee \sim o$$

$$\frac{f}{\quad}$$

$$\therefore o$$

### 59. Write the following argument in symbolic form.

**If she doesn't get on the plane, she will regret it. She does not regret it.**

**Therefore, she got on the plane**

#### Solution:

Let  $p$  represent the statement "She got on the plane." Let  $r$  represent the statement "She will regret it." Then the symbolic form of the argument is

$$\sim p \rightarrow r$$

$$\frac{\sim r}{\quad}$$

$$\therefore p$$

### Arguments and Truth Tables

The following truth table procedure can be used to determine whether an argument is valid or invalid.

#### Truth Table Procedure to Determine the Validity of an Argument

1. Write the argument in symbolic form.
2. Construct a truth table that shows the truth value of each premise and the truth value of the conclusion for all combinations of truth values of the component statements.
3. If the conclusion is true in every row of the truth table in which all the premises are true, the argument is valid. If the conclusion is false in any row in which all the premises are true, the argument is invalid.

**60. If Aristotle was human, then Aristotle was mortal. Aristotle was human. Therefore, Aristotle was mortal. Using procedure show that argument is valid or not?**

**Solution:**

Let  $h$  represent the statement "Aristotle was human" and  $m$  represent the statement "Aristotle was mortal." In symbolic form, the argument is

$$h \rightarrow m$$

$$\frac{h}{\quad}$$

$$\therefore m$$

Construct a truth table as shown below.

$h$	$m$	First Premise $h \rightarrow m$	Second Premise $h$	Conclusion $m$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	F	F

Row 1 is the only row in which all the premises are true, so it is the only row that we examine. Because the conclusion is true in row 1, the argument is valid.

**61. Determine whether the following argument is valid or invalid.**

**If it rains, then the game will not be played. It is not raining. Therefore, the game will be played.**

**Solution:**

If we let  $r$  represent "it rains" and  $g$  represent "the game will be played," then the symbolic form is

$$r \rightarrow \sim g$$

$$\frac{\sim r}{\quad}$$

$$\therefore g$$

The truth table for this argument is as follows:

$r$	$g$	First Premise $r \rightarrow \sim g$	Second Premise $\sim r$	Conclusion $g$
T	T	F	F	T
T	F	T	F	F
F	T	T	T	T
F	F	T	T	F

Because the conclusion in row 4 is false and the premises are both true, we know the argument is invalid.

**62. Determine whether the following argument is valid or invalid.**

**If the stock market rises, then the bond market will fall.**

**The bond market did not fall.**

**∴ The stock market did not rise.**

**Solution:**

Let  $r$  represent the statement "The stock market rises." Let  $f$  represent the statement "The bond market will fall." Then the symbolic form of the argument is

$$r \rightarrow \sim f$$

$$\frac{\sim f}{\quad}$$

$$\therefore \sim r$$

The truth table for this argument is as follows:

$r$	$f$	First Premise $r \rightarrow f$	Second Premise $\sim f$	Conclusion $\sim r$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Because the conclusion is true in row 4, the argument is valid.

**63. Determine whether the following argument is valid or invalid.**

**If I am going to run the marathon, then I will buy new shoes.**

**If I buy new shoes, then I will not buy a television.**

**∴ If I buy a television, I will not run the marathon.**

**Solution:**

Label the statements

$m$ : I am going to run the marathon.

$s$ : I will buy new shoes.

$t$ : I will buy a television.

The symbolic form of the argument is given as;

$$m \rightarrow s$$

$$s \rightarrow \sim t$$

$$\therefore t \rightarrow \sim m$$

The truth table for this argument is as follows:

			First Premise	Second Premise	Conclusion
$m$	$s$	$t$	$m \rightarrow s$	$s \rightarrow \sim t$	$t \rightarrow \sim m$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

The only rows in which both premises are true are rows 2, 6, 7, and 8. Because the conclusion is true in each of these rows, the argument is valid.

**64. Determine whether the following argument is valid or invalid.**

**If I arrive before 8 A.M., then I will make the flight.**

**If I make the flight, then I will give the presentation.**

**∴ If I arrive before 8 A.M., then I will give the presentation.**

**Solution:**

Let  $a$  represent the statement "I arrive before 8 A.M." Let  $f$  represent the statement "I will make the flight." Let  $p$  represent the statement "I will give the presentation." Then the symbolic form of the argument is

$$a \rightarrow f$$

$$f \rightarrow p$$

$$\therefore a \rightarrow p$$

The truth table for this argument is as follows:

			First Premise	Second Premise	Conclusion
$a$	$f$	$p$	$a \rightarrow f$	$f \rightarrow p$	$a \rightarrow p$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

The only rows in which all the premises are true are rows 1, 5, 7, and 8. In each of these rows the conclusion is also true. Thus the argument is a valid argument.

**Standard Forms:** Some arguments can be shown to be valid if they have the same symbolic form as an argument that is known to be valid. For instance, we have shown that the argument

$$h \rightarrow m$$

$$\frac{h}{\quad}$$

$$\therefore m$$

is valid. This symbolic form is known as **modus ponens** or the **law of detachment**. All arguments that have this symbolic form are valid. Table shows **four symbolic forms** and the name used to identify each form. Any argument that has a symbolic form identical to one of these symbolic forms is a valid argument.

Modus Ponens	Modus Tollens	Law of Syllogism	Disjunctive Syllogism
$p \rightarrow q$	$p \rightarrow q$	$p \rightarrow q$	$p \vee q$
$\frac{p}{\quad}$	$\frac{\sim q}{\quad}$	$\frac{p \rightarrow r}{\quad}$	$\frac{\sim p}{\quad}$
$\therefore q$	$\therefore \sim p$	$\therefore p \rightarrow r$	$\therefore q$

The law of syllogism can be extended to include more than two conditional premises. For example, if the premises of an argument are  $a \rightarrow b, b \rightarrow c, \dots, y \rightarrow z$  then a valid conclusion for the argument is  $a \rightarrow z$ . We will refer to any argument of this form with more than two conditional premises as the **extended law of syllogism**. Table shows two **symbolic forms** associated with **invalid arguments**. Any argument that has one of these symbolic forms is invalid.

Fallacy of the Converse	Fallacy of the Inverse
$p \rightarrow q$	$p \rightarrow q$
$\frac{q}{\quad}$	$\frac{\sim p}{\quad}$
$\therefore p$	$\therefore \sim q$

65. Use a standard form to determine whether the following argument is valid or invalid.

The program is interesting or I will watch the basketball game.

The program is not interesting.

∴ I will watch the basketball game.

**Solution:** Label the statements.

$i$ : The program is interesting.

$w$ : I will watch the basketball game.

In symbolic form the argument is:

$$i \vee w$$

$$\frac{\sim i}{\quad}$$

$$\therefore w$$

This symbolic form matches the standard form known as disjunctive syllogism.

Thus the argument is valid.

**66. Use a standard form to determine whether the following argument is valid or invalid.**

**If I go to Florida for spring break, then I will not study.**

**I did not go to Florida for spring break.**

**∴ I studied.**

**Solution:**

Let  $f$  represent "I go to Florida for spring break." Let  $\sim s$  represent "I will not study." Then the symbolic form of the argument is

$$\begin{array}{l} f \rightarrow \sim s \\ \sim f \\ \hline \end{array}$$

∴  $s$

This argument has the form of the fallacy of the inverse. Thus the argument is invalid.

**67. Determine whether the following argument is valid.**

**If the movie was directed by George Lucas (l), then I want to see it (w). If I want to see a movie, then the movie's production costs must have exceeded 20 million dollars (c). The movie's production costs were less than 20 million dollars.**

**Therefore, the movie was not directed by George Lucas.**

**Solution:**

In symbolic form the argument is:

$$\begin{array}{l} l \rightarrow w \\ w \rightarrow c \\ \sim c \\ \hline \end{array}$$

∴  $\sim l$

Applying the law of syllogism to Premises 1 and 2 produces

$$\begin{array}{l} l \rightarrow w \\ w \rightarrow c \\ \sim c \\ \hline \end{array}$$

∴  $l \rightarrow c$

Combining the above conclusion  $l \rightarrow c$  with Premise 3 gives us

$$\begin{array}{l} l \rightarrow c \\ \sim c \\ \hline \end{array}$$

∴  $\sim l$

This sequence of valid arguments has produced the conclusion given in the original argument. Thus the original argument is a valid argument.

**68. Determine whether the following argument is valid.**

**I start to fall asleep if I read a math book. I drink a soda whenever I start to fall asleep. If I drink a soda, then I must eat a candy bar. Therefore, I eat a candy bar whenever I read a math book.**

**Solution:**

Let  $r$  represent "I read a math book." Let  $f$  represent "I start to fall asleep." Let  $d$  represent "I drink a soda." Let  $e$  represent "I eat a candy bar." Then the symbolic form of the argument is

$$r \rightarrow f$$

$$f \rightarrow d$$

$$d \rightarrow e$$

$$\therefore r \rightarrow e$$

The argument has the form of the extended law of syllogism. Thus the argument is valid.

## Categorical Propositions

In logic, categorical propositions are statements that express a relationship between two categories or classes of things. They are a fundamental concept in Aristotelian logic and are used to make assertions about the properties or characteristics of a particular group or category.

A categorical proposition typically has the following structure:

Subject-Predicate-Quantity-Quality

**Subject:** The category or class being referred to (e.g., "All humans").

**Predicate:** The property or characteristic being attributed to the subject (e.g., "are mortal").

**Quantity:** The extent or scope of the statement (e.g., "all," "some," "no").

**Quality:** The affirmation or negation of the statement (e.g., "are" or "are not").

### Categorical Propositions and Venn Diagrams

Now that we have discussed general relationships between sets, we are ready to study propositions that make claims about sets. For example, the proposition all whales are mammals makes a very specific claim—namely, that the set whales is a subset of the set mammals. Propositions of this type are called **categorical propositions** because they claim a particular relationship between two categories or sets.

Like all propositions, categorical propositions must have the structure of a complete sentence. Categorical propositions also have another important general feature. Of the two sets in a categorical proposition, one set appears in the subject of the sentence and the other appears in the predicate. For example, in the proposition all whales are mammals, the set whales is the subject set and the set mammals is the predicate set.

We usually use the letter  $S$  to represent the subject set and  $P$  for the predicate set, so we can rewrite all whales are mammals as all  $S$  are  $P$ , where  $S$  = whales and  $P$  = mammals

Categorical propositions come in the following four standard forms.

The Four Standard Categorical Propositions			
Form	Example	Subject Set ( $S$ )	Predicate Set ( $P$ )
All $S$ are $P$	All whales are mammals	Whales	Mammals
No $S$ are $P$	No fish are mammals	Fish	Mammals
Some $S$ are $P$	Some doctors are women	Doctors	Women
Some $S$ are not $P$	Some teachers are not men	Teachers	men

### Venn Diagrams for Categorical Propositions

We can use Venn diagrams to make visual representations of categorical propositions. Figures 1 to 5 show the diagrams for each of the four sample propositions in the box above. Note the following key features of these diagrams:

- **Figure 1** shows the Venn diagram for the proposition all whales are mammals. The circle for the set  $S$  = whales is drawn inside the circle for the set  $P$  = mammals, because  $S$  is a subset of  $P$  in this case. All propositions of the form all  $S$  are  $P$  have the same basic Venn diagram.
- **Figure 2** shows the Venn diagram for the proposition no fish are mammals. In this case, the sets  $S$  = fish and  $P$  = mammals have no common members (they are disjoint). The Venn diagram therefore shows two separated circles.
- **Figure 3** shows the Venn diagram for the proposition some doctors are women, which requires overlapping circles for  $S$  = doctors and  $P$  = women. However, the overlapping circles alone do not represent the proposition, because they don't tell us which regions have members. In this case, the proposition asserts that there are some people (at least one) who are both doctors and women. We indicate this fact by putting an  $X$  in the overlapping region of the diagram. Note that the proposition does not tell us whether there are also doctors who are not women, nor does it tell us whether there are women who are not doctors. Without further information, we do not know whether the non-overlapping regions contain any members.
- **Figure 4** shows the Venn diagram for the proposition some teachers are not men, which also requires overlapping circles. This proposition asserts that there are some people (at least one) who are in the  $S$  = teachers circle but not in the  $P$  = men circle. Therefore, we put an  $X$  in the non-overlapping region of the teachers circle. Note that no claim is made about whether there are teachers who are also men or men who are not teachers.

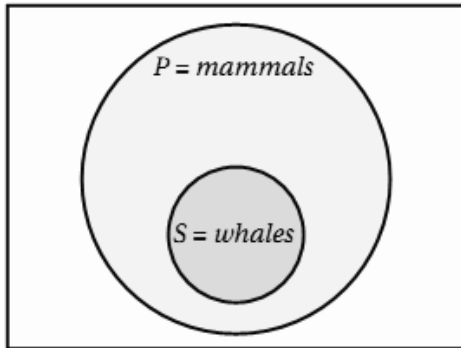


Figure (1)

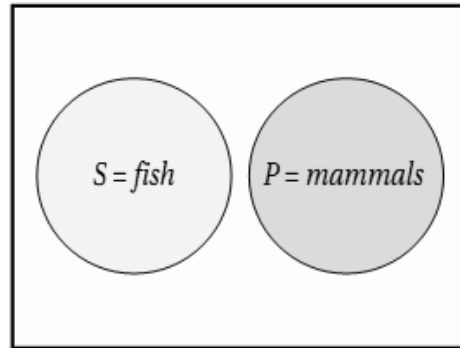


Figure (2)

The X indicates that the overlapping Region has at least one member

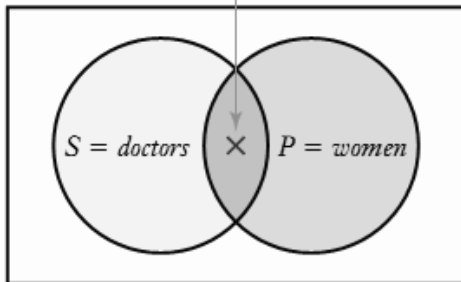


Figure (3)

The X indicates the region claimed to have at least one member

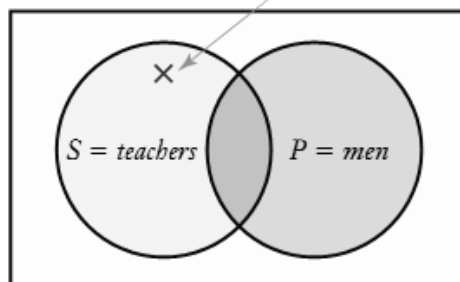


Figure (4)

### Putting Categorical Propositions in Standard Form

Many statements in everyday speech make claims about relationships between two categories, but don't look precisely like one of the four standard forms for categorical propositions. It's often useful to rephrase such statements in one of the standard forms. For example, the statement all diamonds are valuable can be rephrased to read all diamonds are things of value. Now the statement has the form all S are P, where S = diamonds and P = things of value. Following examples illustrate how categorical propositions can be applied to mathematical statements, using quantifiers like "all", "some", "no", and "there exists" to make assertions about sets, functions, and relations.

#### Universal Affirmative (A)

- All integers are rational numbers.
- For all real numbers  $x$ ,  $x^2 \geq 0$
- Every even number is divisible by 2.

#### Universal Negative (E)

- No integer is equal to its own square.
- There is no real number  $x$  such that  $x^2 + 1 = 0$ .
- No odd number is divisible by 2.

**Particular Affirmative (I)**

- Some integers are prime numbers.
- There exists a real number  $x$  such that  $x^2 = 2$ .
- Some even numbers are perfect squares.

**Particular Negative (O)**

- Some integers are not rational numbers.
- There exists a real number  $x$  such that  $x^2 < 0$ .
- Some odd numbers are not prime.

**Examples with Sets**

- Let  $A = \{1, 2, 3, 4, 5\}$  and  $B = \{2, 4, 6, 8, 10\}$ . Then:
- All elements of  $A$  are less than 6. (A)
- Some elements of  $A$  are not in  $B$ . (I)
- No element of  $A$  is greater than 5. (E)
- Some elements of  $B$  are not in  $A$ . (O)

**Examples with Functions**

- Let  $f(x) = x^2$  and  $g(x) = 2x$ . Then:
- For all real numbers  $x$ ,  $f(x) \geq 0$ . (A)
- There exists a real number  $x$  such that  $f(x) = g(x)$ . (I)
- No real number  $x$  satisfies  $f(x) = -1$ . (E)
- Some real numbers  $x$  satisfy  $f(x) \neq g(x)$ . (O)

Following examples illustrate how categorical propositions can be applied to various fields of study, including education, environmental science, history, economics, philosophy, sociology, psychology, and business.

**Universal Affirmative (A)**

All students who graduate from a university have a degree. In this example, the subject is "students who graduate from a university", and the predicate is "have a degree". This proposition is asserting that every single student who graduates from a university has a degree.

**Universal Negative (E)**

No fossil fuel is renewable. In this example, the subject is "fossil fuel", and the predicate is "renewable". This proposition is asserting that not a single fossil fuel is renewable.

**Particular Affirmative (I)**

Some ancient civilizations built pyramids. In this example, the subject is "ancient civilizations", and the predicate is "built pyramids". This proposition is asserting that at least one ancient civilization built pyramids.

**Particular Negative (O)**

Some countries do not have a minimum wage. In this example, the subject is "countries", and the predicate is "have a minimum wage". This proposition is asserting that there exists at least one country that does not have a minimum wage.

**Universal Affirmative (A)**

All moral actions are guided by reason.

In this example, the subject is "moral actions", and the predicate is "guided by reason". This proposition is asserting that every single moral action is guided by reason.

**Universal Negative (E)**

No social institution is completely egalitarian. In this example, the subject is "social institution", and the predicate is "completely egalitarian". This proposition is asserting that not a single social institution is completely egalitarian.

**Particular Affirmative (I)**

Some people are born with a talent for music. In this example, the subject is "people", and the predicate is "born with a talent for music". This proposition is asserting that at least one person is born with a talent for music.

**Particular Negative (O)**

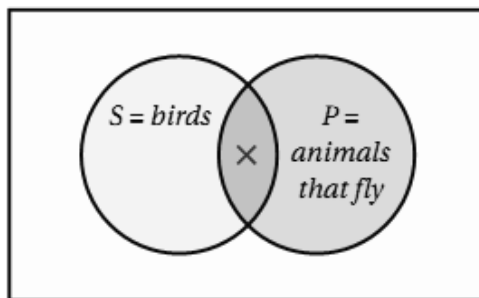
Some companies do not prioritize profit over social responsibility. In this example, the subject is "companies", and the predicate is "prioritize profit over social responsibility". This proposition is asserting that there exists at least one company that does not prioritize profit over social responsibility.

**69. Rephrase each of the following statements in one of the four standard forms for categorical propositions. Then draw the Venn diagram.**

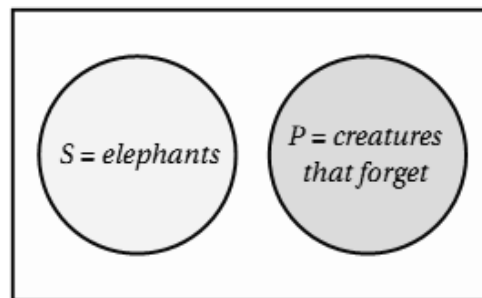
- a. Some birds can fly.      b. Elephants never forget.

**Solution:**

- a. Some birds can fly can be rephrased as some birds are animals that can fly. It now has the form some S are P where S = birds and P = animals that can fly. Figure (a) shows the Venn diagram, with an X in the overlapping region to indicate the claim that there are some birds that can fly.
- b. Elephants never forget can be rephrased as no elephants are creatures that forget. This proposition has the form no S are P shown in Figure (b), where S = elephants and P = creatures that forget.



(a)

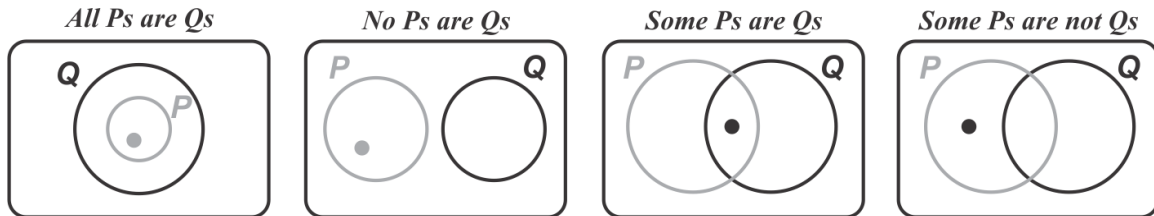


(b)

### Euler Diagrams

Many arguments involve sets whose elements are described using the quantifiers all, some, and none. The mathematician Leonhard Euler used diagrams to determine whether arguments that involved quantifiers were valid or invalid.

The following figures show **Euler diagrams** that illustrate the **four possible relationships** that can exist **between two sets**.



Euler used diagrams to illustrate logic concepts. Some 100 years later, John Venn extended the use of Euler's diagrams to illustrate many types of mathematics. In this section, we will construct diagrams to determine the validity of arguments. We will refer to these diagrams as Euler diagrams.

**70. Use an Euler diagram to determine whether the following argument is valid or invalid.**

**All college courses are fun.**

**This course is a college course.**

**∴ This course is fun.**

**Solution:**

The first premise indicates that the set of college courses is a subset of the set of fun courses. We illustrate this subset relationship with an Euler diagram, as shown in Figure 1. The second premise tells us that "this course" is an element of the set of college courses. If we use  $c$  to represent "this course," then  $c$  must be placed inside the set of college courses, as shown in Figure 2.

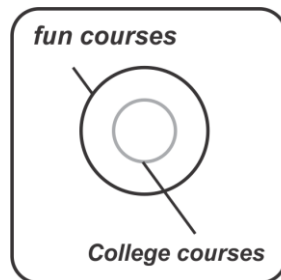


Fig 1

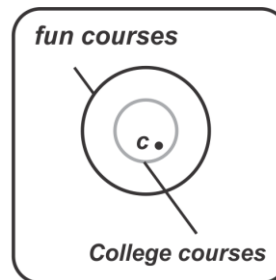


Fig 2

Figure 2 illustrates that  $c$  must also be an element of the set of fun courses. Thus the argument is valid.

**71. Use an Euler diagram to determine whether the following argument is valid or invalid.**

**All lawyers drive BMWs.**

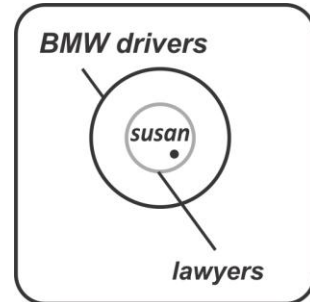
**Susan is a lawyer.**

∴ **Susan drives a BMW.**

**Solution:** Euler diagram shows that the argument is valid.

**Note**

If an Euler diagram can be drawn so that the conclusion does not necessarily follow from the premises, then the argument is invalid.



**72. Use an Euler diagram to determine whether the following argument is valid or invalid.**

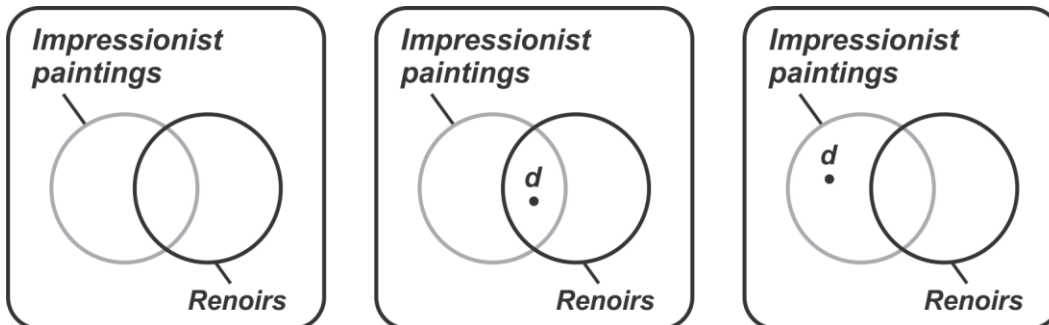
**Some impressionist paintings are Renoirs.**

**Dance at Bougival is an impressionist painting.**

∴ **Dance at Bougival is a Renoir.**

**Solution:**

The Euler diagram in Figure 1 illustrates the premise that some impressionist paintings are Renoirs. Let *d* represent the painting Dance at Bougival. Figures 2 and 3 show that *d* can be placed in one of two region.



Although Figure 2 supports the argument, Figure 3 shows that the conclusion does not necessarily follow from the premises, and thus the argument is invalid.

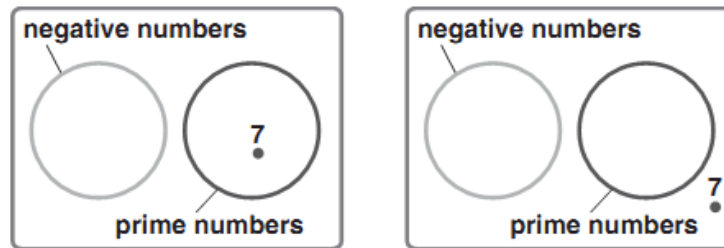
**73. Use an Euler diagram to determine whether the following argument is valid or invalid.**

**No prime numbers are negative.**

**The number 7 is not negative.**

**∴ The number 7 is a prime number.**

**Solution:** From the given premises we can conclude that 7 may or may not be a prime number. Thus the argument is invalid.



**74. If one particular example can be found for which the conclusion of an argument is true when its premises are true, must the argument be valid?**

**Solution:** No. To be a valid argument, the conclusion must be true whenever the premises are true. Just because the conclusion is true for one specific example, it does not mean the argument is a valid argument.

**75. Use an Euler diagram to determine whether the following argument is valid or invalid.**

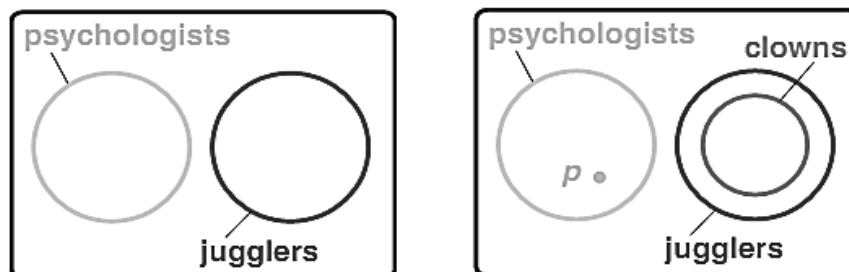
**No psychologist can juggle.**

**All clowns can juggle.**

**∴ No psychologist is a clown.**

**Solution:**

The Euler diagram in Figure 1 shows that the set of psychologists and the set of jugglers are disjoint sets. Figure 2 shows that because the set of clowns is a subset of the set of jugglers, no psychologists  $p$  are elements of the set of clowns. Thus the argument is valid.



76. Use an Euler diagram to determine whether the following argument is valid or invalid.

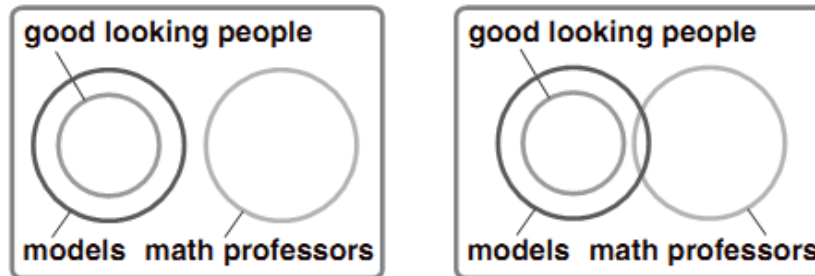
No mathematics professors are good-looking.

All good-looking people are models.

∴ No mathematics professor is a model.

**Solution:**

From the given premises we can construct two possible Euler diagrams.



From the rightmost Euler diagram we can determine that the argument is invalid.

### Euler Diagrams and the Extended Law of Syllogism

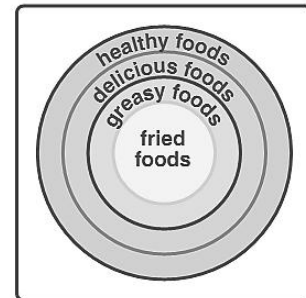
77. Use an Euler diagram to determine whether the following argument is valid or invalid.

All fried foods are greasy.

All greasy foods are delicious.

All delicious foods are healthy.

∴ All fried foods are healthy.



**Solution:**

The figure illustrates that every fried food is an element of the set of healthy foods, so the argument is valid.

78. Use an Euler diagram to determine whether the following argument is valid or invalid.

All squares are rhombi.

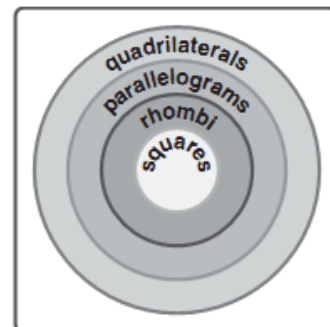
All rhombi are parallelograms.

All parallelograms are quadrilaterals.

∴ All squares are quadrilaterals.

**Solution:**

The following Euler diagram illustrates that all squares are quadrilaterals, so the argument is a valid argument.



### Using Euler Diagrams to Form Conclusions

79. Use an Euler diagram and all of the premises in the following argument to determine a valid conclusion for the argument.

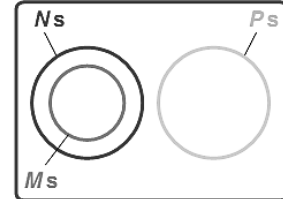
All Ms are Ns.

No Ns are Ps.

$\therefore ?$

**Solution:**

The first premise indicates that the set of Ms is a subset of the set of Ns. The second premise indicates that the set of Ns and the set of Ps are disjoint sets. The following Euler diagram illustrates these set relationships. An examination of the Euler diagram allows us to conclude that no Ms are Ps.



80. Use an Euler diagram and all of the premises in the following argument to determine a valid conclusion for the argument.

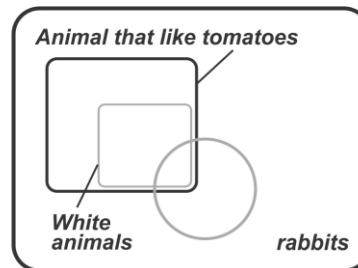
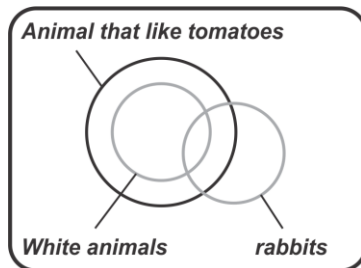
Some rabbits are white.

All white animals like tomatoes.

$\therefore ?$

**Solution:**

The following Euler diagrams illustrate two possible cases. In both cases we see that all white rabbits like tomatoes.



## Quantifiers

The word or symbol, which conveys the idea of quantity or numbers, is called quantifier. In mathematics and logic, quantifiers specify the scope of a predicate or statement.

- Universal Quantifier ( $\forall$ ): "For all"
- Existential Quantifier ( $\exists$ ): "There exists"
- In a statement, the word *some* and the phrases *there exists* and *at least one* are called existential quantifiers. Existential quantifiers are used as prefixes to assert the existence of something.
- In a statement, the words *none*, *no*, *all*, and *every* are called universal quantifiers.
- The universal quantifiers *none* and *no* deny the existence of something, whereas the universal quantifiers *all* and *every* are used to assert that every element of a given set satisfies some condition.

### Universal Quantification

The universal quantification of  $P(x)$  is the statement “ $P(x)$  for all values of  $x$  in the domain.”

The notation  $\forall xP(x)$  denotes the universal quantification of  $P(x)$ . Here  $\forall$  is called the universal quantifier. We read  $\forall xP(x)$  as “for all  $xP(x)$ ” or “for every  $xP(x)$ .” An element for which  $P(x)$  is false is called a counterexample to  $\forall xP(x)$ .

Statement	When True?	When False?
$\forall xP(x)$	$P(x)$ is true for every $x$ .	There is an $x$ for which $P(x)$ is false.
$\exists xP(x)$	There is an $x$ for which $P(x)$ is true.	$P(x)$ is false for every $x$ .

**81. Let  $P(x)$  be the statement “ $x + 1 > x$ .” What is the truth value of the quantification  $\forall xP(x)$ , where the domain consists of all real numbers?**

**Solution:**

Because  $P(x)$  is true for all real numbers  $x$ , the quantification  $\forall xP(x)$  is true.

**Remark**

- Generally, an implicit assumption is made that all domains of discourse for quantifiers are nonempty. Note that if the domain is empty, then  $\forall xP(x)$  is true for any propositional function  $P(x)$  because there are no elements  $x$  in the domain for which  $P(x)$  is false.
- A statement  $\forall xP(x)$  is false, where  $P(x)$  is a propositional function, if and only if  $P(x)$  is not always true when  $x$  is in the domain. One way to show that  $P(x)$  is not always true when  $x$  is in the domain is to find a counterexample to the statement  $\forall xP(x)$ . Note that a single counterexample is all we need to establish that  $\forall xP(x)$  is false.

**82. Let  $Q(x)$  be the statement “ $x < 2$ .” What is the truth value of the quantification  $\forall xQ(x)$ , where the domain consists of all real numbers?**

**Solution:**

$Q(x)$  is not true for every real number  $x$ , because, for instance,  $Q(3)$  is false. That is,  $x = 3$  is a counterexample for the statement  $\forall xQ(x)$ . Thus,  $\forall xQ(x)$  is false.

**83. Suppose that  $P(x)$  is “ $x^2 > 0$ .” To show that the statement  $\forall xP(x)$  is false where the universe of discourse consists of all integers.**

**Solution**

Suppose that  $P(x)$  is “ $x^2 > 0$ .” To show that the statement  $\forall xP(x)$  is false where the universe of discourse consists of all integers, we give a counterexample. We see that  $x = 0$  is a counterexample because  $x^2 = 0$  when  $x = 0$ , so that  $x^2$  is not greater than 0 when  $x = 0$ .

**84. What does the statement  $\forall xN(x)$  mean if  $N(x)$  is “Computer  $x$  is connected to the network” and the domain consists of all computers on campus?**

**Solution:**

The statement  $\forall xN(x)$  means that for every computer  $x$  on campus, that computer  $x$  is connected to the network. This statement can be expressed in English as “Every computer on campus is connected to the network.”

**85. What is the truth value of  $\forall x(x^2 \geq x)$  if the domain consists of all real numbers? What is the truth value of this statement if the domain consists of all integers?**

**Solution:**

The universal quantification  $\forall x(x^2 \geq x)$ , where the domain consists of all real numbers, is false. For example,  $\left(\frac{1}{2}\right)^2 \not\geq \left(\frac{1}{2}\right)$ .

Note that  $x^2 \geq x$  if and only if  $x^2 - x = x(x - 1) \geq 0$ . Consequently,  $x^2 \geq x$  if and only if  $x \leq 0$  or  $x \geq 1$ . It follows that  $\forall x(x^2 \geq x)$  is false if the domain consists of all real numbers (because the inequality is false for all real numbers  $x$  with  $0 < x < 1$ ).

However, if the domain consists of the integers,  $\forall x(x^2 \geq x)$  is true, because there are no integers  $x$  with  $0 < x < 1$ .

### Existential Quantification

The existential quantification of  $P(x)$  is the proposition

“There exists an element  $x$  in the domain such that  $P(x)$ .”

We use the notation  $\exists xP(x)$  for the existential quantification of  $P(x)$ .

Here  $\exists$  is called the existential quantifier.

**86. Let  $P(x)$  denote the statement “ $x > 3$ .” What is the truth value of the quantification  $\exists xP(x)$ , where the domain consists of all real numbers?**

**Solution:**

Because “ $x > 3$ ” is sometimes true—for instance, when  $x = 4$ —the existential quantification of  $P(x)$ , which is  $\exists xP(x)$ , is true.

Observe that the statement  $\exists xP(x)$  is false if and only if there is no element  $x$  in the domain for which  $P(x)$  is true. That is,  $\exists xP(x)$  is false if and only if  $P(x)$  is false for every element of the domain. We illustrate this observation in next Question.

**87. Let  $Q(x)$  denote the statement “ $x = x + 1$ .” What is the truth value of the quantification  $\exists xQ(x)$ , where the domain consists of all real numbers?**

**Solution:**

Because  $Q(x)$  is false for every real number  $x$ , the existential quantification of  $Q(x)$ , which is  $\exists xQ(x)$ , is false.

## Predicates

In mathematics and logic, a predicate is a statement that contains variables and asserts a property or relationship about those variables. Predicates are used to define sets, functions, and relationships.

### Types of Predicates

- Universal predicate ( $\forall$ ): "For all"
- Existential predicate ( $\exists$ ): "There exists"
- Equality predicate ( $=$ ): "Equal to"

**88. Let  $P(x)$  be the predicate "x is an even number." What is  $P(4)$ ?**

**Solution:** True

**89. Let  $Q(x)$  be the predicate "x > 5." What is  $Q(3)$ ?**

**Solution:** False

**90.  $\exists x (x^2 = 16)$**

**Solution:** True ( $x = \pm 4$ )

**91.  $\forall x (x + 0 = x)$**

**Solution:** True (additive identity)

**92. Let  $P(x)$  denote the statement "x > 3" What are the truth values of  $P(4)$  and  $P(2)$ ?**

**Solution:**

We obtain the statement  $P(4)$  by setting  $x = 4$  in the statement "x > 3." Hence,  $P(4)$ , which is the statement "4 > 3," is true. However,  $P(2)$ , which is the statement "2 > 3," is false.

**93. Let  $A(x)$  denote the statement "Computer x is under attack by an intruder." Suppose that of the computers on campus, only CS2 and MATH1 are currently under attack by intruders. What are truth values of  $A(\text{CS1})$ ,  $A(\text{CS2})$ , and  $A(\text{MATH1})$ ?**

**Solution:**

We obtain the statement  $A(\text{CS1})$  by setting  $x = \text{CS1}$  in the statement "Computer x is under attack by an intruder." Because CS1 is not on the list of computers currently under attack, we conclude that  $A(\text{CS1})$  is false. Similarly, because CS2 and MATH1 are on the list of computers under attack, we know that  $A(\text{CS2})$  and  $A(\text{MATH1})$  are true. We can also have statements that involve more than one variable. For instance, consider the statement "x = y + 3." We can denote this statement by  $Q(x, y)$ , where x and y are variables and Q is the predicate. When values are assigned to the variables x and y, the statement  $Q(x, y)$  has a truth value.

**94. Let  $Q(x, y)$  denote the statement "x = y + 3." What are the truth values of the propositions  $Q(1, 2)$  and  $Q(3, 0)$ ?**

**Solution:**

To obtain  $Q(1, 2)$ , set  $x = 1$  and  $y = 2$  in the statement  $Q(x, y)$ . Hence,  $Q(1, 2)$  is the statement "1 = 2 + 3," which is false. The statement  $Q(3, 0)$  is the proposition "3 = 0 + 3," which is true.

95. Let  $A(c, n)$  denote the statement “Computer  $c$  is connected to network  $n$ ,” where  $c$  is a variable representing a computer and  $n$  is a variable representing a network. Suppose that the computer MATH1 is connected to network CAMPUS2, but not to network CAMPUS1. What are the values of  $A(\text{MATH1}, \text{CAMPUS1})$  and  $A(\text{MATH1}, \text{CAMPUS2})$ ?

**Solution:**

Because MATH1 is not connected to the CAMPUS1 network, we see that  $A(\text{MATH1}, \text{CAMPUS1})$  is false. However, because MATH1 is connected to the CAMPUS2 network, we see that  $A(\text{MATH1}, \text{CAMPUS2})$  is true.

Similarly, we can let  $R(x, y, z)$  denote the statement “ $x + y = z$ .” When values are assigned to the variables  $x$ ,  $y$ , and  $z$ , this statement has a truth value.

96. What are the truth values of the propositions  $R(1, 2, 3)$  and  $R(0, 0, 1)$ ?

**Solution:**

The proposition  $R(1, 2, 3)$  is obtained by setting  $x = 1$ ,  $y = 2$ , and  $z = 3$  in the statement  $R(x, y, z)$ . We see that  $R(1, 2, 3)$  is the statement “ $1 + 2 = 3$ ,” which is true. Also note that  $R(0, 0, 1)$ , which is the statement “ $0 + 0 = 1$ ,” is false.

**Key Concepts**

1. Quantifiers ( $\forall, \exists$ )
2. Variables ( $x, y, z$ )
3. Predicates ( $P(x), Q(x), R(x, y)$ )
4. Logical operators ( $\wedge, \vee, \sim$ )
5. Set notation ( $\{x \mid P(x)\}$ )

## Logical Fallacies

A fallacy is a deceptive argument—an argument in which the conclusion is not well supported by the premises. While logical fallacies are errors in reasoning that undermine the validity of an argument, leading to misleading or false conclusions. These fallacies occur when arguments are based on flawed assumptions, invalid inferences, or misleading evidence. In logic, the term **argument** refers to a reasoned or thoughtful process. Specifically, an argument uses a set of facts or assumptions, called **premises**, to support a **conclusion**. A **fallacy** is a deceptive argument, an argument in which the conclusion is not well supported by the premises.

### Types of Logical Fallacies

#### Informal Fallacies (error in the content of an argument)

- Ad Hominem: Attacking the person, not the argument.
- Straw Man: Misrepresenting or exaggerating an opposing argument.
- False Dilemma: Presenting only two options when more exist.
- Slippery Slope: Assuming a chain of events without evidence.
- Begging the Question: Assuming the truth of the conclusion.

**Formal Fallacies (error in the logical structure of an argument)**

- Appeal to Authority: Using authority to support an unjustified claim.
- Appeal to Emotion: Using emotions rather than logic.
- Appeal to Popularity: Arguing that popularity validates an argument.
- False Cause: Assuming causation without evidence.
- Slanting: Selectively presenting information.

**Fallacies of Presumption**

- Assuming the Conclusion: Using a conclusion as a premise.
- Shifting the Burden of Proof: Requiring others to disprove an argument.
- Lack of Evidence: Arguing from ignorance.

**Fallacies of Ambiguity**

- Equivocation: Using ambiguous language.
- Vagueness: Using unclear language.

**Fallacies of Relevance**

- Red Herring: Introducing irrelevant information.
- Non Sequitur: Drawing unrelated conclusions.

**Characteristics of Logical Fallacies**

- Misleading or false premises
- Flawed assumptions
- Invalid inferences
- Misleading evidence
- Emotional appeals

**Consequences of Logical Fallacies**

- Misleading conclusions
- Invalid arguments
- Poor decision-making
- Miscommunication
- Erosion of critical thinking

**Avoiding Logical Fallacies**

- Use clear language.
- Support claims with evidence.
- Address counterarguments.
- Avoid emotional appeals.
- Encourage critical thinking.

**97. Write a note on logical fallacy Appeal to Popularity.****Solution:**

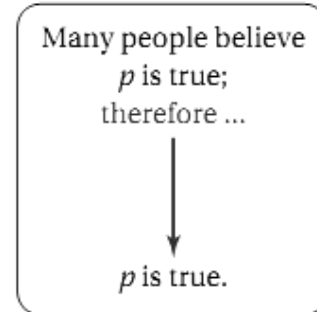
**Appeal to Popularity:** “Ford makes the best pickup trucks in the world. More people drive Ford pickups than any other light truck.”

**Analysis** The first step in dealing with any argument is recognizing which statements are premises and which are conclusions. This argument tries to make the case that Ford makes the best pickup trucks in the world, so this statement is its conclusion. The only evidence it offers to support this conclusion is the statement more people drive Ford pickups than any other light truck. This is the argument's only premise. Overall, this argument has the form

**Premise:** More people drive Ford pickups than any other light truck.

**Conclusion:** Ford makes the best pickup trucks in the world.

Note that the original written argument states the conclusion before the premise. Such “backward” structures are common in everyday speech and are perfectly legitimate as long as the argument is well reasoned. In this case, however, the reasoning is faulty. The fact that more people drive Ford pickups does not necessarily mean that they are the best trucks. This argument suffers from the fallacy of appeal to popularity (or appeal to majority), in which the fact that large numbers of people believe or act some way is used inappropriately as evidence that the belief or action is correct. We can represent the general form of this fallacy with a diagram in which the letter  $p$  stands for a particular statement (Figure). In this case,  $p$  stands for the statement Ford makes the best pickup trucks in the world.



### 98. Write a note on logical fallacy False Cause.

**Solution:**

**False Cause:** “I placed the quartz crystal on my forehead, and in five minutes my headache was gone. The crystal made my headache go away.”

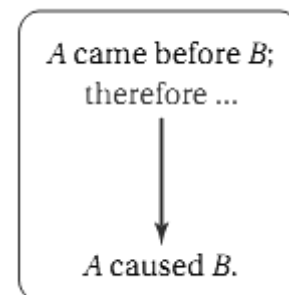
**Analysis** We identify the premises and conclusion of this argument as follows:

**Premise:** I placed the quartz crystal on my forehead.

**Premise:** Five minutes later my headache was gone.

**Conclusion:** The crystal made my headache go away.

The premises tell us that one thing (crystal on forehead) happened before another (headache went away), but they don't prove any connection between them. That is, we cannot conclude that the crystal caused the headache to go away. This argument suffers from the fallacy of false cause, in which the fact that one event came before another is incorrectly taken as evidence that the first event caused the second event. We can represent this fallacy with a diagram in which  $A$  and  $B$  represent two different events (Figure). In this case,  $A$  is the event of putting the crystal on the forehead and  $B$  is the event of the headache going away.



**99. Write a note on logical fallacy Appeal to ignorance.****Solution:**

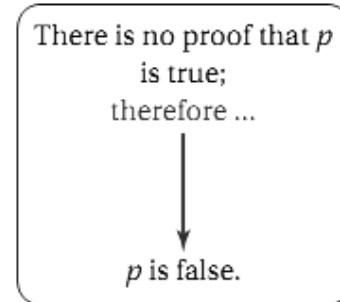
**Appeal to ignorance:** “Scientists have not found any concrete evidence of aliens visiting Earth. Therefore, anyone who claims to have seen a UFO must be hallucinating.”

**Analysis** If we strip the argument to its core, it says this:

**Premise:** There’s no proof that aliens have visited Earth.

**Conclusion:** Aliens have not visited Earth.

The fallacy should be clear: A lack of proof of alien visits does not mean that visits have not occurred. This fallacy is called appeal to ignorance because it uses ignorance (lack of knowledge) about the truth of a proposition to conclude the opposite (Figure). We sometimes sum up this fallacy with the statement “An absence of evidence is not evidence of absence.”

**100. Write a note on logical fallacy Hasty generalization.****Solution:**

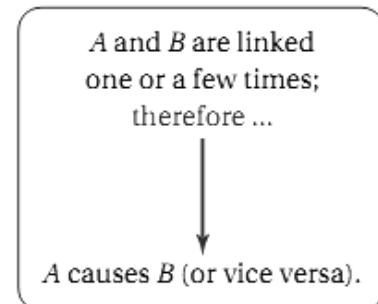
**Hasty generalization:** “Two cases of childhood leukemia have occurred along the street where the high-voltage power lines run. The power lines must be the cause of these illnesses.”

**Analysis** The premise of this argument cites two cases of leukemia, but two cases are not enough to establish a pattern, let alone to conclude that the power lines caused the illnesses.

The fallacy here is hasty generalization, in which a conclusion is drawn from an inadequate number of cases or cases that have not been sufficiently analyzed.

If any connection between power lines and leukemia exists, it would have to be established with far more evidence than is provided in this argument. (In fact, decades of research have found no connection between power lines and illness.)

We can represent this fallacy with a diagram in which  $A$  and  $B$  represent two linked events (Figure).



**101. Write a note on logical fallacy Limited Choice.****Solution:**

**Limited Choice:** “You don’t support the President, so you are not a patriotic American.”

**Analysis** This argument has the form

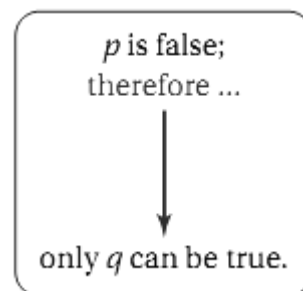
**Premise:** You don’t support the President.

**Conclusion:** You are not a patriotic American.

The argument suggests that there are only two types of Americans: patriotic ones who support the President and unpatriotic ones who don’t. But there are many other possibilities, such as being patriotic while disliking a particular President.

This fallacy is called limited choice (or false dilemma) because it artificially precludes choices that ought to be considered. Figure shows one common form of this fallacy.

Limited choice also arises with questions such as “Have you stopped smoking?” Because both yes and no answers imply that you smoked in the past, the question precludes the possibility that you never smoked. (In legal proceedings, questions of this type are disallowed because they attempt to “lead the witness.”) Another simple and common form of this fallacy is “You’re wrong, so I must be right.”

**102. Write a note on logical fallacy Appeal to Emotion.****Solution:**

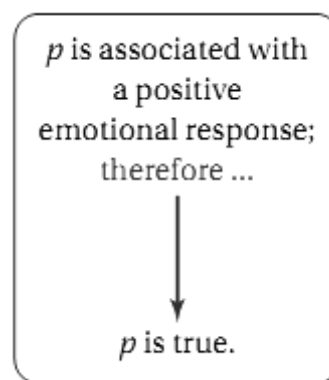
**Appeal to Emotion:** In ads for Michelin tires, a picture of a baby is shown with the words “because so much is riding on your tires.”

**Analysis** If we can consider this an argument at all, it has the form

**Premise:** You love your baby.

**Conclusion:** You should buy Michelin tires.

The advertisers hope that the love you feel for a baby will make you want to buy their tires. This attempt to evoke an emotional response as a tool of persuasion represents the fallacy of appeal to emotion. Figure shows its form when the emotional response is positive. Sometimes the appeal is to negative emotions. For example, the statement if my opponent is elected, your tax burden will rise tries to convince you that electing the other candidate will lead to consequences you won’t like. (In this negative form, the fallacy is sometimes called appeal to force.)



**103. Write a note on logical fallacy Personal attack.****Solution:****Personal attack**

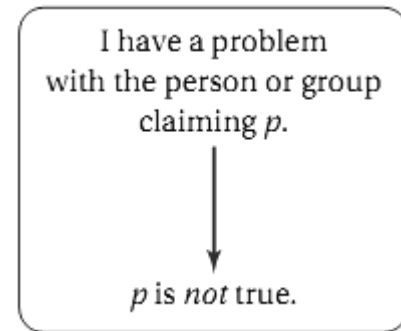
**Gwen:** You should stop drinking because it's hurting your grades, endangering people when you drink and drive, and destroying your relationship with your family.

**Merle:** I've seen you drink a few too many on occasion yourself!

**Analysis** Gwen's argument is well reasoned, with premises offering strong support for her conclusion that Merle should stop drinking.

Merle rejects this argument by noting that Gwen sometimes drinks too much herself. Even if Merle's claim is true, it is irrelevant to Gwen's point. Merle has resorted to attacking Gwen personally rather than arguing logically, so we call this fallacy personal attack (Figure). (It is also called *ad hominem*, Latin for "to the person.")

The fallacy of personal attack can also apply to groups. For example, someone might say, "This new bill will be an environmental disaster because its sponsors received large campaign contributions from oil companies." This argument is fallacious because it doesn't challenge the provisions of the bill, but only questions the motives of the sponsors.

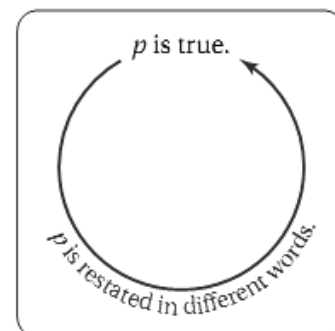
**104. Write a note on logical fallacy Circular reasoning.****Solution:**

**Circular reasoning:** "Society has an obligation to provide health insurance because health care is a right of citizenship."

**Analysis** This argument states the conclusion (society has an obligation to provide health insurance) before the premise (health care is a human right).

But if you read carefully, you'll recognize that the premise and the conclusion both say essentially the same thing, as social obligations are generally based on definitions of accepted rights.

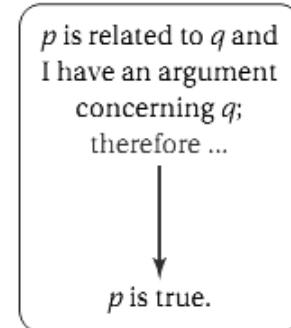
This argument therefore suffers from circular reasoning (Figure).



**105. Write a note on logical fallacy Diversion (red herring).****Solution:**

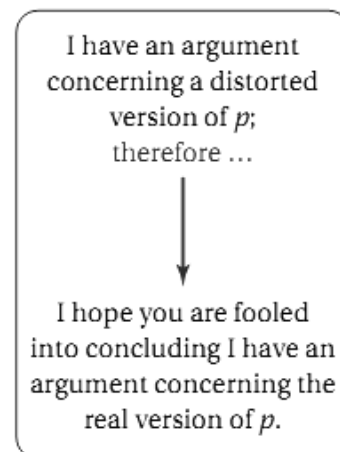
**Diversion (red herring):** “We should not continue to fund cloning research because there are so many ethical issues involved. Decisions are based on ethics, and we cannot afford to have too many ethical loose ends.”

**Analysis** The argument begins with its conclusion—we should not continue to fund cloning research. However, the discussion is all about ethics. This argument represents the fallacy of diversion (Figure) because it attempts to divert attention from the real issue (funding for cloning research) by focusing on another issue (ethics). The issue to which attention is diverted is sometimes called a red herring. (A herring is a fish that turns red when rotten. Use of the term red herring to mean a diversion can be traced back to the 19th century, when British fugitives discovered that they could divert bloodhounds from their pursuit by rubbing a red herring across their trail.)

**106. Write a note on logical fallacy Straw Man.****Solution:**

**Straw Man:** Suppose that the mayor of a large city proposes decriminalizing drug possession in order to reduce overcrowding in jails and save money on enforcement. His challenger in the upcoming election says, “The mayor doesn’t think there’s anything wrong with drug use, but I do.”

**Analysis** The mayor did not say that drug use is acceptable. His proposal for decriminalization is designed to solve another problem—overcrowding of jails—and tells us nothing about his general views on drug use. The speaker has distorted the mayor’s views. Any argument based on a distortion of someone’s words or beliefs is called a straw man (Figure). The term comes from the idea that the speaker has used a poor representation of a person’s beliefs in the same way that a straw man is a poor representation of a real man. A straw man is similar to a diversion. The primary difference is that a diversion argues against an unrelated issue, while the straw man argues against a distorted version of the real issue.



**Exercise**

1. In Exercises a – j, determine whether each sentence is a statement.
  - a) West Virginia is west of the Mississippi River.
  - b) 1031 is a prime number.
  - c) The area code for Storm Lake, Iowa, is 512.
  - d) Some negative numbers are rational numbers.
  - e) Have a fun trip.
  - f) Do you like to read?
  - g) All hexagons have exactly five sides.
  - h) If  $x$  is a negative number, then  $x^2$  is a positive number.
  - i) Mathematics courses are better than history courses.
  - j) Every real number is a rational number.
2. In Exercises a – d, write the negation of each statement.
  - a) The Giants lost the game.
  - b) The lunch was served at noon.
  - c) The game did not go into overtime.
  - d) The game was not shown on ABC.
3. Which of these sentences are propositions? What are the truth values of those that are propositions?
  - a) Boston is the capital of Massachusetts.
  - b) Miami is the capital of Florida.
  - c)  $2 + 3 = 5$ .
  - d)  $5 + 7 = 10$ .
  - e)  $x + 2 = 11$ .
  - f) Answer this question.
4. Which of these are propositions? What are the truth values of those that are propositions?
  - a) Do not pass go.
  - b) What time is it?
  - c) There are no black flies in Maine.
  - d)  $4 + x = 5$ .
  - e) The moon is made of green cheese.
  - f)  $2^n \geq 100$ .
5. What is the negation of each of these propositions?
  - a) Linda is younger than Sanjay.
  - b) Mei makes more money than Isabella.
  - c) Moshe is taller than Monica.
  - d) Abby is richer than Ricardo.

6. What is the negation of each of these propositions?
  - a) Janice has more Facebook friends than Juan.
  - b) Quincy is smarter than Venkat.
  - c) Zelda drives more miles to school than Paola.
  - d) Briana sleeps longer than Gloria.
7. What is the negation of each of these propositions?
  - a) Mei has an MP3 player.
  - b) There is no pollution in New Jersey.
  - c)  $2 + 1 = 3$ .
  - d) The summer in Maine is hot and sunny.
8. What is the negation of each of these propositions?
  - a) Jennifer and Teja are friends.
  - b) There are 13 items in a baker's dozen.
  - c) Abby sent more than 100 text messages yesterday.
  - d) 121 is a perfect square.
9. What is the negation of each of these propositions?
  - a) Steve has more than 100 GB free disk space on his laptop.
  - b) Zach blocks e-mails and texts from Jennifer.
  - c)  $7.11.13 = 999$ .
  - d) Diane rode her bicycle 100 miles on Sunday.
10. Suppose that Smartphone A has 256 MB RAM and 32 GB ROM, and the resolution of its camera is 8 MP; Smartphone B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; and Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP. Determine the truth value of each of these propositions.
  - a) Smartphone B has the most RAM of these three smartphones.
  - b) Smartphone C has more ROM or higher resolution camera than Smartphone B.
  - c) Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.
  - d) If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.
  - e) Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.
11. Suppose that during the most recent fiscal year, the annual revenue of Acme Computer was 138 billion dollars and its net profit was 8 billion dollars, the annual revenue of Nadir Software was 87 billion dollars and its net profit was 5 billion dollars, and the annual revenue of Quixote Media was 111 billion dollars and its net profit was 13 billion dollars. Determine the truth value of each of these propositions for the most recent fiscal year.

- a) Quixote Media had the largest annual revenue.  
 b) Nadir Software had the lowest net profit and Acme Computer had the largest annual revenue.  
 c) Acme Computer had the largest net profit or Quixote Media had the largest net profit.  
 d) If Quixote Media had the smallest net profit, then Acme Computer had the largest annual revenue.  
 e) Nadir Software had the smallest net profit if and only if Acme Computer had the largest annual revenue.
- 12.** Let  $p$  and  $q$  be the propositions  
 $p$ : You drive over 65 miles per hour.  
 $q$ : You get a speeding ticket.  
 Write these propositions using  $p$  and  $q$  and logical connectives (including negations).
- a) You do not drive over 65 miles per hour.  
 b) You drive over 65 miles per hour, but you do not get a speeding ticket.  
 c) You will get a speeding ticket if you drive over 65 miles per hour.  
 d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.  
 e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.  
 f) You get a speeding ticket, but you do not drive over 65 miles per hour.  
 g) Whenever you get a speeding ticket, you are driving over 65 miles per hour.
- 13.** State the converse, contrapositive, and inverse of each of these conditional statements.
- a) If it snows today, I will ski tomorrow.  
 b) I come to class whenever there is going to be a quiz.  
 c) A positive integer is a prime only if it has no divisors other than 1 and itself.
- 14.** State the converse, contrapositive, and inverse of each of these conditional statements.
- a) If it snows tonight, then I will stay at home.  
 b) I go to the beach whenever it is a sunny summer day.  
 c) When I stay up late, it is necessary that I sleep until noon.
- 15.** Construct truth table of converse, inverse and contra positive of given conditional.

Conditional	Converse	Inverse	Contra Positive
$\sim p \rightarrow q$			
$q \rightarrow p$			
$\sim p \rightarrow \sim q$			
$\sim q \rightarrow \sim p$			

- 16.** Construct truth table for the following statements.
- $(p \rightarrow \sim p) \vee (p \rightarrow q)$
  - $(p \wedge \sim p) \rightarrow q$
  - $\sim(p \rightarrow q) \leftrightarrow (p \wedge \sim q)$
- 17.** Determine whether each of the following is a tautology or not?
- $(p \wedge q) \rightarrow p$
  - $p \rightarrow (p \vee q)$
  - $\sim(p \rightarrow q) \rightarrow p$
  - $\sim q \wedge (p \rightarrow q) \rightarrow \sim p$
- 18.** Show that each of these conditional statements is a tautology by using truth tables.
- $(p \wedge q) \rightarrow p$
  - $p \rightarrow (p \vee q)$
  - $\sim p \rightarrow (p \rightarrow q)$
  - $(p \wedge q) \rightarrow (p \rightarrow q)$
  - $\sim(p \rightarrow q) \rightarrow p$
  - $\sim(p \rightarrow q) \rightarrow \sim q$
- 19.** Show that each of these conditional statements is a tautology by using truth tables.
- $[\sim p \wedge (p \vee q)] \rightarrow q$
  - $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
  - $[p \wedge (p \rightarrow q)] \rightarrow q$
  - $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$
- 20.** Determine whether each of the following is a tautology, a contingency or an absurdity?
- $p \wedge \sim p$
  - $p \rightarrow (q \rightarrow p)$
  - $q \vee (\sim q \vee p)$
- 21.** Prove that  $p \vee (\sim p \wedge \sim q) \vee (p \wedge q) \equiv p \vee (\sim p \wedge \sim q)$
- 22.** Show that  $p \leftrightarrow q$  and  $(p \wedge q) \vee (\sim p \wedge \sim q)$  are logically equivalent.
- 23.** Show that  $\sim(p \leftrightarrow q)$  and  $p \leftrightarrow \sim q$  are logically equivalent.
- 24.** Show that  $p \rightarrow q$  and  $\sim q \rightarrow \sim p$  are logically equivalent.
- 25.** Show that  $\sim p \leftrightarrow q$  and  $p \leftrightarrow \sim q$  are logically equivalent.
- 26.** Show that  $\sim(p \oplus q)$  and  $p \leftrightarrow q$  are logically equivalent.
- 27.** Show that  $\sim(p \leftrightarrow q)$  and  $\sim p \leftrightarrow q$  are logically equivalent.
- 28.** Show that  $(p \rightarrow q) \wedge (p \rightarrow r)$  and  $p \rightarrow (q \wedge r)$  are logically equivalent.
- 29.** Show that  $(p \rightarrow r) \wedge (q \rightarrow r)$  and  $(p \vee q) \rightarrow r$  are logically equivalent.
- 30.** Show that  $(p \rightarrow q) \vee (p \rightarrow r)$  and  $p \rightarrow (q \vee r)$  are logically equivalent.
- 31.** Show that  $(p \rightarrow r) \vee (q \rightarrow r)$  and  $(p \wedge q) \rightarrow r$  are logically equivalent.
- 32.** Show that  $\sim p \rightarrow (q \rightarrow r)$  and  $q \rightarrow (p \vee r)$  are logically equivalent.

33. Show that  $p \leftrightarrow q$  and  $(p \rightarrow q) \wedge (q \rightarrow p)$  are logically equivalent.
34. Show that  $p \leftrightarrow q$  and  $\sim p \leftrightarrow \sim q$  are logically equivalent.
35. Show that  $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$  is a tautology.
36. Show that  $(p \vee q) \wedge (\sim p \vee r) \rightarrow (q \vee r)$  is a tautology.
37. Show that  $(p \rightarrow q) \rightarrow r$  and  $p \rightarrow (q \rightarrow r)$  are not logically equivalent.
38. Show that  $(p \wedge q) \rightarrow r$  and  $(p \rightarrow r) \wedge (q \rightarrow r)$  are not logically equivalent.
39. Show that  $(p \rightarrow q) \rightarrow (r \rightarrow s)$  and  $(p \rightarrow r) \rightarrow (q \rightarrow s)$  are not logically equivalent.
40. Let  $P(x)$  denote the statement “ $x \leq 4$ .” What are these truth values?  
 a)  $P(0)$                       b)  $P(4)$                       c)  $P(6)$
41. Let  $P(x)$  be the statement “The word  $x$  contains the letter a.” What are these truth values?  
 a)  $P(\text{orange})$                       b)  $P(\text{lemon})$   
 c)  $P(\text{true})$                       d)  $P(\text{false})$
42. Let  $Q(x, y)$  denote the statement “ $x$  is the capital of  $y$ .” What are these truth values?  
 a)  $Q(\text{Denver, Colorado})$   
 b)  $Q(\text{Detroit, Michigan})$   
 c)  $Q(\text{Massachusetts, Boston})$   
 d)  $Q(\text{New York, New York})$
43. State the value of  $x$  after the statement if  $P(x)$  then  $x := 1$  is executed, where  $P(x)$  is the statement “ $x > 1$ ,” if the value of  $x$  when this statement is reached is  
 a)  $x = 0$ .      b)  $x = 1$ .      c)  $x = 2$ .
44. Let  $P(x)$  be the statement “ $x$  spends more than five hours every weekday in class,” where the domain for  $x$  consists of all students. Express each of these quantifications in English.  
 a)  $\exists x P(x)$                       b)  $\forall x P(x)$                       c)  $\exists x \sim P(x)$                       d)  $\forall x \sim P(x)$
45. Let  $N(x)$  be the statement “ $x$  has visited North Dakota,” where the domain consists of the students in your school. Express each of these quantifications in English.  
 a)  $\exists x N(x)$                       b)  $\forall x N(x)$                       c)  $\sim \exists x N(x)$   
 d)  $\exists x \sim N(x)$                       e)  $\sim \forall x N(x)$                       f)  $\forall x \sim N(x)$
46. Construct the following theorems to logical form and prove them by constructing truth tables.
- a)  $(A \cap B)' = A' \cup B'$   
 b)  $(A \cup B) \cup C = A \cup (B \cup C)$   
 c)  $(A \cap B) \cap C = A \cap (B \cap C)$   
 d)  $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

- 47.** In Exercises i–viii, use a truth table to determine whether the argument is valid or invalid.

<p>a) <math>\frac{p \vee \sim q}{\sim q}</math></p> <p><math>\therefore p</math></p> <p><math>p \rightarrow \sim q</math></p>	<p>b) <math>\frac{\sim p \wedge q}{\sim p}</math></p> <p><math>\therefore q</math></p> <p><math>\sim p \rightarrow \sim q</math></p>	<p>c) <math>\frac{p \rightarrow \sim q}{\sim q}</math></p> <p><math>\therefore p</math></p> <p><math>\sim p \rightarrow q</math></p>
<p>d) <math>\frac{p}{\sim q}</math></p> <p><math>(p \rightarrow q) \wedge (\sim p \rightarrow q)</math></p>	<p>e) <math>\frac{\sim q}{\sim q}</math></p> <p><math>\therefore \sim q</math></p>	<p>f) <math>\frac{p}{(p \vee q) \wedge (p \wedge q)}</math></p> <p><math>\therefore \sim q</math></p>
<p>g) <math>\frac{q}{\therefore p}</math></p>	<p>h) <math>\frac{p}{\therefore q}</math></p>	

- 48.** In Exercises a–f, use the indicated letters to write the argument in symbolic form. Then use a truth table to determine whether the argument is valid or invalid.

- a) If you finish your homework ( $h$ ) you may attend the reception ( $r$ ). You did not finish your homework. Therefore, you cannot go to the reception.
- b) The X Games will be held in Oceanside ( $o$ ) if and only if the city of Oceanside agrees to pay \$100,000 in prize money ( $a$ ). If San Diego agrees to pay \$200,000 in prize money ( $s$ ), then the city of Oceanside will not agree to pay \$100,000 in prize money. Therefore, if the X Games were held in Oceanside, then San Diego did not agree to pay \$200,000 in prize money.
- c) If I can't buy the house ( $\sim b$ ) then at least I can dream about it ( $d$ ). I can buy the house or at least I can dream about it. Therefore, I can buy the house.
- d) If the winds are from the east ( $e$ ) then we will not have a big surf ( $\sim s$ ). We do not have a big surf. Therefore, the winds are from the east.
- e) If I master college algebra ( $c$ ) then I will be prepared for trigonometry I am prepared for trigonometry ( $t$ ). Therefore, I mastered college algebra.
- f) If it is a blot ( $b$ ) then it is not a clot ( $\sim c$ ). If it is a zlot ( $z$ ) then it is a clot. It is a blot. Therefore, it is not a zlot.

- 49.** In Exercises a–d, draw an Euler diagram that illustrates the relationship between the given sets. Also use a dot to show an element of the first set that satisfies the given relationship.

- a) All cats ( $C$ ) are nimble ( $N$ ).
- b) Some mathematicians ( $M$ ) are extroverts ( $E$ ).
- c) Some actors ( $A$ ) are not famous ( $F$ ).
- d) No alligators ( $A$ ) are trustworthy ( $T$ ).

- 50.** In Exercises i–xvi, use an Euler diagram to determine whether the argument is valid or invalid.
- i.** All frogs are poetical.  
Kermit is a frog.  
∴Kermit is poetical.
  - ii.** All Oreo cookies have a filling.  
All Fig Newtons have a filling.  
∴All Fig Newtons are Oreo cookies.
  - iii.** Some plants have flowers.  
All things that have flowers are beautiful.  
∴Some plants are beautiful.
  - iv.** No squares are triangles.  
Some triangles are equilateral.  
∴No squares are equilateral.
  - v.** No rocker would do the Mariachi.  
All baseball fans do the Mariachi.  
∴No rocker is a baseball fan.
  - vi.** Nuclear energy is not safe.  
Some electric energy is safe.  
∴No electric energy is nuclear energy.
  - vii.** Some birds bite.  
All things that bite are dangerous.  
∴Some birds are dangerous.
  - viii.** All fish can swim.  
That barracuda can swim.  
∴That barracuda is a fish.
  - ix.** All men behave badly.  
Some hockey players behave badly.  
∴Some hockey players are men.
  - x.** All grass is green.  
That ground cover is not green.  
∴That ground cover is not grass.
  - xi.** Most teenagers drink soda.  
No CEOs drink soda.  
No CEO is a teenager.
  - xii.** Some students like history.  
Vern is a student.  
∴Vern likes history.

- xiii.** No mathematics test is fun.  
All fun things are worth your time.  
∴ No mathematics test is worth your time.
- xiv.** All prudent people shun sharks.  
No accountant is imprudent.  
∴ No accountant fails to shun sharks.
- xv.** All candidates without a master's degree will not be considered for the position of director.  
All candidates who are not considered for the position of director should apply for the position of assistant.  
∴ All candidates without a master's degree should apply for the position of assistant.
- xvi.** Some whales make good pets. Some good pets are cute.  
Some cute pets bite.  
∴ Some whales bit