INTRODUCTION TO ENUMERATION AND ITS APPLICATIONS

CHAPTER 1

Welcome to Quantitative Skills and Reasoning! Just what are quantitative skills and reasoning? The simple answer is working with numbers, making sense of data, and using your brain to figure things out. This will cover fundamental concepts from problem solving, statistics, probability, graphs, logic, sets, measurements, and finance.

In this chapter we will learn about;

- Introduction to quantitative reasoning
- Types of quantitative reasoning
- Overview of Mathematics
- Arithmetic and basic arithmetic operations
- Types of standard numbers system
- Base numbers system and its applications
- Contributions of mathematicians and statisticians especially Muslim scholars
- Inductive reasoning, deductive reasoning and abductive reasoning
- Exercises about e introduction to enumeration and its applications

What is Reasoning?

Reasoning is the ability to assess things rationally by applying logic based on new or existing information when making a decision or solving a problem. It allows you to weigh the benefits and disadvantages of two or more courses of action before choosing the one with the most benefit or the one that suits your needs.

Types of Reasoning

- **Deductive Reasoning:** Reasoning that uses formal logic and observations to prove a theory or hypothesis. It can be used to apply a general law to a specific case or test an induction. Its results typically have a logical certainty.
- **Inductive Reasoning:** Inductive reasoning is the process of reaching a general conclusion by examining specific examples. It uses theories and assumptions to validate observations. It can be used to apply a specific law to a general. Its results are not always certain because it uses conclusions from observations to make generalizations. It is helpful for extrapolation, prediction and part – to – whole arguments.

- **Analogical Reasoning:** Form of thinking that finds similarities between two or more things and then use those characteristics to find other qualities common to them. It is based on brain tendency. It can help you expand your understanding by looking for similarities between different things.
- **Abductive Reasoning:** Type of reasoning that uses an observation or set of observations to reach a logical conclusion. It is similar to inductive reasoning; however it permits making best guesses to arrive at the simplest conclusions.
- **Cause and Effect Reasoning:** Type of thinking in which you show the linkage between two events. It explains what may happen if an action takes place or why things happen when some conditions are present.
- **Critical Thinking:** It involves extensive rational thought about a specific object in order to come to a definitive conclusion. It is helpful in logic, computing and social sciences.
- **Decompositional Reasoning:** It is the process of breaking things into constituent parts to understand the function of each component and how it contributes to the operation of the item as a whole. It is helpful in logic, computing, game theory, product development, marketing and social sciences.

Quantitative Skills

Any skills that use or manipulate numbers are called quantitative skills. They help to make sense of numerical, categorical or ordinal data and scientific concepts. It is helpful in statistics, economics, algebra, finance, business, logic and social sciences.

Quantitative Reasoning / Quantitative Literacy / Enumeration

Quantitative Reasoning is the ability to assess mathematical ideas or things rationally by applying logic based on new or existing information when making a decision or solving a problem. It is application of mathematical concepts or skills to solve real world problems.

Importance of Quantitative Skills / Enumeration

Enumeration is simply the application of critical thinking skills like analysis and interpretation along with mathematical basics like algebra to quantitative information. It refers to the ability to solve quantitative reasoning problems, or to making judgment derived from quantitative reasoning in a variety of context. It helps to make sense of numerical, categorical or ordinal data and scientific concepts. It is helpful in statistics, economics, algebra, finance, business, logic and social sciences.

Quantitative Reasoning Examples

- **Statistical Analysis:** Analysts apply quantitative reasoning when they assess large dataset to derive meaningful conclusions. They use statistical methods like regression analysis and hypothesis testing to interpret data and distinguish patterns.
- **Financial Planning:** A financial planner utilizes quantitative reasoning for a client’s investment strategy. This involves analyzing expected returns, tax implications and risk factors.

What is Mathematics?

The branch of science that deals with the numbers is called Mathematics. The world "Mathematics" is derived from the Greek word "Mathematikos" which means "inclined to learn".

Mathematics is based on deductive reasoning though man's first experience with mathematics was of an inductive nature. This means that the foundation of mathematics is the study of some logical and philosophical notions. We elaborate in simple terms that the deductive system involves four things:

Known Branches of Mathematics

- **Logic:** The Study of Principles of Reasoning.
- **Arithmetic:** Method for operating on numbers.
- **Algebra:** Method for working with unknown quantities.
- **Geometry:** The study of size and shape.
- **Trigonometry:** The study of triangles and their uses.
- **Probability:** The study of chance.
- **Statistics:** Method for analyzing data.
- **Calculus:** The study of quantities that change.

Number : A number is a mathematical object used to count, measure, & label. It is the mathematical notation for representing numbers of a given set by using digits or other symbols in a consistent manner. It provides a unique representation of every number and represents the arithmetic and algebraic structure of the figures.

Number System

A system of writing to express numbers. It presents unique representation of numbers.

Types of Standard Numbers

1. **Natural Numbers**

   Common counting numbers. Natural numbers are also called "counting numbers" which contains the set of positive integers that start at 1 and continue infinitely. The set of natural numbers is represented by the letter "N".i.e. N = {1,2,3,4,5….}.

2. **Whole Numbers**

   In math, whole numbers are positive integers, including zero, that do not have any decimal or fractional parts. The symbol for whole numbers is "W".

   i.e. W = {0, 1, 2, 3, 4, 5...}.

3. **Integers**

   Integers, also known as whole numbers or round numbers or positive or negative numbers that don't have fractional or decimal parts. The symbol for integers is Z.

   i.e. Z = {...,−1,−2,0,1,2,...}.

4. **Rational Numbers**

   The set of rational numbers includes all the integers, each of which can be written as a quotient with the integer as the numerator and one as the denominator.

Rational number, in arithmetic, a number that can be represented as the quotient p/q of two integers such that q ≠ 0. i.e. $Q = \left\{ x \middle| x = \frac{p}{q}, q \neq 0, p, q \in Z \right\}$.

5. Irrational Numbers

Irrational numbers are real numbers that cannot be written as a fraction of two integers, or in the form of p/q, where p and q are integers and q ≠ 0.

Decimal Representation of Rational and Irrational Numbers

- **i) Terminating decimals:** A decimal which has only a finite number of digits in its decimal part, is called a terminating decimal. Thus 202.04, 0.0000415, 100000.41237895 are examples of terminating decimals. Since a terminating decimal can be converted into a common fraction, so every terminating decimal represents a rational number.
- **ii) Recurring Decimals:** A decimal in which one or more digits repeat indefinitely is called a recurring or periodic decimal. e.g., 1.333…, 21.134134… are both recurring decimals.
- **iii) Non-terminating and Non-recurring decimal:** A non-terminating, non-recurring decimal is a decimal which neither terminates nor it is recurring. It is not possible to convert such a decimal into a common fraction.

Thus a non-terminating, non-recurring decimal represents an irrational number.

Examples:

i) $.25\left(=\frac{25}{100}\right)$ is a rational number.

ii) $.333...\left(\frac{1}{3}=\right)$ is a recurring decimal, it is a rational number.

iii) $2.\overline{3}\,(=2.333...)$ is a rational number.

iv) $0.142857142857...\left(=\frac{1}{7}\right)$ is a rational number.

v) 0.01001000100001 ... is a non-terminating, non-periodic decimal, so it is an irrational number.

vi) 214.121122111222 1111 2222 ... is also an irrational number.

vii) 1.4142135 ... is an irrational number.

viii) 7.3205080 ... is an irrational number.

ix) 1.709975947 ... is an irrational number.

x) 3.141592654...is an important irrational number called it π (Pi)which denotes the constant ratio of the circumference of any circle to length of its diameter

i.e., $$\pi = \frac{circumference\ of\ any\ circle}{length\ of\ its\ diameter}$$

6. **Real Numbers**
   Real numbers can be positive or negative and include fractions, integers and irrational numbers. They can be used in arithmetic operations and represented on a number line. Real numbers include rational and irrational numbers. i.e. $R = Q \cup Q'$.
7. **Prime Numbers**
   Prime numbers are natural numbers that are divisible by only 1 and the number itself. In other words, prime numbers are positive integers greater than 1 with exactly two factors, 1 and the number itself. Some of the prime numbers include
   2, 3, 5, 7, 11, 13, etc.
8. **Composite Numbers**
   A composite number is a natural number or a positive integer which has more than two factors. For example, 15 has factors 1, 3, 5 and 15, hence it is a composite number.
9. **Complex Numbers**
   A complex number is a number that has both real and imaginary parts, and is written in the form $C = \{a + bi : a, b \in R\}$. For example $2 + 0i = 2, 1 + 3i.$
10. **Even & Odd Numbers**
    Even numbers are numbers that can be divided into two equal parts, while odd numbers are numbers that cannot.
    Even numbers: End in 0, 2, 4, 6, or 8.
    Odd numbers: End in 1, 3, 5, 7, or 9.

Arithmetic

Arithmetic is a field of mathematic that studies the characteristics of classical operations on numbers, such as addition, subtraction, multiplication, division, and exponentiation and root extraction.

Arithmetic Operations

Arithmetic is the fundamental of mathematics that includes the operations of numbers. These operations are addition, subtraction, multiplication and division. It is one of the most important branches of mathematics that lays the foundation of the subject for students.

Addition: Combines objects into a larger collection, or increases a value. It is represented by the plus sign (+) and answer is called the sum. For example, 4 + 7 = 11

Subtraction: Finds the difference between numbers or quantities, or decreases a value. It is represented by the minus sign (-) and the answer is called the difference.
For example, 9 - 7 = 2.

Multiplication: Multiplication is represented by the multiplication signs $(\times)$ or $(*)$.
For example, 8 multiplied by 4 is equal to 32, which can be written as $8 \times 4 = 32$.

Division: Division is a method of dividing or distributing a number into equal parts. For example, 16 divided by 4 is equal to 4, which can be written as 16 ÷ 4 = 4.

Base Number System

A base number system, also known as a radix or numeral system, is a mathematical notation that represents numbers using a specific set of digits or symbols. Each base number system has its own unique characteristics and applications, and is used in various fields such as mathematics, computer science, and engineering.

Some common base number systems are as follows:

Base 2 - Binary Number System

The binary number system is a base-2 number system that uses only two digits: 0 and 1.

Base 8 - Octal Number System

The octal number system is a base-8 number system that uses eight digits: 0-7.

Base 10 - Decimal Number System

The decimal number system is a base-10 number system that uses ten digits: 0-9.

Base 16 - Hexadecimal Number System

The hexadecimal number system is a base-16 number system that uses ten digits: 0-15.

Note that usually, the hexadecimal digits used are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F, where the letters A through F represent the digits corresponding to the numbers 10 through 15 (in decimal notation).

1. **What is the decimal expansion of the integer that has $(1\ 0101\ 1111)_2$ as its binary expansion?**

Solution: We have

$$(101011111)_2 = 1.2^8 + 0.2^7 + 1.2^6 + 0.2^5 + 1.2^4 + +1.2^3 + 1.2^2 + 1.2^1 + 1.2^0 = 351$$

2. **What is the decimal expansion of the number with octal expansion $(7016)_8$?**

Solution: Using the definition of a base b expansion with b = 8 tells us that

$$(7016)_8 = 7.8^3 + 0.8^2 + 1.8^1 + 6.8^0 = 3598$$

3. **What is the decimal expansion of the number with hexadecimal expansion $(2AE0B)_{16}$?**

Solution: Using the definition of a base b expansion with b = 16 tells us that

$$(2AEOB)_{16} = 2.16^4 + 10.16^3 + 14.16^2 + 0.16^1 + 11.16^0 = 175627$$

4. **Find the octal expansion of $(12345)_{10}$.**

Solution:

8	12345
8	1543 - 1
8	192 – 7
8	24 – 0
	3 – 0

Hence,$(12345)_{10} = (30071)_8$

5. Find the hexadecimal expansion of $(177130)_{10}$.

Solution:

16	173130
16	11070 - 10
16	691 – 14
16	43 – 3
	2 – 11

Hence, $(177130)_{10} = (2B3EA)_{16}$

(Recall that the integers 10, 11, and 14 correspond to the hexadecimal digits A, B, and E, respectively.)

6. Find the binary expansion of $(241)_{10}$.

Solution:

2	241
2	120 - 1
2	60 – 0
2	30 – 0
2	15 – 0
2	7 – 1
2	3 – 1
2	1 – 1

Hence,$(241)_{10} = (1111\ 0001)_2$

Here, a table is given for the integers 0 to 15 with their expansions in Decimal, Binary, Octal and Hexadecimal systems.

Hexadecimal, Octal and binary Representation of the Integers 0 through 15.																
Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hexadecimal	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Octal	0	1	2	3	4	5	6	7	10	11	12	13	14	15	16	17
Binary	0	1	10	11	100	101	110	111	1000	1001	1010	1011	1100	1101	1110	1111

7. Find the octal expansions of $(11\ 1110\ 1011\ 1100)_2$.

Solution:

To convert $(11\ 1110\ 1011\ 1100)_2$ into octal notation we group the binary digits into blocks of three, adding initial zeros at the start of the leftmost block if necessary.These blocks, from left to right, are 011, 111, 010, 111, and 100, corresponding to 3, 7, 2, 7,and 4, respectively.

Consequently, $(11\ 1110\ 1011\ 1100)_2 = (37274)_8$.

8. Find the hexadecimal expansions of $(11\ 1110\ 1011\ 1100)_2$.

Solution:

To convert $(11\ 1110\ 10111100)_2$ into hexadecimal notation we group the binary digits into blocks of four, adding initialzeros at the start of the leftmost block if necessary. These blocks, from left to right, are 0011,1110, 1011, and 1100, corresponding to the hexadecimal digits 3, E, B, and C, respectively.

Consequently, $(11\ 1110\ 1011\ 1100)_2 = (3EBC)_{16}$.

9. Find the binary expansions of $(765)_8$ and $(A8D)_{16}$.

Solution:

To convert $(765)_8$ into binary notation, we replace each octal digit by a block of three binarydigits. These blocks are 111, 110, and 101.
Hence, $(765)_8 = (1\ 1111\ 0101)_2$. To convert $(A8D)_{16}$into binary notation, we replace each hexadecimal digit by a block of four binary digits. Theseblocks are 1010, 1000, and 1101.
Hence, $(A8D)_{16} = (1010\ 1000\ 1101)_2$.

Contributions of Mathematicians and Statisticians especially Muslim Scholars

Mathematicians and statisticians have profoundly impacted human knowledge and innovation. Mathematicians have laid the foundational framework for groundbreaking discoveries, shaping our understanding of the world. They've developed powerful tools and theories, driving advancements in medicine, finance, technology and more. Statisticians have revolutionized decision-making processes, empowering policymakers, businesses and researchers with data-driven insights.

Together, mathematicians and statisticians form a formidable alliance, harnessing numerical prowess and data analysis to tackle pressing global challenges. Their synergy yields transformative breakthroughs, reshaping our understanding of complexity, uncertainty and change. From optimizing processes to mitigating risks, their work drives progress in various industries, including finance, environmental science, public health and economics. Here is the list with era, date of birth, and date of death:

Mathematicians

Isaac Newton (1643-1727)

Developing binomial theorem and new theories of infinite series. Developed calculus, laws of motion, and universal gravitation. Published "Philosophiæ Naturalis Principia Mathematica" (1687). Laid the foundation for classical mechanics and modern physics.

Archimedes (c. 287 BC - c. 212 BC)

Deriving an approximation of pi (π), defining and investigating the Archimedean spiral, and devising a system using exponentiation for expressing very large numbers. Discovered the principle of buoyancy and developed fluid mechanics. Made significant contributions to geometry and the study of spheres, cylinders, and cones. Invented various machines, including the Claw of Archimedes and the Archimedes' screw

Euclid(fl. 300 BC)

Using his system of proofs, Euclid proved some well-known items in mathematics, such as the Pythagorean Theorem. He demonstrated how to calculate the volumes of solids, such as cones and pyramids, and described geometric number sequences. Authored the famous book "Elements," systematizing geometry and establishing axioms. He developed the concept of theorems and proofs and Introduced the concept of irrational numbers.

Pierre-Simon Laplace (1749-1827)

Laplace formulated Laplace's equation, and pioneered the Laplace transform which appears in many branches of mathematical physics, a field that he took a leading role in forming. The Laplacian differential operator, widely used in mathematics, is also named after him. He wrote the book *The System of the World*. Developed probability theory and the concept of expected value. Made significant contributions to celestial mechanics and the study of planetary motion. Also published "A Philosophical Essay on Probabilities" (1812).

Albert Einstein (1879-1955)

Developed the theory of special relativity and the famous equation $E = mc^2$.

Introduced the concept of spacetime and the speed of light as a universal constant.

Made significant contributions to the development of quantum mechanics.

Albert Einstein's most popular book is Relativity: The Special and General Theory.

Statisticians

Ronald Fisher (1890-1962)

Developed modern statistical inference and experimental design.

Introduced the concept of null hypothesis testing. Made significant contributions to the development of maximum likelihood estimation.

Karl Pearson (1857-1936)

Developed the correlation coefficient and principal component analysis.

Introduced the concept of the p-value. Published "The Grammar of Science" (1892).

William Gosset (1876-1937)

Developed the t-distribution and statistical hypothesis testing. Introduced the concept of the t-test. Made significant contributions to quality control and statistical process control.

John Turkey (1915-2000)

Developed exploratory data analysis and the Fast Fourier Transform. Introduced the concept of the box plot. Made significant contributions to statistical graphics and visualization.

Florence Nightingale (1820-1910)

Developed statistical graphics and applied statistics to medicine. Introduced the concept of the polar area chart. Made significant contributions to hospital sanitation and public health.

Muslim Scholars

Muslim Scholars played a pivotal role in shaping the Islamic Golden Age. Their groundbreaking contributions transformed various fields of study. Contribution of some scholars is as follows:

Muhammad ibn Musa al-Khwarizmi

Muhammad ibn Musa al-Khwarizmi (780AD–850AD) was a Persian mathematician, astronomer, astrologer geographer and a scholar in the House of Wisdom in Baghdad. He was born in Persia of that time around 780. Al-Khwarizmi was one of the learned men who worked in the House of Wisdom. The House of Wisdom was a scientific research and teaching center. Al-Khwarizmi developed the concept of the algorithm in mathematics Al-Khwarizmi's algebra is regarded as the foundation and cornerstone of the sciences. He is known as the "father of algebra". His note able work is "Al-Kitab-almukhtasar fi hisab al-jabar wal-muqabala", text on algebra.

Muhammad ibn Musa al-Khwarizmi died in c. 850 being remembered as one of the most seminal scientific minds of early Islamic culture.

Ibn al-Haytham

Ibn al-Haytham Latinised as Alhazen (965AD–1040AD) was a medieval mathematician, astronomer, and physicist of the Islamic Golden Age from present-day Iraq. Referred to as "the father of modern optics". He made significant contributions to the principles of optics and the use of scientific experiments. He made significant contributions to geometry algebra and number theory. His most influential work is titled Kitāb al-Manāẓir "Book of Optics" in Latin Edition. Ibn al-Haytham, who lived a thousand years ago, is finally being recognized as the world's first true scientist.

Omar Khayyam

Omar Khayyam (1048AD–1131AD) was a Persian mathematician, astronomer, and poet. He made great contributions to these areas. He lived during the period of the Seljuk dynasty, around the time of the First Crusade. As a mathematician, he is most notable for his work on the classification and solution of cubic equations. He is best known for his work in geometric algebra, the Jalil calendar, and his poetry collected as, The Rubaiyat.

Ibrahim ibn Sinan

Ibrahim ibn Sinan (980AD-1037AD) was born in Baghdad. He was a mathematician and astronomer who belonged to a family of scholars originally from Harran in northern Mesopotamia. He belonged to a religious sect of star worshippers known as the Sabians of Harran. Ibrahim ibn Sinan studied geometry, in particular tangents to circles. He made advances in the quadrature of the parabola and the theory of integration, generalizing the work of Archimedes, which was unavailable at the time. Ibrahim ibn Sinan is often considered to be one of the most important mathematicians of his time.

Sharaf al-Din al-Tusi

Sharaf al-Din al-Tusi (1135AD-1213AD) was an Iranian mathematician and astronomer of the Islamic Golden Age (during the Middle Ages). Sharaf al-Tusi was an Islamic mathematician who wrote a treatise on cubic equations. Al-Tusi is best known for his mathematically impressive study of the conditions under which cubic equations have a positive real root and of numerical methods for finding a solution of such equations. He made significant contributions to development of Algebraic geometry and cubic equation.

Abu Rayhan Muhammad ibn Ahmad al-Biruni

Abu Rayhan Muhammad ibn Ahmad al-Biruni (973AD–1048AD) known as al-Biruni, was a Khwarazmian Iranian scholar and polymath during the Islamic Golden Age. He has been called variously "Father of Comparative Religion", "Father of modern geodesy", Founder of Indology and the first anthropologist. Al-Biruni was well versed in physics, mathematics, astronomy, and natural sciences, and also distinguished himself as a historian, chronologist, and linguist. In 1017, he travelled to the Indian subcontinent and wrote a treatise on Indian culture entitled Tārīkh al-Hind ("The History of India"). Al-Biruni developed many instruments for astronomy and geography measurements. He was also a very good encyclopedia writer. His famous achievements were, studying geography of India, accurately measuring Earth's radius, comparing different calendars & He enabled direction of Qibla.

Inductive and Deductive Reasoning

Inductive and detective reasoning are two fundamental approaches to logical thinking, empowering us to navigate the complexities of information, arrive at informed conclusions, and uncover new insights.

Inductive Reasoning: Inductive reasoning is the process of reaching a general conclusion by examining specific examples.

Deductive Reasoning: Deductive reasoning is the process of reaching a conclusion by applying general assumptions, procedures, or principles.

10. Use inductive reasoning to predict the most probable next number in each of the following lists. a. 3, 6, 9, 12, 15? b. 1, 3, 6, 10, 15?

Solution:

a. Each successive number is 3 larger than the preceding number. Thus we predict that the most probable next number in the list is 3 larger than 15, which is **18**.

b. The first two numbers differ by 2. The second and the third numbers differ by 3. It appears that the difference between any two numbers is always 1 more than the preceding difference. Since 10 and 15 differ by 5, we predict that the next number in the list will be 6 larger than 15, which is **21**.

11. Use inductive reasoning to predict the most probable next number in each of the following lists. a. 5, 10, 15, 20, 25, ? b. 2, 5, 10, 17, 26, ?

Solution:

a. Each successive number is 5 larger than the preceding number. Thus we predict that the next number in the list is 5 larger than 25, which is 30.

b. The first two numbers differ by 3. The second and third numbers differ by 5. It appears that the difference between any two numbers is always 2 more than the preceding difference. Since 17 and 26 differ by 9, we predict that the next number will be 11 more than 26, which is 37.

12. Consider the following procedure: Pick a number. Multiply the number by 8, add 6 to the product, divide the sum by 2, and subtract 3.Complete the above procedure for several different numbers. Use inductive reasoning to make a conjecture about the relationship between the size of the resulting number and the size of the original number.

Solution: Suppose we pick 5 as our original number. Then the procedure would produce the following results:

Original number:	5
Multiply by 8:	$8\times5=40$
Add 6:	$40+6=46$
Divide by 2:	$46\div2=23$
Subtract 3:	$23-3=20$

We started with 5 and followed the procedure to produce 20. Starting with 6 as our original number produces a final result of 24. Starting with 10 produces a final result of 40. Starting with 100 produces a final result of 400. In each of these cases the resulting number is four times the original number. We conjecture that following the given procedure will produce a resulting number that is four times the original number.

13. Consider the following procedure: Pick a number. Multiply the number by 9, add 15 to the product, divide the sum by 3, and subtract 5. Complete the above procedure for several different numbers. Use inductive reasoning to make a conjecture about the relationship between the size of the resulting number and the size of the original number.

Solution:

If the original number is 2, then $\frac{2\times9+15}{3}-5=6$ which is three times the original number. If the original number is 7, then $\frac{7\times9+15}{3}-5=21$ which is three times the original number. If the original number is -12 then $\frac{-12\times9+15}{3}-5=-36$

Which is three times the original number. It appears, by inductive reasoning, that the procedure produces a number that is three times the original number.

14. Examine the table below. Identify a pattern and then use that pattern to find the missing terms in the sequence.

Term	1	2	3	4	5	6	7	8
Value	1	3	9	27	81			

Solution:

With this problem we see that the pattern to get the next number in the sequence is to multiply the previous term in the sequence by 3. So to find the 6th, 7th, and 8th terms in the sequence we will use this pattern. The 5th term is 81. The 6th term is $3\times 81=243$, the 7th term is $3\times 243=729$, and the 8th term is $3\times 729=2187$.

15. Examine the table below. Identify a pattern and then use that pattern to find the missing terms in the sequence.

Term	1	2	3	4	5	6	7	8
Value	58	46	34	22	10			

Solution:

With this problem we see that the pattern to get the next number in the sequence is to subtract 12 from the previous term in the sequence. To find the 6th, 7th, and 8th terms in the sequence we will use this pattern. The 5th term is 10. The 6th term is $10-12=-2$, 7th term is $-2-12=-14$, and 8th term is $-14-12=-26$.

16. Examine the table below. Identify a pattern and then use that pattern to find the missing terms in the sequence.

Term	1	2	3	4	5	6	7	8
Value	5	10	30	120	240	720	2880	

Solution:

With this sequence we see to go from 5 to 10 we multiply by 2. To go from 10 to 30 we multiply by 3. To go from 30 to 120 we multiply by 4. Then we see that this pattern repeats to get the next three terms in the sequence. $2\times 120=240,\quad 3\times 240=720$ and $4\times 720=2880$. So we will use this same pattern to get the 8th, 9th, and 10th terms. The 8th term is $2\times 2880=5760$, the 9th term is $3\times 5760=2880$, and the 10th term is $4\times 17280=69120$.

17. Use the data in the table on the preceding page and inductive reasoning to answer each of the following.

Length of pendulum in units	Period of pendulum in heart beats
1	1
4	2
9	3
16	4

a. **If a pendulum has a length of 25 units, what is its period?**

b. **If the length of a pendulum is quadrupled, what happens to its period?**

Solution:

a. In the table, each pendulum has a period that is the square root of its length. Thus we conjecture that a pendulum with a length of 25 units will have a period of 5 heartbeats.

b. In the table, a pendulum with a length of 4 units has a period that is twice that of a pendulum with a length of 1 unit. A pendulum with a length of 16 units has a period that is twice that of a pendulum with a length of 4 units. It appears that quadrupling the length of a pendulum doubles its period.

18. A tsunami is a sea wave produced by an under-water earthquake. The velocity of a tsunami as it approaches land depends on the height of the tsunami. Use the table at the left and inductive reasoning to answer each of the following questions.

Height of Tsunami in feet	Velocity of Tsunami in feet per second
4	6
9	9
16	12
25	15
36	18
49	21
64	24

a. What happens to the height of a tsunami when its velocity is doubled?

b. What should be the height of a tsunami if its velocity is 30 feet per second?

Solution:

a. It appears that when the velocity of a tsunami is doubled, it height is quadrupled.

b. A tsunami with a velocity of 30 feet per second will have a height that is four times that of a tsunami with a speed of 15 feet per second. Thus, we predict a height of $4 \times 25 = 100$ feet for a tsunami with a velocity of 30 feet per second.

19. The last four times I have driven downtown at 6pm there has been traffic. Use inductive reasoning to draw your conclusion.

Solution:

My conclusion is that there is always traffic downtown around 6pm.

20. Consider the statement and determine if it is inductive or deductive:

"Every month has 30days in it. July is month. Therefore it has 30days in it. "

Solution:

This statement starts with a generalization and it’s then applied to a specific case. This follows the pattern of deductive reasoning. The statements are not necessarily true, but if every month has 30 days in it, then it would be true.

21. Use deductive reasoning to show that the following procedure produces a number that is four times the original number. Procedure: Pick a number. Multiply the number by 8, add 6 to the product, divide the sum by 2, and subtract 3.

Solution:

Let n represent the original number.

Multiply the number by 8: $8n$

Add 6 to the product: $8n+6$

Divide the sum by 2: $\frac{8n+6}{2}=4n+3$

Subtract 3: $4n+3-3=4n$

We started with n and ended with 4n. The procedure given in this example produces a number that is four times the original number.

22. Use deductive reasoning to show that the following procedure produces a number that is three times the original number.

Procedure: Pick a number. Multiply the number by 6, add 10 to the product, divide the sum by 2, and subtract 5. Hint: Let n represent the original number.

Solution:

Let n represent the original number.

Multiply the number by 6: $6n$

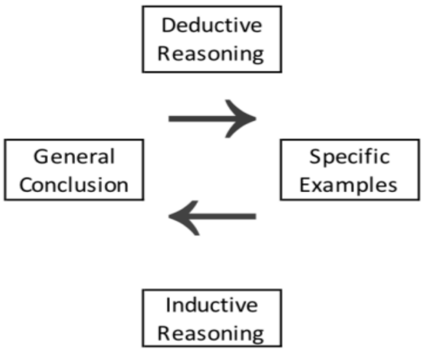
Add 1 to the product: $6n+10$

Divide the sum by 2: $\frac{6n+10}{2}=3n+5$

Subtract 5: $3n+5-5=3n$

The procedure always produces a number that is three times the original number.

Inductive Reasoning versus Deductive Reasoning



- For Inductive Reasoning we start with examples or cases, and then draw general conclusions.
- For Deductive Reasoning we start with a general statement and apply it to examples or cases.

  In next Example we analyze arguments to determine whether they use inductive or deductive reasoning.

23. Determine whether each of the following arguments is an example of inductive reasoning or deductive reasoning.

a. During the past 10 years, a tree has produced plums every other year. Last year the tree did not produce plums, so this year the tree will produce plums.

b. All home improvements cost more than the estimate. The contractor estimated my home improvement will cost $35,000. Thus my home improvement will cost more than $35,000.

Solution:

a. This argument reaches a conclusion based on specific examples, so it is an example of inductive reasoning.

b. Because the conclusion is a specific case of a general assumption, this argument is an example of deductive reasoning.

24. Determine whether each of the following arguments is an example of inductive reasoning or deductive reasoning.

a. All Janet Evanovich novels are worth reading. The novel *To the Nines* is a Janet Evanovich novel. Thus *To the Nines* is worth reading.

b. I know I will win a jackpot on this slot machine in the next 10 tries, because it has not paid out any money during the last 45 tries.

Solution:

a. The conclusion is a specific case of a general assumption, so the argument is an example of deductive reasoning.

b. The argument reaches a conclusion based on specific examples, so the argument is an example of inductive reasoning.

25. Consider the statement and determine if it is inductive or deductive:

" Pizza Hut has a lunch buffet. Stevi B’s has a lunch buffet. Therefore all pizza restaurants have a lunch buffet."

Solution:

This statement starts with two examples about pizza restaurants having lunch buffets. Based on these examples a generalization is made. This follows the pattern of inductive reasoning.

26. Consider the statement and determine if it is inductive or deductive:

"All pro wrestlers have a catch phrase. Macho Man Randy Savage was a pro wrestler. Therefore he had a catch phrase. ”

Solution:

This statement starts with a generalization about pro wrestlers having catch phrases. It’s then applied to the specific case of Macho Man Randy Savage. This follows the pattern of deductive reasoning.

27. A movie director tells her producer (who pays for the movie) not to worry-her film will be a hit. As evidence, she cites the following facts: She's hired big stars for the lead roles, she has a great advertising campaign planned, and it's a sequel to her last hit movie. Explain why this argument is inductive, and evaluate its strength.

Solution:

Each of the three pieces of evidence is a specific characteristic of her movie.
She uses them to support the more general conclusion that her movie will be a hit.
Because the conclusion is more general than the premises, the argument is inductive. In this case, her argument is relatively weak. As all producers know, even the best planned movies can flop.

28. Evaluate the following argument, and discuss the truth of its conclusion.
Geological evidence shows that, for thousands of years, the San Andreas Fault has suffered a major earthquake at least once every hundred years. Therefore, we should expect another earthquake on the fault during the next one hundred years.

Solution:

This argument is inductive because it cites many specific past events as evidence that another earthquake will occur. The fact that the pattern has held for thousands of years suggests a strong likelihood that it will continue to hold. The argument does not prove that another earthquake will occur, but it makes another earthquake seem quite likely. The argument is strong.

Abductive Reasoning

Type of reasoning that uses an observation or set of observations to reach a logical conclusion. It is similar to inductive reasoning; however it permits making best guesses to arrive at the simplest conclusions. It is a form of logical reasoning that involves making an educated guess or hypothesis based on incomplete or limited information.

It involves:

1. Observing a phenomenon or pattern
2. Identifying possible explanations
3. Selecting the most plausible explanation
4. Testing and refining the hypothesis

Abductive reasoning is essential in mathematics, science, and problem-solving.

29. What is the next number in the sequence: 2, 4, 8, 16, ?

Solution:

32 (recognizing a geometric progression)

30. A bakery sells 250 loaves of bread per day. If each loaf costs $2, how much money does the bakery make daily?

Solution:

$500 (assuming each loaf sells at the given price)

31. A car travels 250 miles in 5 hours. What is its average speed?

Solution:

50 mph (using distance = rate × time)

32. What is the sum of the interior angles of a triangle?

Solution:

180° (using geometric properties)

33. A survey shows 3/5 of students prefer pizza. If 100 students participated, how many prefer pizza?

Solution:

60 (applying proportionality)

34. Solve for x: 2x + 5 = 11

Solution:

x = 3 (using algebraic manipulation)

35. A rectangle has a perimeter of 24 cm. If its length is 8 cm, what is its width?

Solution:

4 cm (using perimeter = 2(length + width))

36. What is the probability of rolling a 6 on a fair six-sided die?

Solution:

1/6 (using probability theory)

37. A water tank fills at 0.5 liters/minute. How long to fill a 30-liter tank?

Solution:

60 minutes (using rate × time)

38. Find the missing value: 3, 6, 12, ?, 48

Solution:

24 (recognizing a geometric progression)

39. What is the next number in the sequence: 1, 2, 4, 7, 11, ?

Solution:

16 (recognizing a quadratic progression)

40. A snail moves 3 cm/hour. How far will it move in 5 hours?

Solution:

15 cm (using rate × time)

41. Solve for x: x^2 + 5x - 6 = 0

Solution:

x = -6 or x = 1 (using quadratic formula)

42. A circle has a circumference of 20π cm. What is its radius?

Solution:

10 cm (using circumference = 2πr)

43. What is the sum of the exterior angles of a polygon?

Solution:

360° (using geometric properties)

44. A survey shows 2/3 of students prefer math. If 150 students participated, how many prefer math?

Solution:

100 (applying proportionality)

45. Find the missing value: 2, 6, 12, 20, ?

Solution: 30 (recognizing a quadratic progression)

46. A cylinder has a volume of 40π cm^3. If its height is 10 cm, what is its radius?

Solution: 2 cm (using volume $= \pi r^2 h$)

47. What is the probability of drawing an ace from a standard deck of cards?

Solution: 4/52 = 1/13 (using probability theory)

48. Solve for x: 3x - 2 = 14

Solution: x = 16/3 (using algebraic manipulation)

49. Explain the concept of abductive reasoning and its significance in mathematical problem-solving.

Solution: Abductive reasoning involves making educated guesses or hypotheses based on incomplete information. In mathematics, it's essential for:

1. Pattern recognition: Identifying relationships between numbers or shapes.
2. Hypothesis formation: Proposing solutions to problems.
3. Logical inference: Drawing conclusions from available data.

Abductive reasoning facilitates mathematical discovery, fosters critical thinking, and enhances problem-solving skills.

50. Discuss how abductive reasoning differs from deductive and inductive reasoning.

Solution: Abductive reasoning differs from:

1. **Deductive reasoning:** Abductive reasoning involves uncertainty, whereas deductive reasoning draws definitive conclusions.
2. Inductive reasoning: Abductive reasoning focuses on hypothesis formation, whereas inductive reasoning seeks generalizations.

Abductive reasoning bridges the gap between deductive and inductive reasoning, enabling mathematicians to navigate uncertainty.

51. Describe a real-world scenario where abductive reasoning is essential.

Solution:

Medical diagnosis: Doctors use abductive reasoning to:

1. Identify symptoms
2. Formulate hypotheses
3. Test and refine diagnoses

Abductive reasoning enables doctors to make informed decisions amidst uncertainty, saving lives.

52. Analyze the role of abductive reasoning in resolving mathematical paradoxes.

Solution: Abductive reasoning helps resolve paradoxes by:

1. Identifying underlying assumptions
2. Formulating alternative hypotheses
3. Evaluating evidence

Examples: Russell's Paradox, the Liar Paradox.

Exercise

1. Is the following statement inductive or deductive reasoning?
"All Noble prize winners get a monetary award. Jennifer Doudna won a Noble Prize, so she must have received money."
2. Is the following statement inductive or deductive reasoning?
"My friend and my brother graduated from Harvard and immediately got great jobs. Therefore, everyone who graduates from Harvard will immediately get a great job. "
3. Find a counter example to disprove the hypothesis: If two even numbers are divided, the quotient is a whole number.
4. Find a counter example to disprove the hypothesis: If a number is added to itself, the sum is greater than the original number.
5. Describe the pattern found in the following sequence of numbers and then find in the next two values: 1, 2, 4, 7, 11, 16.
6. Describe the pattern found in the following sequence of days and then find in the next two values: Monday, Thursday, Sunday, Wednesday, Saturday.
7. In Exercises i–x, use inductive reasoning to predict the most probable next number in each list.
   i. 4, 8, 12, 16, 20, 24, ?
   ii. 5, 11, 17, 23, 29, 35, ?
   iii. 3, 5, 9, 15, 23, 33, ?
   iv. 1, 8, 27, 64, 125, ?
   v. 1, 4, 9, 16, 25, 36, 49, ?
   vi. 80, 70, 61, 53, 46, 40, ?
   vii. $\frac{3}{5}, \frac{5}{7}, \frac{7}{9}, \frac{9}{11}, \frac{11}{13}, \frac{13}{15}, ?$
   viii. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, ?$
   ix. 2,7, -3,2,-8, -3, -13, -8, -18,?
   x. 1, 5, 12, 22, 35,?
8. Determine whether the argument is an example of inductive reasoning or deductive reasoning.
   i. Andrea enjoyed reading the Dark Tower series by Stephen King, so I know she will like his next novel.
   ii. All pentagons have exactly five sides. Figure A is a pentagon. Therefore, Figure A has exactly five sides.
   iii. Every English setter likes to hunt. Duke is an English setter, so Duke likes to hunt.
   iv. Cats don't eat tomatoes. Scat is a cat. Therefore, Scat does not eat tomatoes.
   v. A number is a "neat" number if the sum of the cubes of its digits equals the number. Therefore, 153 is a "neat" number.
   vi. The Atlanta Braves have won five games in a row.

9. Convert the decimal expansion of each of these integersto a binary expansion.

a) 231
b) 4532
c) 97644

10. Convert the binary expansion of each of these integers toa decimal expansion.

a) $(1\ 1111)_2$
b) $(10\ 0000\ 0001)_2$
c) $(1\ 0101\ 0101)_2$
d) $(110\ 1001\ 0001\ 0000)_2$

11. Convert the octal expansion of each of these integers toa binary expansion.

a) $(572)_8$
b) $(1604)_8$
c) $(423)_8$
d) $(2417)_8$

12. Convert the binary expansion of each of these integers toan octal expansion.

a) $(1111\ 0111)_2$
b) $(1010\ 1010\ 1010)_2$
c) $(111\ 0111\ 0111\ 0111)_2$
d) $(101\ 0101\ 0101\ 0101)_2$

13. Convert the hexadecimal expansion of each of these integers to a binary expansion.

a) $(80E)_{16}$
b) $(135AB)_{16}$
c) $(ABBA)_{16}$
d) $(DEFACED)_{16}$