Multiple Choice Questions For BSc / BS / PPSC

Chapters:

1. *Complex Numbers*
2. *Groups*
3. *Matrices*
4. *System of Linear Equations*
5. *Determinants*
6. *Metric Spaces*
7. *Number Theory*
8. *Ordinary Differential Equations*
9. *Infinite Series*

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For detailed solutions of these, visit

YouTube Channel: Suppose Math

https://www.youtube.com/supposemath

1. If z is any complex number, then $\overline{z} - z$ equals:
   - A. 2 Im(z)
   - B. -2 Im(z)
   - C. 2 Im(z)i
   - D. -2 Im(z)i
2. Complex numbers with 0 as real part are called:
   - A. imaginary numbers
   - B. pure non real numbers
   - C. pure imaginary numbers
   - D. pure complex numbers
3. The argument of which of the following number is not defined?
   - A. 0
   - B. 1
   - C. 1/0
   - D. i
4. If θ is the principal argument Arg(z) of a complex number z, then:
   - A. $0 \leq \theta \leq 2\pi$
   - B. $-\pi \leq \theta \leq \pi$
   - C. $-\pi \leq \theta < \pi$
   - D. $-\pi < \theta \leq \pi$
5. For $k \in \mathbb{Z}$, the relationship between arg(z) and Arg(z) is:
   - A. arg(z)= Arg(z)+$2k\pi$
   - B. Arg(z)= arg(z)+$2k\pi$
   - C. arg(z)= Arg(z)$-2k\pi$
   - D. All of these

6. Which of the following is unique?
   - A. $\text{Arg}(z)$
   - B. $\arg(z)$
   - C. Both A and B
   - D. None of these
7. We can write $r(\cos\theta + i\sin\theta)$ as:
   - A. $rsic\theta$
   - B. $rcsi\theta$
   - C. $rcis\theta$
   - D. $r\cos\theta$
8. The value of $\arg(5)$ is:
   - A. 0°
   - B. 90°
   - C. 180°
   - D. 270°
9. The value of $\arg(-5)$ is:
   - A. 0°
   - B. 90°
   - C. 180°
   - D. 270°
10. The value of $\arg(5i)$ is:
    - A. 0°
    - B. 90°
    - C. 180°
    - D. 270°
11. The value of $\arg(-5i)$ is:
    - A. 0°
    - B. -90°
    - C. 180°
    - D. 270°

12. The value of $\text{Arg}(-5i)$ is:
    A. $0°$
    B. $90°$
    C. $180°$
    D. $270°$

13. The value of $\text{Arg}(-5)$ is:
    A. $0°$
    B. $90°$
    C. $180°$
    D. $270°$

14. The equation of a circle with center at origin and radius 2 is:
    A. $|z| = 2$
    B. $|z| = 4$
    C. $|z| = \sqrt{2}$
    D. None of these

15. Which of the following is not true?
    A. $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$
    B. $\text{Arg}(z_1 z_2) = \text{Arg}(z_1) + \text{Arg}(z_2)$
    C. $z\overline{z} = |z|^2$
    D. $\arg(\frac{z_1}{z_2}) = \arg(z_1)$ - $\arg(z_2)$

16. The least value of $|z_1 + z_2|$ is:
    A. $||z_1| + |z_2||$
    B. $||z_1||z_2||$
    C. $||z_1|/|z_2||$
    D. $||z_1| - |z_2||$

17. The inequality $||z_1| - |z_2|| \leq |z_1 + z_2| \leq |z_1| + |z_2|$ is called:
    A. Triangle Inequality
    B. Minkowski Inequality
    C. Cauchy-Schwarz Inequality
    D. Holder's Inequality

18. The principal argument of any complex number can not be:

    A. $\frac{7\pi}{8}$

    B. $\frac{7\pi}{6}$

    C. $\frac{\pi}{2}$

    D. $-\frac{\pi}{2}$

19. If $|z| = 2i(1-i)(2-4i)(3+i)$, then $|z|$ equals:

    A. 20

    B. -20

    C. 40

    D. -40

20. $z = a + ib$ is pure imaginary if and only if:

    A. $z = -\overline{z}$

    B. $z = \overline{z}$

    C. $z = -z$

    D. $z = z^{-1}$

21. If $z_1 = 24 + 7i$ and $|z_2| = 6$, then the least value of $|z_1 + z_2|$ is:

    A. 31

    B. 19

    C. -19

    D. -13

22. $\frac{|az+b|}{|\bar{b}z+\overline{a}|}=1$, for $|z|=?$

    A. 1

    B. 0

    C. 2

    D. -1

23. Locus of the points satisfying $\text{Re}(i\overline{z}) = 3$ is:

    A. a line parallel to x-axis

    B. a line parallel to y-axis

    C. a circle

    D. a parabola

24. For all integers n, we have:

    A. $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

    B. $(\cos\theta + i\sin\theta)^n = \cos n\theta - i\sin n\theta$

    C. $(\cos\theta - i\sin\theta)^n = \cos n\theta + i\sin n\theta$

    D. $(\cos\theta + i\sin\theta)^{-n} = \cos n\theta + i\sin n\theta$

25. The value of $\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^6$ is:

    A. 0

    B. $\frac{1}{2}$

    C. 1

    D. -1

26. For any integers n, we have $(\sin x + i\cos x)^n =$

    A. $\sin n(\frac{\pi}{2} - x) + i\cos n(\frac{\pi}{2} - x)$

    B. $\cos n(\frac{\pi}{2} - x) + i\sin n(\frac{\pi}{2} - x)$

    C. $\sin n(\frac{\pi}{2} + x) + i\cos n(\frac{\pi}{2} + x)$

    D. $\sin n(\frac{\pi}{2} + x) + i\cos n(\frac{\pi}{2} + x)$

27. If $x = \cos\theta + i\sin\theta$, then the value of $\frac{1}{x} =$

    A. $\cos\theta + i\sin\theta$

    B. $\sin\theta + i\cos\theta$

    C. $\cos\theta - i\sin\theta$

    D. $\sin\theta - i\cos\theta$

28. If $x = \cos\theta + i\sin\theta$, then the value of $\frac{1}{x^n} =$

    A. $\cos n\theta + i\sin n\theta$

    B. $\sin n\theta + i\cos n\theta$

    C. $\cos n\theta - i\sin n\theta$

    D. $\sin n\theta - i\cos n\theta$

29. If $x = \cos\theta + i\sin\theta$, then the value of $x^n + \frac{1}{x^n} =$

    A. $2i\sin n\theta$

    B. $2i\cos n\theta$

    C. $2\cos n\theta$

    D. $2\sin n\theta$

30. If $x = \cos\theta + i\sin\theta$, then the value of $x^n - \frac{1}{x^n}$=

A. $2i\sin nx$

B. $2i\cos nx$

C. $2\cos nx$

D. $2\sin nx$

31. If $|z| = r$ and $\arg(z) = \theta$, then all the nth roots of z are:

A. $r^{\frac{1}{n}} cis(\frac{2k\pi+\theta}{n})$

B. $r^{\frac{1}{n}} cis(\frac{2\pi+\theta}{kn})$

C. $r^{\frac{1}{n}} cis(\frac{2\pi+k\theta}{n})$

D. $r^{\frac{1}{n}} cis(\frac{2k\pi+\theta}{kn})$

32. $1, \omega, \omega^2, ..., \omega^{n-1}$ are nth roots of:

A. zero

B. unity

C. 2i

D. None of these

33. If z is a root of w, then which of following is also a root of w?

A. 1

B. $-z$

C. $\overline{z}$

D. z^{-1}

34. Three cube roots of $8i$ are:

A. $2, 2\omega, 2\omega^2$

B. $2i, 2i\omega, 2i\omega^2$

C. $-2, -2\omega, -2\omega^2$

D. $-2i, -2i\omega, -2i\omega^2$

35. Sum of four fourth roots of unity is:

A. 0

B. 1

C. i

D. -1

36. $\frac{(\cos\theta+i\sin\theta)^n}{(\cos\phi+i\sin\phi)^m}$ equals:

A. $\cos(m\theta+n\phi)+i\sin(m\theta+n\phi)$

B. $\cos(n\theta+m\phi)+i\sin(n\theta+m\phi)$

C. $\cos(m\theta-n\phi)+i\sin(m\theta-n\phi)$

D. $\cos(n\theta-m\phi)+i\sin(n\theta-m\phi)$

37. $\frac{(\cos\alpha-i\sin\alpha)^{11}}{(\cos\beta+i\sin\beta)^9}$ equals:

A. $\cos(11\alpha+9\beta)+i\sin(11\alpha+9\beta)$

B. $\cos(11\alpha-9\beta)+i\sin(11\alpha-9\beta)$

C. $\cos(-11\alpha+9\beta)+i\sin(-11\alpha+9\beta)$

D. $\cos(-11\alpha-9\beta)+i\sin(-11\alpha-9\beta)$

38. For a complex number z, $\frac{e^{iz}-e^{-iz}}{i(e^{iz}+e^{-iz})}=$

A. $\cot z$

B. $\tan z$

C. $\coth z$

D. $\tanh z$

39. $\sin^2 z+\cos^2 z=$

A. 1

B. -1

C. 0

D. $2\sin z\cos z$

40. $\sin iz=$

A. $\sinh z$

B. $\sinh iz$

C. $i\sin z$

D. $i\sinh z$

41. $\cos iz=$

A. $\cosh z$

B. $\cosh iz$

C. $i\cos z$

D. $i\cosh z$

42. $\tan iz=$

    A. $\tanh z$

    B. $\tanh iz$

    C. $i\tan z$

    D. $i\tanh z$

43. $\sinh iz=$

    A. $\sin z$

    B. $i\sin z$

    C. $\sinh z$

    D. $i\sinh z$

44. $\cosh iz=$

    A. $\cos z$

    B. $i\cos z$

    C. $\cosh z$

    D. $i\cosh z$

45. $\tanh iz=$

    A. $\tan z$

    B. $i\tan z$

    C. $\tanh z$

    D. $i\tanh z$

Important Points

(i). e^z is never zero.

(ii). For $z = x + iy$, $|e^z| = e^x$.

(iii). $|e^{i\theta}| = 1$, where $\theta \in \mathbb{R}$.

(iv). $e^z = 1$ if and only if $z = 2k\pi i$, where $k \in \mathbb{Z}$.

(v). $e^{z_1} = e^{z_2}$ if and only if $z_1 - z_2 = 2k\pi i$, where $k \in \mathbb{Z}$.

46. Multiplication of a vector z by ... rotates the vector z counterclockwise through an angle of measure α.

A. e^{α}

B. $e^{-\alpha}$

C. $e^{i\alpha}$

D. $e^{-i\alpha}$

47. $-3-4i=$

A. $5e^{i\tan^{-1}\frac{4}{3}}$

B. $5e^{i(-\tan^{-1}\frac{4}{3})}$

C. $5e^{i(\pi-\tan^{-1}\frac{4}{3})}$

D. $5e^{i(\pi+\tan^{-1}\frac{4}{3})}$

48. For any complex number z, $\log z=$

A. $\ln|z|+i\arg z$

B. $\ln z+i\arg|z|$

C. $\ln|z|+i\arg|z|$

D. All of these

49. Which number(s) has(have) no complex logarithm?

A. 0

B. Negative real numbers

C. Non positive real numbers

D. None of these

50. For any complex number z, Log$z=$

A. $\ln|z|+i\ \text{Arg}\ z$

B. $\ln z+i\ \text{Arg}\ |z|$

C. $\ln|z|+i\ \text{Arg}\ |z|$

D. All of these

51. The value of $\text{Log}(-i)$ is:

A. $\frac{\pi}{2}i$

B. $\frac{3\pi}{2}i$

C. $-\frac{\pi}{2}i$

D. $-\frac{3\pi}{2}i$

52. If x is any negative real number, then Log$x$ is:
   A. $\ln x + i\pi$
   B. $\ln x - i\pi$
   C. $\ln(-x) + i\pi$
   D. $\ln(-x) - i\pi$

53. $\log(e^z)$=
   A. z
   B. $z + 2n\pi$
   C. $z + 2n\pi i$
   D. e^z

54. If z is a positive real number, then
   A. Log(z)=log(z)
   B. Log(z)=log(z)+ $2n\pi$
   C. log(z)=Log(z)+ $2n\pi$
   D. None of these

55. $\sinh^{-1} z$=
   A. $\log(z + \sqrt{z^2 + 1})$
   B. $\log(z - \sqrt{z^2 + 1})$
   C. $\log(z + \sqrt{z^2 - 1})$
   D. $\log(z - \sqrt{z^2 - 1})$

56. $\cosh^{-1} z$=
   A. $\log(z + \sqrt{z^2 + 1})$
   B. $\log(z - \sqrt{z^2 + 1})$
   C. $\log(z + \sqrt{z^2 - 1})$
   D. $\log(z - \sqrt{z^2 - 1})$

57. $\sin^{-1} z$=
   A. $i \log(iz + \sqrt{1 + z^2})$
   B. $-i \log(iz - \sqrt{1 - z^2})$
   C. $-i \log(iz + \sqrt{1 + z^2})$
   D. $-i \log(iz + \sqrt{1 - z^2})$

58. If z and w are complex numbers, then z^w=
   A. $\exp(z \log w)$
   B. $z \exp(\log w)$
   C. $\exp(w \log z)$
   D. $w \exp(\log z)$

59. If z and w are complex numbers, then the principal value of z^w is:
   A. $\exp(z\text{Log}w)$
   B. $z \exp(\text{Log}w)$
   C. $\exp(w\text{Log}z)$
   D. $w \exp(\text{Log}z)$

60. The principal value of i^i is:
   A. $e^{\frac{\pi}{2}}$
   B. $-e^{\frac{\pi}{2}}$
   C. $e^{-\frac{\pi}{2}}$
   D. $-e^{-\frac{\pi}{2}}$

61. The principal value of $(-1)^i$ is:
   A. e^{π}
   B. $e^{-\pi}$
   C. $-e^{\pi}$
   D. $-e^{-\pi}$

62. The principal value of $(-i)^{-i}$ is:
   A. $e^{\frac{\pi}{2}}$
   B. $-e^{\frac{\pi}{2}}$
   C. $e^{-\frac{\pi}{2}}$
   D. $-e^{-\frac{\pi}{2}}$

63. If a is a positive real number, then the principal value of a^i is:
   A. $\cos(\ln a) + i \sin(\ln a)$
   B. $\cos(a) + i \sin(a)$
   C. $\sin(a) + i \cos(a)$
   D. $\sin(\ln a) + i \cos(\ln a)$

64. $\text{Log}(1-i)=$

A. $\frac{1}{2}\ln 2+\frac{\pi i}{4}$

B. $\frac{1}{2}\ln 2-\frac{\pi i}{4}$

C. $\frac{1}{2}\ln 2+\frac{3\pi i}{4}$

D. $\frac{1}{2}\ln 2-\frac{3\pi i}{4}$

65. $(-1+i)^{i+\sqrt{3}}=$

A. $\exp[(i-\sqrt{3})\log(-1-i)]$

B. $\exp[(-1+i)\log(i+\sqrt{3})]$

C. $\exp[(i+\sqrt{3})\log(-1+i)]$

D. $\exp[(i+\sqrt{3})\log(-1-i)]$

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Multiple Choice Questions
For BA, BSc (Mathematics)

1. Which of the following is not a binary operation on $\mathbb{R}$?
   - A. $+$
   - B. $-$
   - C. $\times$
   - D. $\div$
2. An element $b \in G$ is inverse of $a \in G$ if:
   - A. $ab = ba$
   - B. $ab = ab^2$
   - C. $ba = a^2b$
   - D. $ab = ba = e$
3. An element x of a group G is said to be ... if $x^2 = x$.
   - A. Nilpotent
   - B. Involutory
   - C. Idempotent
   - D. Square
4. The only idempotent element in a group is:
   - A. Inverse
   - B. Identity
   - C. Both A and B
   - D. None of these
5. Which of the following is a group under multiplication?
   - A. $\mathbb{Z}$
   - B. $\mathbb{Q}$
   - C. $\mathbb{R}$
   - D. $\mathbb{Q} - \{0\}$

6. A group is abelian if its Cayley's table is ... about its main diagonal.
   A. Symmetric
   B. Skew symmetric
   C. Hermitian
   D. Skew Hermitian

7. The set of all the nth roots of unity, $C_n = \{e^{\frac{2k\pi i}{n}},\ k = 0, 1, ..., n-1\}$ is a group under:
   A. Addition
   B. Subtraction
   C. Multiplication
   D. Division

8. In the group of Quaternions $\{\pm I, \pm i, \pm j, \pm k\}$, which of the following is not true?
   A. $jk = i$
   B. $ik = -j$
   C. $j^2 = -I$
   D. None of these

9. In the group $\mathbb{Z}_5$, the inverse of $\overline{3}$ is:
   A. $\overline{1}$
   B. $\overline{2}$
   C. $\overline{3}$
   D. $\overline{4}$

10. Which of the following holds in a group.
    A. Cancellation
    B. Associative
    C. Both A and B
    D. None of these

11. For $a, b \in G$, we have $(ab)^{-1}$=
    A. ab
    B. $a^{-1}b^{-1}$
    C. $b^{-1}a^{-1}$
    D. ba

12. The number of elements in a group is called its:
    A. degree
    B. order
    C. power
    D. None of these

13. The least positive integer n, such that $a^n =$... is called order of a.
    A. e
    B. a
    C. a^{-1}
    D. None of these

14. Let $a \in G$ has order n. Then, for any integer k, $a^k = e$ if and only if ..., where q is an integer.
    A. $q = nk$
    B. $n = qk$
    C. $k = nq$
    D. None of these

15. If $|a| = 5$, then for what value of n, $a^n = e$?
    A. 10
    B. 15
    C. 20
    D. All of these

16. The set $\{\overline{1}, \overline{3}, \overline{5}, \overline{7}\}$ is a group under:
    A. Addition
    B. Multiplication
    C. Addition modulo 8
    D. Multiplication modulo 8

17. The set $\{\overline{1}, \overline{3}, \overline{5}, \overline{7}\}$ is a group under Multiplication modulo 8. The inverse of $\overline{5}$ is:
    A. $\overline{1}$
    B. $\overline{3}$
    C. $\overline{5}$
    D. $\overline{7}$

18. The set $\{\overline{1}, \overline{3}, \overline{5}, \overline{7}\}$ is a group under Multiplication modulo 8. The order of $\overline{5}$ is:
    A. 1
    B. 2
    C. 3
    D. 4

19. Let G be a group and $a, b \in G$, which of the following is true?
    A. $|a| = |a^{-1}|$
    B. $|ab| = |ba|$
    C. $|a| = |bab^{-1}|$
    D. All of these

20. Every group of ... order contains at least one element of order 2.
    A. Prime
    B. Even
    C. Odd
    D. Composite

21. Let G be a group and the order of $x \in G$ is odd. Then there exists an element $y \in G$ such that:
    A. $y = x$
    B. $y^2 = x$
    C. $y = x^2$
    D. $y = x^3$

22. Which of the following are not groups? (*Free to choose more than one options*).
    A. The set of positive rational numbers under multiplication
    B. The set of complex numbers z such that $|z| = 1$, under multiplication
    C. The set $\mathbb{Z}$ of all integers under the binary operation $\star$ defined by

$$a \star b = a - b, \quad \forall\ a, b\ \in \mathbb{Z}$$

    D. The set $\mathbb{Q}'$ of all irrational numbers under multiplication
    E. $\mathbb{R}^+ = \{x \in \mathbb{R} : x > 0\}$ under multiplication
    F. $\mathbb{R}^- = \{x \in \mathbb{R} : x < 0\}$ under multiplication
    G. $E = \{e^x : x \in \mathbb{R}\}$ under multiplication

23. Let G be a group and $x^2 = e$, for all $x \in G$, then G is:
    A. Abelian
    B. Non Abelian
    C. Commutative
    D. Both A and C

24. Which of the following is false? (*Free to choose more than one options*).
    A. A group can have more than one identity element.
    B. The null set can be considered to be a group.
    C. There may be groups in which the cancellation law fails.
    D. Every set of numbers which is group under addition is also a group under multiplication and vice versa.
    E. The set $\mathbb{R}$ of real numbers is a group under subtraction.
    F. The set of all nonzero integers is a group under division.
    G. To each element of a group, there corresponds only one inverse element.

25. Let G be a group. Which of the following is not unique in G?
    A. identity
    B. inverse of an element
    C. idempotent
    D. None of these

26. The set $GL_2(\mathbb{R})$ is the collection of all 2×2 matrices with real entries whose determinant is:
    A. Zero
    B. Nonzero
    C. Unit
    D. 1

27. $(\mathbb{Z}, +)$ is a subgroup of:
    A. $(\mathbb{Z}, +)$
    B. $(\mathbb{R}, +)$
    C. $(\mathbb{C}, +)$
    D. All of these

28. Every group has at least ... subgroups.
    A. 1
    B. 2
    C. 3
    D. 4

29. A non empty subset of a group G is a subgroup of G if and only if for $a, b \in H$, we have:
    A. $ba^{-1} \in H$
    B. $ab^{-1} \in H$
    C. $ab \in H$
    D. Both A and B

30. The ... of subgroups is a subgroup.
    A. Intersection
    B. Union
    C. Difference
    D. Symmetric difference

31. If every element of a group G is a power of one and the same element, then G is called:
    A. Infinite
    B. Finite
    C. Cyclic
    D. Symmetric

32. Every subgroup of a cyclic group is:
    A. Abelian
    B. Normal
    C. Cyclic
    D. Trivial

33. Let G be a group of order 18, then G must have a unique subgroup of order:
    A. 5
    B. 6
    C. 7
    D. 8

34. Every cyclic group is:
   - A. Abelian
   - B. Normal
   - C. Finite
   - D. Infinite

35. Every cyclic group of even order has a unique subgroup of order:
   - A. 2
   - B. 3
   - C. 4
   - D. 5

36. The number of subgroups of a cyclic group of order 12 is:
   - A. 3
   - B. 4
   - C. 5
   - D. 6

37. Group of order ... has not a proper non-trivial subgroup?
   - A. 46
   - B. 47
   - C. 48
   - D. 50

38. An infinite cyclic group has exactly ... generators.
   - A. 1
   - B. 2
   - C. 3
   - D. 4

39. The order of $\overline{3}$ in the group $\{\overline{0}, \overline{1}, \overline{2}, \overline{3}\}$ is:
   - A. 1
   - B. 2
   - C. 3
   - D. 4

40. Let G be a group, H be a subgroup of G and $a \in G$, then which of the following is a subgroup of G?

    A. aH

    B. Ha

    C. Ha^{-1}

    D. aHa^{-1}

41. If H and K are subgroups of a group G, then which of the following need not to be a subgroup of G?

    A. $H \cup K$

    B. $H \cap K$

    C. He

    D. eK

42. Let G be a group and $G =< a >$, for some $a \in G$, then a is called ... of G.

    A. Involutory

    B. Idempotent

    C. Generator

    D. None of these

43. Let G be a finite group of order n generated by $a \in G$. Then $a^i = a^j$ if and only if:

    A. $n|(i-j)$

    B. $n|(i+j)$

    C. $i = j$

    D. None of these

44. Let G be an infinite group generated by $a \in G$. Then $a^i = a^j$ if and only if:

    A. $n|(i-j)$

    B. $n|(i+j)$

    C. $i = j$

    D. None of these

45. Let G be a cyclic group of order 18. How many subgroups of G are of order 6?

    A. 1

    B. 2

    C. 3

    D. None of these

46. A partition of a set A is the collection of subsets $\{A_i : i \in I\}$ of A such that

    A. $A = \cup\{A_i : i \in I\}$ and $A_i \cap A_j = \phi$, where $i, j \in I$ and $i \neq j$.

    B. $A = \cup\{A_i : i \in I\}$ and $A_i \cap A_j = \phi$, where $i, j \in I$ and $i = j$.

    C. $A = \cup\{A_i : i \in I\}$ and $A_i \cap A_j \neq \phi$, where $i, j \in I$ and $i \neq j$.

    D. $A = \cap\{A_i : i \in I\}$ and $A_i \cap A_j = \phi$, where $i, j \in I$ and $i \neq j$.

47. Let H be a subgroup of G. Then the set of all left cosets of H in G defines a ...on G.

    A. Equivalence relation

    B. Partition

    C. Transitive relation

    D. All of these

48. The number of distinct left cosets of a subgroup H of a group G is called the ... of H in G, and it is denoted by $[G : H]$.

    A. Index

    B. Cardinality

    C. Order

    D. Partition

49. The index of $\{\overline{0}, \overline{2}, \overline{4}\}$ in $\{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}\}$ is:

    A. 1

    B. 2

    C. 3

    D. 4

50. The index of $\{0, \pm 2, \pm 4, ...\}$ in the group $(\mathbb{Z}, +)$ is:

    A. 0

    B. 1

    C. 2

    D. ∞

51. "Both the order and index of a subgroup of a finite group divides the order of the group" is the statement of:

    A. Division Algorithm

    B. Lagrange Theorem

    C. Euclid Theorem

    D. Cayley Theorem

52. The order of an element of a finite group divides:
    - A. the order of group
    - B. the order of subgroup
    - C. the index of every subgroup
    - D. None of these

53. A group of order ... is always cyclic.
    - A. 7
    - B. 8
    - C. 9
    - D. 10

54. A finite group of ... order is necessarily cyclic.
    - A. Prime
    - B. Even
    - C. Odd
    - D. Composite

55. Which of the following abelian group is not cyclic?
    - A. $(\mathbb{Z}, +)$
    - B. $(\mathbb{Q}, +)$
    - C. $(\mathbb{R}, +)$
    - D. Both B and C

56. Let G be a group of order 90. G can have a subgroup of order:
    - A. 30
    - B. 40
    - C. 50
    - D. 60

57. Let G be a cyclic group of order n generated by a. Then for any $1 \leq k < n$, the order of a^k is:
    - A. $\frac{k}{gcd(n,k)}$
    - B. $\frac{n}{lcm(n,k)}$
    - C. $\frac{n}{gcd(n,k)}$
    - D. $\frac{k}{lcm(n,k)}$

58. Let G be a cyclic group of order 24 generated by a. Then the order of a^{10} is:
    - A. 6
    - B. 14
    - C. 18
    - D. 24

59. Let H and K be two finite subgroups of a group G whose orders are relatively prime, then $H \cap K$ equals:
    - A. $\{e, a\}$
    - B. $H \cup K$
    - C. HK
    - D. $\{e\}$

60. Let X be a nonempty set. A bijective function $f : X \to X$ is called a ... on X.
    - A. Homomorphism
    - B. Isomorphism
    - C. Endomorphism
    - D. Permutation

61. The set of all permutations on a set X is denoted by:
    - A. SX
    - B. XS
    - C. S_X
    - D. X_S

62. The set S_n is a group under the operation of ... of permutations.
    - A. Addition
    - B. Subtraction
    - C. Multiplication
    - D. Composition

63. The order of symmetric group of degree n is:
    - A. n
    - B. $n!$
    - C. $\frac{n!}{2}$
    - D. $(\frac{n}{2})!$

64. Composition of permutations is not:

A. Associative

B. Closed

C. Commutative

D. All of these

65. If $f_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ and $f_2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$, then $f_1 \circ f_2$ equals:

A. $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$

B. $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$

C. $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$

D. $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$

66. A permutation of the form $\begin{pmatrix} a_1 & a_2 & ... & a_k \\ a_2 & a_3 & ... & a_1 \end{pmatrix}$ is called a ... of length k.

A. Permutation

B. Cycle

C. Transposition

D. Matrix

67. If two cycles act on mutually disjoint sets, then they:

A. can commute

B. must commute

C. don't commute

D. None of these

68. If $\alpha = (1\ 2\ 3)$ and $\beta = (5\ 7\ 8)$, then:

A. $\alpha\beta = I$

B. $\beta\alpha = I$

C. $\alpha\beta = \beta\alpha$

D. $\alpha\beta \neq \beta\alpha$

69. Every permutation of degree n can be written as a ... of cyclic permutations acting on mutually disjoint sets.

    A. Sum
    B. Difference
    C. Product
    D. Quotient

70. A cycle of length 2 is called a :

    A. Permutation
    B. Transposition
    C. Cycle
    D. Matrix

71. Every cyclic permutation can be expressed as a ... of transposition.

    A. Sum
    B. Difference
    C. Product
    D. Quotient

72. A permutation α in S_n is said to be ... permutation if it can be written as a product of an even number of transposition.

    A. Even
    B. Odd
    C. Composite
    D. Cyclic

73. Every transposition is an ... permutation.

    A. Even
    B. Odd
    C. Composite
    D. Cyclic

74. A cycle of even length is an ... permutation.

    A. Even
    B. Odd
    C. Composite
    D. Cyclic

75. The product of two even permutations is ... permutation.
    - A. Even
    - B. Odd
    - C. Composite
    - D. Cyclic

76. The product of two odd permutations is ... permutation.
    - A. Even
    - B. Odd
    - C. Composite
    - D. Cyclic

77. The product of an even and an odd permutations is ... permutation.
    - A. Even
    - B. Odd
    - C. Composite
    - D. Cyclic

78. If α is an odd permutation and τ is a transposition, then $\alpha\tau$ is ... permutation.
    - A. Even
    - B. Odd
    - C. Both A and B
    - D. None of these

79. For $n \geq 2$, the number of even permutations in S_n is ... the number of odd permutations in S_n.
    - A. Equal to
    - B. Not equal to
    - C. Greater than
    - D. Lesser than

80. The set of even permutations in S_n is denoted by:
    - A. A_n
    - B. E_n
    - C. $S_{\frac{n}{2}}$
    - D. None of these

81. The number of elements in alternating group A_n is:

A. n

B. $\frac{n}{2}$

C. $n!$

D. $\frac{n!}{2}$

82. The order of a cyclic permutation of length m is:

A. m

B. $\frac{m}{2}$

C. $m!$

D. $\frac{m!}{2}$

83. The order of $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 2 & 3 & 4 & 1 & 7 & 9 & 6 & 5 & 8 & 10 \end{pmatrix}$ is:

A. 10

B. 12

C. 15

D. 20

84. Inverse of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 5 & 4 & 1 & 7 & 2 & 6 \end{pmatrix}$ is:

A. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{pmatrix}$

B. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 2 & 7 & 1 & 4 & 5 & 3 \end{pmatrix}$

C. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 6 & 1 & 3 & 2 & 7 & 5 \end{pmatrix}$

D. All of these

85. A ring R is an abelian group under:

A. Addition

B. Subtraction

C. Multiplication

D. Division

86. Which of the following is a ring under usual addition and multiplication?
    A. $\mathbb{Z}$
    B. $\mathbb{Q}$
    C. $\mathbb{R}$
    D. All of these

87. If $(R, +, \cdot)$ is a ring with additive identity 0, then for all $a, b \in R$, we have:
    A. $a0 = 0a = 0$
    B. $a(-b) = (-a)b = -ab$
    C. $(-a)(-b) = ab$
    D. All of these

88. The multiplicative identity (if it exists) is called:
    A. Unit
    B. Unity
    C. Identity
    D. None of these

89. An element of a ring whose multiplicative inverse exists, is called:
    A. Unit
    B. Unity
    C. Identity
    D. None of these

90. Let R be a ring with unity. If every nonzero element of R is unit, then R is called:
    A. Division ring
    B. Skew field
    C. Integral domain
    D. Both A and B

91. A commutative division ring is called:
    A. Integral Domain
    B. Skew field
    C. Field
    D. Commutative ring

92. Which of the following is(are) field(s)?
    A. $\mathbb{Q}$
    B. $\mathbb{R}$
    C. $\mathbb{C}$
    D. All of these

93. $\mathbb{Z}_n$ is a field if and only if n is:
    A. Prime
    B. Composite
    C. Even
    D. Odd

94. Which of the following are true? (Free to choose more than one option).
    A. Every field is a ring.
    B. Every ring has a multiplicative identity.
    C. Multiplication in a field is commutative.
    D. The nonzero elements of a field form a group under multiplication.
    E. Addition in every ring is commutative.
    F. Every element in a ring has an additive inverse.

95. Which of the following is a field?
    A. $\mathbb{Z}$
    B. $\mathbb{Z}_8$
    C. $\mathbb{Z}_{13}$
    D. None of these

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1. If a matrix has 3 columns and 6 rows then the order of matrix is:
   - A. 3×6
   - B. 18
   - C. 6×3
   - D. 3×3
2. If order of a matrix A is 3×6, then each row of A consists ... elements.
   - A. 3
   - B. 6
   - C. 18
   - D. None of these
3. A matrix $A = [a_{ij}]_{m \times n}$ is square if:
   - A. $m = n$
   - B. $m \neq n$
   - C. $m < n$
   - D. $m > n$
4. A matrix that is not square is:
   - A. Rectangular
   - B. Identity
   - C. Diagonal
   - D. Scalar
5. A matrix $A = [a_{ij}]_{m \times n}$ is row matrix if:
   - A. $n = 1$
   - B. $n \neq 1$
   - C. $m = 1$
   - D. $m \neq 1$

6. In a square matrix $A = [a_{ij}]_{n\times n}$, the elements $a_{11}, a_{22}, ...a_{nn}$ are called ... elements.
   - A. Diagonal
   - B. Scalar
   - C. Identity
   - D. Unit

7. A square matrix $A = [a_{ij}]_{n\times n}$ is called upper triangular if $a_{ij} = 0$ for all:
   - A. $i > j$
   - B. $i < j$
   - C. $i \geq j$
   - D. $i \leq j$

8. A matrix, all of whose elements are zero except those in the main diagonal, is called a ... matrix.
   - A. Unit
   - B. Identity
   - C. Scalar
   - D. Diagonal

9. Which of the following is a diagonal matrix?
   - A. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 6 \\ 0 & 8 & 0 \end{bmatrix}$
   - B. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 6 & 8 & 0 \end{bmatrix}$
   - C. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
   - D. None of these

10. Every scalar matrix is a ... matrix.
   - A. Unit
   - B. Identity
   - C. Diagonal
   - D. All of these

11. If $A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ Then which of the following is true for A?

    A. $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

    B. $A = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}$

    C. $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

    D. None of these

12. If A and B are matrices of orders $m \times n$ and $p \times q$ respectively, then the product AB is possible if:

    A. $n = p$

    B. $n = q$

    C. $m = q$

    D. $m = p$ and $n = q$

13. If A and B are matrices of orders 4×5 and 5×7 respectively, then the order of AB is:

    A. 5×5

    B. 4×7

    C. 5×4

    D. 7×5

14. Let $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{n\times p}$, then $(i, j)th$ element of AB is:

    A. $\sum_{k=1}^{n} a_{ik}b_{kj}$

    B. $\sum_{k=1}^{n} a_{ki}b_{kj}$

    C. $\sum_{k=1}^{n} a_{ik}b_{jk}$

    D. $\sum_{k=1}^{n} a_{ki}b_{jk}$

15. If A and B are two nonzero matrices. Is it possible to have $AB = 0$?

    A. Yes

    B. No

16. Which law does not hold in matrices?

    A. Associative law of multiplication

    B. Distributive law of multiplication over addition

    C. Cancellation law

    D. Both A and B

17. If the matrices A, B and C are conformable for the sums and multiplications, then which of the following is correct?
    A. $A(BC) = (AB)C$
    B. $A(B + C) = AB + AC$
    C. $k(AB) = (kA)B$
    D. All of these

18. If order of A is 8×7, then the order of AA^t is:
    A. 7×8
    B. 7×7
    C. 8×8
    D. Product is not possible

19. If the matrices A and B are conformable for the sum and the product, then:
    A. $(AB)^t = B^t A^t$
    B. $(A^t)^t = A$
    C. $(kA)^t = kA^t$
    D. All of these

20. A square matrix A for which $A^{k+1} = A$, (k being a positive integer), is called a ... matrix.
    A. Nilpotent
    B. Periodic
    C. Involutory
    D. Idempotent

21. If $A^6 = A$, then the period of A is:
    A. 5
    B. 6
    C. 7
    D. Not period

22. A matrix of period 1 is:
    A. Nilpotent
    B. Involutory
    C. Idempotent
    D. Involutory

23. A square matrix A for which $A^p = 0$ (p being a positive integer), is called ...
    A. Nilpotent
    B. Involutory
    C. Idempotent
    D. Involutory

24. A square matrix A such that ... is called an involutory matrix.
    A. $A^2 = A$
    B. $A^2 = I$
    C. $A^2 = -A$
    D. $A^2 = -I$

25. For any square real matrix A, the matrix $A - A^t$ is:
    A. Symmetric
    B. Skew Symmetric
    C. Hermitian
    D. None of these

26. For a complex square matrix A, the matrix $A + (\overline{A})^t$ is:
    A. Symmetric
    B. Skew symmetric
    C. Hermitian
    D. Skew Hermitian

27. If A is a square matrix over $\mathbb{C}$ and $A(\overline{A})^t = 0$, then which of the following is true?
    A. $A = 0$
    B. $A^t = 0$
    C. $\overline{A} = 0$
    D. All of these

28. If A is a square matrix and B is left inverse of A, then:
    A. B can be right inverse of A
    B. B must be right inverse of A
    C. B must not be right inverse of A
    D. There is no relation between A and B

29. A square matrix, whose inverse exists, is called:
    - A. Singular
    - B. Nonsingular
    - C. Invertible
    - D. Both B and C

30. If A and B are nonsingular matrices of the same order, then $(AB)^{-1}$ equals:
    - A. AB
    - B. $A^{-1}B^{-1}$
    - C. BA
    - D. $B^{-1}A^{-1}$

31. A matrix obtained by applying an elementary row operation on I_n is called:
    - A. Invertible
    - B. Non Invertible
    - C. Elementary
    - D. Secondary

32. Every elementary matrix E is:
    - A. Singular
    - B. Nonsingular
    - C. Non invertible
    - D. Symmetric

33. A square matrix A of order n is nonsingular if and only if A is row equivalent to:
    - A. I_n
    - B. $-I_n$
    - C. A^2
    - D. $-A$

34. If an $m \times n$ matrix B is obtained from an $m \times n$ matrix A by a finite number of elementary row and column operations, then B is said to be ... to A.
    - A. Equal
    - B. Equivalent
    - C. Similar
    - D. Not equal

35. Every nonzero $m \times n$ matrix is equivalent to an $m \times n$ matrix $D = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$. Then D is called ... form of A.

A. Normal

B. Canonical

C. Both A and B

D. None of these

36. The rank of matrix $A \begin{bmatrix} 4 & 1 & 8 \\ 0 & 7 & 7 \\ 0 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ is:

A. 1

B. 2

C. 3

D. 4

37. The rank of matrix $A \begin{bmatrix} 1 & 3 \\ 0 & -2 \\ 5 & -1 \\ -2 & 3 \end{bmatrix}$ is:

A. 1

B. 2

C. 3

D. 4

38. If A is invertible and $AB = 0$, then:

A. $A = 0$

B. $B = 0$

C. $B \neq 0$

D. B is nonsingular

39. If A and B are square matrices of order n, then $AB - BA$ is:

A. Symmetric

B. Hermitian

C. Skew Symmetric

D. All of these

40. If $A = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$, then A^{50} equals:

    A. $\begin{bmatrix} 50 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$

    B. $\begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$

    C. $\begin{bmatrix} 25 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$

    D. $\begin{bmatrix} 25 & 0 \\ 50 & 1 \end{bmatrix}$

41. If a matrix A is symmetric as well as skew symmetric, then A is:

    A. Identity

    B. Nill

    C. Idempotent

    D. Diagonal

42. If $A^2 - A - I = 0$, then the inverse of A is:

    A. $A + I$

    B. $A - I$

    C. $I - A$

    D. $-A - I$

43. If A and B are square matrices of same order and $A^2 - B^2 = (A + B)(A - B)$, then which of the following must be true?

    A. $A = B$

    B. $AB = BA$

    C. Either A or B is a zero matrix

    D. Either A or B is an identity matrix

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1. A system of linear equations $Ax = b$ is called non homogeneous if:

   A. $b = 0$

   B. $b \neq 0$

   C. $A = 0$

   D. $A \neq 0$

2. If rank(A)=rank(A_b), then the system $Ax = b$:

   A. is consistent

   B. can have unique solution

   C. can have infinite solutions

   D. All of these

3. Let $Ax = b$ be a system of 3 linear equations in 7 variables, then which of the following can be the maximum value of rank(A_b)?

   A. 3

   B. 4

   C. 6

   D. 7

4. Let A be a matrix of order 4×5 and rank(A)=rank(A_b)=3, then the system $Ax = b$ has:

   A. unique solution

   B. no solution

   C. infinitely many solutions

   D. None of these

5. The system $\begin{bmatrix} -3 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ has:

   A. unique solution

   B. no solution

   C. infinitely many solutions

   D. None of these

6. If the augmented matrix of a system is $\begin{bmatrix} 1 & 2 & 1 & 0 \\ 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 \end{bmatrix}$, then the system has:
   - A. unique solution
   - B. no solution
   - C. infinitely many solutions
   - D. None of these

7. Let A be a 4×4 matrix and the system $Ax = b$ has infinitely many solutions, then:
   - A. $\text{rank}(A) = 4$
   - B. $\text{rank}(A) \neq 4$
   - C. $\text{rank}(A) < 4$
   - D. $\text{rank}(A) > 4$

8. If $Ax = b$ does not have any solution, then the system is called:
   - A. consistent
   - B. inconsistent
   - C. Both A and B
   - D. None of these

9. Every homogeneous system of linear equations:
   - A. is consistent
   - B. is inconsistent
   - C. has only trivial solution
   - D. has infinitely many solutions

10. For what value of λ, the system

$$\begin{aligned} (1-\lambda)x_1 - x_2 &= 0 \\ x_1 + (1-\lambda) &= 0 \end{aligned}$$

has non trivial solution?
   - A. 0
   - B. 2
   - C. 3
   - D. 4

11. In Gauss Elimination method, we need to reduce the augmented matrix into:
    - A. Echelon form
    - B. Reduced echelon form
    - C. Both A and B
    - D. None of these

12. A system $Ax = 0$ of n equations and n unknowns has a unique solution if A is:
    - A. singular
    - B. non singular
    - C. non invertible
    - D. None of these

13. The system $Ax = b$ of m equations and n unknowns has solution (is consistent) if rank(A) ... rank(A_b).
    - A. $=$
    - B. $\neq$
    - C. $>$
    - D. $<$

14. The system $Ax = b$ of m equations and n unknowns has no solution (is inconsistent) if rank(A) ... rank(A_b).
    - A. $=$
    - B. $\neq$
    - C. $>$
    - D. $<$

15. The system

$$x_1 + 2x_2 = 1$$
$$2x_1 + x_2 = 2$$

has a solution:
    - A. $(1, 1)$
    - B. $(1, 2)$
    - C. $(2, 1)$
    - D. $(1, 0)$

16. In Gauss-Jordan elimination method, we reduce the augmented matrix into:
    A. Echelon form
    B. Reduced echelon form
    C. Both A and B
    D. None of these

17. If a system of 2 equations and 2 unknowns has no solution, then the graph look like:
    A. Intersecting lines
    B. Non intersecting lines
    C. Same lines
    D. None of these

18. Which of the following is a linear equation in the variables x, y, z?
    A. $x - 2y = 0$
    B. $x + cosy = z$
    C. $\sin x + \cos y + tanz = 0$
    D. None of these

19. Which one of the following is a linear equation?
    A. $xy = e^{\pi}$
    B. $x + y = e^{\pi}$
    C. $y = \sqrt{3x}$
    D. $x = \sqrt{3y}$

20. If applying row operations to a matrix A of order $n \times n$ results in a row of zeros, then how many solutions does the system $Ax + b = 0$ have?
    A. No solutions
    B. Unique solution
    C. Infinitely many solutions
    D. More information is needed

21. A system of m homogeneous linear equations in n unknowns has a nontrivial solution if:
    A. $m = n$
    B. $m \neq n$
    C. $m < n$
    D. $m > n$

22. A system of m homogeneous linear equations $Ax = 0$ in n unknowns has a nontrivial solution if and only if rank(A):
    A. $= n$
    B. $\neq n$
    C. $= m$
    D. $\neq m$

23. For any matrix A, the collection $\{x : \; Ax = 0\}$ is called ... of A.
    A. Rank
    B. Solution space
    C. Both A and B
    D. None of these

24. A system of m linear equations $Ax = b$ in n unknowns has a unique solution if and only if rank(A)=rank(B) ...
    A. $= m$
    B. $= n$
    C. $\neq m$
    D. $\neq n$

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1. If A is any matrix of order $n \times n$ and k is a non zero real number, then:
   A. $|kA| = k|A|$
   B. $|kA| = |k||A|$
   C. $|kA| = k^2|A|$
   D. $|kA| = k^n|A|$

2. The determinant of a unit matrix is:
   A. 0
   B. 1
   C. -1
   D. ± 1

3. $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} + \begin{vmatrix} a_{11} & b_{12} \\ a_{21} & b_{22} \end{vmatrix} =$
   A. $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & b_{22} \end{vmatrix}$
   B. $\begin{vmatrix} a_{11} & a_{12} + b_{12} \\ a_{21} & a_{22} + b_{22} \end{vmatrix}$
   C. 0
   D. Addition is not possible

4. Let A be a square matrix of order n. A matrix obtained from A by deleting its ith row and jth column is again a matrix of order $n-1$ which is called:
   A. ijth minor of A
   B. ijth cofactor of A
   C. Determinant of A
   D. None of these

5. Let M_{ij} be the ijth minor of a square matrix A of order n. Then ijth cofactor of A is:
   A. $|M_{ij}|$
   B. $-|M_{ij}|$
   C. $\pm|M_{ij}|$
   D. $(-1)^{i+j}|M_{ij}|$

6. Let $A = \begin{bmatrix} 3 & 2 & 1 & -1 \\ 4 & 5 & 1 & 2 \\ -2 & 3 & 0 & 1 \\ 2 & 1 & 3 & 5 \end{bmatrix}$, then 33th cofactor of A is:
   - A. 43
   - B. 34
   - C. 56
   - D. -56

7. $\begin{vmatrix} 1 & 0 & 5 & 6 \\ 0 & 5 & 0 & 8 \\ 0 & 0 & -1 & 8 \\ 0 & 0 & 0 & 3 \end{vmatrix} =$
   - A. 3
   - B. -15
   - C. 28
   - D. -67

8. Let $A = [a_{ij}]$ be an $n \times n$ triangular matrix, then $|A|$ equals:
   - A. $a_{11}a_{22}...a_{nn}$
   - B. $a_{11} + a_{22} + ... + a_{nn}$
   - C. $-a_{11} - a_{22} - ... - a_{nn}$
   - D. There is no formula

9. Let A be a square matrix of order 4×4, then $|A|=$
   - A. $-|A|$
   - B. $|A^t|$
   - C. $-|A^t|$
   - D. 0

10. Row expansion of $|A|$ ... column expansion of $|A|$.
    - A. $=$
    - B. $\neq$
    - C. There is no comparison
    - D. None of these

11. For any $n \times n$ matrices A and B, we have:
    - A. $|AB| = |BA|$
    - B. $|AB| \neq |BA|$
    - C. $|AB| < |BA|$
    - D. $|AB| > |BA|$

12. Let A, B be matrices of order 6 such that $|AB^2| = 144$ and $|A^2B^2| = 72$, then $|A| =$
    - A. 2
    - B. $\frac{1}{2}$
    - C. -2
    - D. $-\frac{1}{2}$

13. For an invertible matrix A, $|A^{-1}|$ equals:
    - A. $|A|$
    - B. $-|A|$
    - C. $|A|^{-1}$
    - D. $-|A|^{-1}$

14. For 2×2 matrices A and B, which of the following equations hold? (Can choose more than one options)
    - A. $|A+B| = |A| + |B|$
    - B. $|A+B|^2 = |(A+B)^2|$
    - C. $|A+B|^2 = |A|^2 + |B|^2$
    - D. $|(A+B)^2| = |A^2 + 2AB + B^2|$

15. $\begin{vmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{vmatrix} =$
    - A. 0
    - B. 1
    - C. -1
    - D. abc

16. If A is an $n \times n$ skew symmetric matrix and n is odd, then $|A|=$
    - A. 0
    - B. 1
    - C. -1
    - D. ± 1

17. If a, b, c are different numbers. For what value of x, the matrix $\begin{bmatrix} 0 & x+b & x^2+c \\ x-b & 0 & x^2-a \\ x^3-c & x+a & 0 \end{bmatrix}$ is singular?

    A. 0

    B. a

    C. b

    D. c

18. If A is a square matrix of odd order, then $|-A|=$

    A. $|A|$

    B. $-|A|$

    C. 0

    D. 1

19. If $\begin{vmatrix} a & -b & 0 \\ 0 & a & b \\ b & 0 & a \end{vmatrix} = 0$, then:

    A. α is a root of unity

    B. β is a root of unity

    C. $\alpha\beta$ is a root of unity

    D. $\frac{\alpha}{\beta}$ is a root of unity

20. If A is an $n \times n$ non singular matrix, then which of the following is true?

    A. $|\text{adj}(A)| = |A|$

    B. $|\text{adj}(A)| = 1$

    C. $|\text{adj}(A)| = |A|^n$

    D. $|\text{adj}(A)| = |A|^{n-1}$

21. Let $A = \begin{bmatrix} k & 4k & 4 \\ 0 & 4 & 4k \\ 0 & 0 & 4 \end{bmatrix}$. If $|A^2| = 16$, then the value of k is:

    A. 1

    B. 4

    C. 16

    D. $\frac{1}{4}$

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1. The property $d(x, y) = d(y, x)$ is named as:
   A. Non negativity
   B. Reflexive
   C. Symmetry
   D. Triangle inequality
2. The property $d(x, y) \leq d(x, z) + d(z, y)$ is named as:
   A. Non negativity
   B. Reflexive
   C. Symmetry
   D. Triangle inequality
3. If (X, d) is a metric space then d is called a ... on X.
   A. Function
   B. Relation
   C. Metric
   D. Metric space
4. If (X, d) is a metric space then X is called:
   A. Metric
   B. Ground Set
   C. Underlying set
   D. Both B and C
5. Which of the following is not a metric on $\mathbb{R}$?
   A. $d(x, y) = |x| + |y|$
   B. $d(x, y) = max\{|x|, |y|\}$
   C. Both A and B
   D. None of these

6. Let (X, d) be a metric space. Which of the following is not a metric on X?
   A. $d_1(x, y) = kd(x, y)$, where k is a positive number
   B. $d_2(x, y) = \frac{d(x,y)}{1+d(x,y)}$
   C. $d_3(x, y) = \frac{kd(x,y)}{1+kd(x,y)}$
   D. $d_4(x, y) = \frac{1-d(x,y)}{1+d(x,y)}$

7. Let (X, d) be a metric space and $x_1, x_2, ..., x_n$ be points of X, then the property

$$d(x_1, x_n) \leq d(x_1, x_2) + d(x_2, x_3) + ... + d(x_{n-1}, x_n)$$

   is called:
   A. Generalized Triangle Inequality
   B. Generalized Non negativity
   C. Generalized Symmetry
   D. Generalized Reflexive

8. The usual (or Euclidean) metric on $\mathbb{R}$ is defined as:
   A. $d(x, y) = |x + y|$
   B. $d(x, y) = |z - y|$
   C. $d(x, y) = |x| + |y|$
   D. $d(x, y) = ||x| - |y||$

9. The usual (or Euclidean) metric on $\mathbb{R}^2$ is defined as ... , where $x = (x_1, x_2)$ and $y = (y_1, y_2)$.
   A. $d(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$
   B. $d(x, y) = |x_1 - y_1| + |x_2 - y_2|$
   C. $d(x, y) = max\{|x_1 - y_1|, |x_2 - y_2|\}$
   D. None of these

10. The taxi-cab metric on $\mathbb{R}^2$ is defined as ... , where $x = (x_1, x_2)$ and $y = (y_1, y_2)$.
   A. $d(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$
   B. $d(x, y) = |x_1 - y_1| + |x_2 - y_2|$
   C. $d(x, y) = max\{|x_1 - y_1|, |x_2 - y_2|\}$
   D. None of these

11. The discrete metric on a non empty set X is defined as:

A. $d(x,y) = \begin{cases} 0 & if\ x = y \\ 1 & if\ x \neq y \end{cases}$

B. $d(x,y) = \begin{cases} 0 & if\ x \neq y \\ 1 & if\ x = y \end{cases}$

C. $d(x,y) = \begin{cases} 0 & if\ x = y \\ -1 & if\ x \neq y \end{cases}$

D. $d(x,y) = \begin{cases} -1 & if\ x = y \\ 0 & if\ x \neq y \end{cases}$

12. Let $x = (x_1, x_2, ..., x_n)$, $y = (y_1, y_2, ..., y_n)$ be any two points of $\mathbb{R}^n$. Then

$$\sum_{k=1}^{n} |x_k y_k| \leq (\sum_{k=1}^{n} |x_k|^2)^{\frac{1}{2}} (\sum_{k=1}^{n} |y_k|^2)^{\frac{1}{2}}.$$

This inequality is called:

A. Cauchy Inequality

B. Cauchy-Schwarz Inequality

C. Minkowski's Inequality

D. Holder's Inequality

13. If $x_1, x_2, ..., x_n$ be real numbers, then $(|x_1| + |x_2| + ... + |x_n|)^{\frac{1}{2}}$...

A. $\leq |x_1|^2 + |x_2|^2 + ... + |x_n|^2$

B. $\leq n(|x_1|^2 + |x_2|^2 + ... + |x_n|^2)$

C. $\geq n(|x_1|^2 + |x_2|^2 + ... + |x_n|^2)$

D. None of these

14. Let $x = (x_1, x_2, ..., x_n)$, $y = (y_1, y_2, ..., y_n)$ be any two points of $\mathbb{R}^n$. Then

$$(\sum_{k=1}^{n} |x_k + y_k|)^{\frac{1}{2}} \leq (\sum_{k=1}^{n} |x_k|^2)^{\frac{1}{2}} + (\sum_{k=1}^{n} |y_k|^2)^{\frac{1}{2}}.$$

This inequality is called:

A. Cauchy Inequality

B. Cauchy-Schwarz Inequality

C. Minkowski's Inequality

D. Holder's Inequality

15. The collection of all continuous real-valued functions defined on a closed interval $[a, b]$ is denoted as:
    A. $C[a, b]$
    B. $L[a, b]$
    C. $D[a, b]$
    D. l^∞

16. Let (X, d) be a metric space and $x, y, z \in X$. Then which of the following is true?
    A. $|d(x, z) - d(y, z)| \leq d(x, y)$
    B. $|d(x, y) - d(x, z)| \leq d(y, z)$
    C. $|d(x, y) - d(y, z)| \leq d(x, z)$
    D. All of these

17. The distance between a point x and subset A of a metric space (X, d) is defined as:
    A. $d(x, A) = inf\{d(x, a) : a \in A)\}$
    B. $d(x, A) = sup\{d(x, a) : a \in A)\}$
    C. $d(x, A) = inf\{d(x, y) : x, y \in A)\}$
    D. $d(x, A) = inf\{|x - 1| : a \in A)\}$

18. The distance between two subsets A, B of a metric space (X, d) is defines as:
    A. $d(A, B) = inf\{d(x, a) : a \in A)\}$
    B. $d(A, B) = inf\{d(x, b) : b \in B)\}$
    C. $d(A, B) = inf\{d(a, b) : a \in A, b \in B)\}$
    D. All of these

19. Let A and B be overlapping subsets of a metric space (X, d), then distance between A and B is:
    A. Not defined
    B. Zero
    C. Infinity
    D. None of these

20. The distance between $A = \{(x, y) \in \mathbb{R}^2 : y = \frac{1}{x}, x \neq 0\}$ and $B = \{(x, y) \in \mathbb{R}^2 : y = 0\}$ is:
    A. Not defined
    B. Zero
    C. Infinity
    D. None of these

21. If A is a subset of a metric space (X, d) such that $\delta(A) < \infty$, then A is called:
    A. Finite
    B. Bounded
    C. Open
    D. Closed

22. Let (X, d) be a metric space and $\delta(X) < \infty$, then d is called ... metric.
    A. Finite
    B. Bounded
    C. Open
    D. Closed

23. An example of a bounded metric is:
    A. Discrete metric on any non empty set
    B. Usual metric on $\mathbb{R}$
    C. Usual metric on $\mathbb{R}^2$
    D. None of these

24. Intersection of many many bounded sets is:
    A. Bounded
    B. Unbounded
    C. Empty
    D. Open

25. Union of finitely many bounded sets is:
    A. Bounded
    B. Not necessarily bounded
    C. Unbounded
    D. Open

26. Let (X, d) be a metric space. If $a \in X$ and $r > 0$, then the open ball centered at a and with radius r is:
    A. $B(a;r) = \{x \in X : d(a,x) \leq r\}$
    B. $B(a;r) = \{x \in X : d(a,x) < r\}$
    C. $\overline{B}(a;r) = \{x \in X : d(a,x) \leq r\}$
    D. $\overline{B}(a;r) = \{x \in X : d(a,x) < r\}$

27. A point $y \in B(a,r)$ if and only if:
    A. $d(a,y) > r$
    B. $d(a,y) \geq r$
    C. $d(a,y) < r$
    D. $d(a,y) \leq r$

28. An open ball in $(\mathbb{R}, d)$ (usual metric) with center a and radius r is:
    A. $(a-r, a+r)$
    B. $[a-r, a+r]$
    C. $(r-a, r+a)$
    D. $[r-a, r+a]$

29. The unit open ball in $(\mathbb{R}^2, d)$ (usual metric) at the origin is:
    A. $\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$
    B. $\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 > 1\}$
    C. $\{(x,y) \in \mathbb{R}^2 : |x| + |y| < 1\}$
    D. $\{(x,y) \in \mathbb{R}^2 : |x| + |y| > 1\}$

30. The unit open ball in $(\mathbb{R}^2, d')$ (Taxi-cab metric) at the origin is:
    A. $\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$
    B. $\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 > 1\}$
    C. $\{(x,y) \in \mathbb{R}^2 : |x| + |y| < 1\}$
    D. $\{(x,y) \in \mathbb{R}^2 : |x| + |y| > 1\}$

31. Let (X, d_0) be a discrete metric space, $a \in X$ and $r > 1$, then $B(a,r)$=
    A. ϕ
    B. $\{a\}$
    C. X
    D. $X - \{a\}$

32. Let (X, d_0) be a discrete metric space, $a \in X$ and $0 < r \leq 1$, then $B(a, r)$=
    A. ϕ
    B. $\{a\}$
    C. X
    D. $X - \{a\}$

33. Let (X, d) be a metric space. A subset $O \subset X$ is called ... if for each $x \in O$, there exists $r > 0$ such that $B(x; r) \subset O$.
    A. Open
    B. Closed
    C. Bounded
    D. Unbounded

34. Any open ball in a metric space is:
    A. Open set
    B. Closed set
    C. Bounded set
    D. Not necessarily a closed set

35. A subset O of a metric space (X, d) is open if and only if O is the ... of open balls.
    A. Union
    B. Intersection
    C. Complement
    D. Any of A, B or C

36. Let (X, d) be a metric space. Then ϕ and X are:
    A. Open
    B. Closed
    C. Both A and B
    D. None of these

37. The arbitrary ... of open sets is an open set.
    A. Union
    B. Intersection
    C. Complement
    D. Symmetric Difference

38. The finite ... of open sets is an open set.
    - A. Union
    - B. Intersection
    - C. Complement
    - D. Symmetric Difference

39. The arbitrary intersection of open sets in a metric space:
    - A. Is open
    - B. Is not necessarily open
    - C. Is closed
    - D. Is not necessarily closed

40. Let $I_n = \{(-\frac{1}{n}, \frac{1}{n}) : n \in \mathbb{N}\}$, then $\cap_{n=1}^{\infty} I_n$ equals:
    - A. $\{\}$
    - B. $\{0\}$
    - C. $\{1\}$
    - D. $(0, 1)$

41. Every subset of a discrete metric space is:
    - A. Open
    - B. Closed
    - C. Open as well as closed
    - D. Not open, nor closed

42. Every finite subset of a metric space is:
    - A. Open
    - B. Closed
    - C. Open as well as closed
    - D. Not open, nor closed

43. Let (X, d) be a metric space and let a be any point of X. A subset N of X is called ... if there exists an open ball $B(a; r)$ such that $B(a; r) \subseteq N$.
    - A. Open set
    - B. Closed set
    - C. Neighborhood of a
    - D. None of these

44. If a subset N of a metric space (X, d) is neighborhood of each of its points, then N is:
   A. Open
   B. Closed
   C. Bounded
   D. Compact

45. If N is a neighborhood of a and $N \subset M$, then M is:
   A. Neighborhood of a
   B. Open
   C. Closed
   D. Bounded

46. If N is a neighborhood of a point a, then a is called ... of N.
   A. Interior point
   B. Exterior point
   C. Limit point
   D. Boundary point

47. For any subset A of a metric space (X, d), interior of A is:
   A. Open
   B. Not necessarily open
   C. Closed
   D. Not necessarily closed

48. For any subset A of a metric space (X, d), which of the following is true?
   A. $A \subseteq A^o$
   B. $A^o \subseteq A$
   C. $A = A^o$
   D. $A \neq A^o$

49. A subset A of a metric space (X, d) is open if and only if:
   A. $A = A^o$
   B. A is neighborhood of each of its points
   C. Both A and B are true
   D. None of these

50. Let $A = [a, b]$ be any subset of $\mathbb{R}$ with usual metric. Then A^o equals:

    A. $[a, b]$

    B. $[a, b)$

    C. $(a, b]$

    D. (a, b)

51. Let $A = [a, b]$ be any subset of $\mathbb{R}$ with discrete metric. Then A^o equals:

    A. $[a, b]$

    B. $[a, b)$

    C. $(a, b]$

    D. (a, b)

52. For any subset A of a metric space (X, d), ... is the largest open subset of A^c.

    A. Interior of A

    B. Exterior of A

    C. Closure of A

    D. Boundary of A

53. For any subset A of a metric space (X, d), interior of A is the ... of all open subsets of A.

    A. Union

    B. Intersection

    C. Symmetric difference

    D. All of these

54. For any subsets A, B of a metric space (X, d), which of the following is false?

    A. $(A^o)^o = A^o$

    B. $A \subseteq B$ implies $A^o \subseteq B^o$

    C. $(A \cap B)^o = A^o \cap B^o$

    D. $(A \cup B)^o = A^o \cup B^o$

55. Consider $\mathbb{Q}$ as a subset of $\mathbb{R}$ with usual metric, then Q^o equals:

    A. ϕ

    B. $\mathbb{Q}$

    C. $\mathbb{Q}'$

    D. $\mathbb{R}$

56. For any two subsets A and B of a metric space (X, d) , $(A \cup B)^o \ldots. A^o \cup B^o$.

    A. $\subseteq$

    B. $\supseteq$

    C. $=$

    D. None of these

57. If $A = \phi$ and $B = \mathbb{R}$, then $A^o \cup B^o$=:

    A. ϕ

    B. $\mathbb{R}$

    C. (a, b)

    D. $[a, b]$

58. Let A be any subset of a metric space (X, d). A point $x \in X$ is called a limit point of A, if for every open ball $B(x; r)$, we have:

    A. $B(x; r) \cap (A - \{x\}) \neq \phi$

    B. $(B(x; r) \cap A) - \{x\} \neq \phi$

    C. $(B(x; r) - \{x\}) \cap A \neq \phi$

    D. All of these

59. The set of all limit points of A, denoted as A^d is called ... of A.

    A. Interior

    B. Derived set

    C. Boundary

    D. Closure

60. Consider $\mathbb{Z}$ as a subset of $\mathbb{R}$ with usual metric, then $\mathbb{Z}^d$=:

    A. ϕ

    B. $\mathbb{Z}$

    C. $\mathbb{Q}$

    D. $\mathbb{R}$

61. A subset K of a metric space (X, d) is .....if K^c is open.

    A. Closed

    B. Interior of K

    C. Closure of K

    D. Boundary of K

62. A set K is closed if and only if
    - A. $K^d \subseteq K$
    - B. $K \subseteq K^d$
    - C. $K = K$
    - D. Any of A, B or C

63. Consider $A = \{1, \frac{1}{2}, \frac{1}{3}...\}$ as a subset of Euclidean metric space $(\mathbb{R}, d)$, then $A^d =$.
    - A. $\{0\}$
    - B. $\{1\}$
    - C. A
    - D. R

64. Consider $A = [a.b]$ as a subset of Euclidean metric space $(\mathbb{R}, d)$, then A^d.
    - A. ϕ
    - B. (a, b)
    - C. $[a, b]$
    - D. $\{a, b\}$

65. If x is a limit point of A, then every neighborhood of x contains ... number of points.
    - A. Finite
    - B. Infinite
    - C. Finite or Infinite
    - D. None of these

66. $\mathbb{Z}$ is ... subset of $\mathbb{R}$ with usual metric.
    - A. Open
    - B. Bounded
    - C. Closed
    - D. Compact

67. $\mathbb{Q}^d =?$
    - A. ϕ
    - B. $\mathbb{Q}$
    - C. $\mathbb{Q}'$
    - D. $\mathbb{R}$

68. $(\mathbb{Q}')^d = ?$

A. ϕ

B. $\mathbb{Q}$

C. $\mathbb{Q}'$

D. $\mathbb{R}$

69. Let (X, d) be a metric space and $a \in X$. For a positive real number r, the closed ball with center at x and radius r is

A. $B(a;r) = \{x \in X : d(a,x) \leq r\}$

B. $B(a;r) = \{x \in X : d(a,x) < r\}$

C. $\overline{B}(a;r) = \{x \in X : d(a,x) \leq r\}$

D. $\overline{B}(a;r) = \{x \in X : d(a,x) < r\}$

70. A closed ball in a metric space is

A. A closed set.

B. Not necessarily a closed set

C. An open set

D. Not an open set

71. Arbitrary intersection of closed sets is

A. A closed set.

B. Not necessarily a closed set

C. An open set

D. Not an open set

72. A point $x \in (X, d)$ is called a ... point if for every $r > 0$, $B(x;r) \cap A \neq \phi$

A. Limit point

B. Adherent point

C. Isolated point

D. Interior point

73. Let (X, d) be a metric space and $A \subseteq X$. A point $x \in A$ is called ... point of A if x is not a limit point of A.

A. Limit point

B. Adherent point

C. Isolated point

D. Interior point

74. A set is called ... if it is closed and has no isolated point
    - A. Perfect
    - B. Closed
    - C. Compact
    - D. Dense

75. The collection of all adherent points of a set A is called ... of A.
    - A. Interior
    - B. Exterior
    - C. Closure
    - D. Boundary

76. If $A = (0, 1)$, then $\overline{A} =$
    - A. $(0, 1)$
    - B. $[0, 1)$
    - C. $(0, 1]$
    - D. $[0, 1]$

77. If $A = \{\frac{1}{n} : n \in \mathbb{N}\}$, then $\overline{A}$=
    - A. A
    - B. $A \cup \{0\}$
    - C. $A - \{0\}$
    - D. ϕ

78. $A \cup A^d =$
    - A. A^o
    - B. $(A')^o$
    - C. $\overline{A}$
    - D. $Fr(A)$

79. $\overline{A}$ is ...
    - A. Open
    - B. Closed
    - C. Compact
    - D. Bounded

80. Which of the follwing is true?

A. $A \subseteq \overline{A}$

B. $\overline{A} \subseteq A$

C. $A \subseteq A^o$

D. $(A')^o = A$

81. The smallest closed superset of A is

A. A^o

B. $ext(A)$

C. A^d

D. $\overline{A}$

82. For any subset A of a metric space (X, d), we have $\overline{\overline{A}}$=

A. A

B. $\overline{A}$

C. A^c

D. A^o

83. Which of the following is false?

A. $\overline{\phi} = \phi$, $\overline{X} = X$

B. $A \subseteq B \Rightarrow \overline{A} \subseteq \overline{B}$

C. $(\overline{A \cup B}) = \overline{A} \cup \overline{B}$

D. $\overline{A \cap B} = \overline{A} \cap \overline{B}$

84. $\overline{A} \cap \overline{A^c}$ =?

A. $\overline{A}$

B. A^d

C. $Fr(A)$

D. A^o

85. Which of the following is true

A. $Fr(A) = \overline{A} - A^o$

B. $\overline{A} = A^o \cup Fr(A)$

C. $Fr(A) \cap A^o = \phi$

D. All of these

86. A is called if and only if
    - A. $Fr(A) \subseteq A$
    - B. $Fr(A) \supseteq A$
    - C. $Fr(A) \subseteq A^c$
    - D. $Fr(A) \supseteq A^c$
87. A is open if ...
    - A. $Fr(A) \subseteq A$
    - B. $Fr(A) \supseteq A$
    - C. $Fr(A) \subseteq A^c$
    - D. $Fr(A) \supseteq A^c$
88. Which of the following is false?
    - A. $ext(A \cup B) = ext(A) \cup ext(B)$
    - B. $ext(A \cap B) = ext(A) \cap ext(B)$
    - C. $ext(ext(A)) \supseteq A^o$
    - D. $A \cap ext(A) = \phi$
89. A subset A of a metric space (X, d) is closed if and only if:
    - A. $A = \overline{A}$
    - B. $A = A^o$
    - C. $A \neq \overline{A}$
    - D. $A \neq A^o$
90. A subset A of a metric space (X, d) is open if and only if:
    - A. $A = \overline{A}$
    - B. $A = A^o$
    - C. $A \neq \overline{A}$
    - D. $A \neq A^o$

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1. For any positive integers a and b, there exists a positive integer n such that $na > b$ is called:
   - A. Archimedean Property
   - B. Division Algorithm
   - C. Density Theorem
   - D. Fundamental Theorem of Arithmetic

2. Let $S \subseteq \mathbb{N}$ having the properties:
   (i) $1 \in S$ and
   (ii) Whenever $k \in S$, then $k+1 \in S$, then
   - A. $S = \mathbb{N}$
   - B. $S \subseteq \mathbb{N}$
   - C. $S \supseteq \mathbb{N}$
   - D. $S \neq \mathbb{N}$

3. $2[1 + 2 + 3 + ... + n]$=
   - A. $\frac{n(n+1)}{2}$
   - B. $\frac{n(n-1)}{2}$
   - C. $n(n+1)$
   - D. $n(n-1)$

4. Given integers a and b with $b \neq 0$, there exist unique integers q and r satisfying
   - A. $a = bq + r$, $0 \leq r < |b|$
   - B. $a = bq + r$, $0 \leq q < |b|$
   - C. $a = bq + r$, $0 \leq r < |a|$
   - D. $a = bq + r$, $0 \leq q < |a|$

5. Which of the following is false?
   - A. $a|a$
   - B. If $a|b$ and $b|c$, then $a|c$
   - C. If $a|b$ and $b|a$, then $a = b$
   - D. If $a|b$ then $a|bc$

6. If $a|b$ and $a|c$, then for any $x, y \in \mathbb{Z}$, we have
   A. $a|(bx + cy)$
   B. $a|(bx - cy)$
   C. $a|bc$
   D. All of these

7. If $a|(b + c)$ and $a|b$, then
   A. $a|c$
   B. $a \nmid c$
   C. $a|(b - c)$
   D. $a \nmid (b - c)$

8. If $a = 73$ and $b = 8$, then
   A. $q = 9$, $r = -1$
   B. $q = 9$, $r = 1$
   C. $q = -9$, $r = 1$
   D. $q = -9$, $r = -1$

9. If $a = -23$ and $b = 7$, then
   A. $q = 4$, $r = 5$
   B. $q = -4$, $r = 5$
   C. $q = 4$, $r = -5$
   D. $q = -4$, $r = -5$

10. We read $a|b$ as
   A. a divides b
   B. b is divisible by a
   C. b is multiple of a
   D. All of these

11. Let $a, b \in \mathbb{Z}$ with $a \neq 0$. Then $a|b$ if for some $c \in \mathbb{Z}$,
   A. $a = bc$
   B. $b = ac$
   C. $c = a + b$
   D. $c = ab$

12. Any integer can be expressed in the form

    A. $2n$ or $2n+1$

    B. $3n$, $3n+1$ or $3n+2$

    C. $4n$, $4n+1$, $4n+2$ or $4n+3$

    D. All of these

13. For any $n \in \mathbb{Z}$, $2.7^n + 3.5^n - 5$ is divisible by

    A. 24

    B. 23

    C. 9

    D. 13

14. The product of any three consecutive integers is divisible by

    A. 4

    B. 5

    C. 6

    D. 7

15. Let a, b be nonzero integers. Then a positive integer d is called ... of a and b if

    (i) $d|a$ and $d|b$

    (ii) If $c|a$ and $c|b$, then $c \leq d$.

    A. G. C. D

    B. L. C. M

    C. H. C. F

    D. Both A and C

    [We denote G. C. D. of a and b as (a, b) or $gcd(a, b)$.]

16. Let a, b be nonzero integers and $(a, b) = 1$, then a, b are called

    A. Prime to each other

    B. Coprime

    C. Relatively prime

    D. All of these

17. The G.C.D of two non zero integers a and b:
    - A. Is always unique
    - B. Is not necessarily unique
    - C. Always exists
    - D. Both A and C

18. If $a|b$, then (a, b)=
    - A. a
    - B. b
    - C. $|a|$
    - D. $|b|$

19. $(8, -40)$=
    - A. 8
    - B. -8
    - C. 2
    - D. -2

20. If $d = (a, b)$, then there exist $x, y \in \mathbb{Z}$ such that:
    - A. $d = ax + by$
    - B. $d = ax - by$
    - C. $d = ay + bx$
    - D. All of these

21. Let $k \in \mathbb{Z}$ and $a, b \in \mathbb{Z} - \{0\}$
    - A. $k(a, b)$
    - B. $|k|(a, b)$
    - C. Both A and B
    - D. None of these

22. If $d = (a, b)$, then
    - A. $(\frac{a}{d}, \frac{b}{d}) = 1$
    - B. $(\frac{a}{d}, \frac{b}{d}) = d$
    - C. $(\frac{a}{b}, \frac{b}{a}) = d$
    - D. $(\frac{a}{b}, \frac{b}{a}) = 1$

23. If $a|bc$ and $(a,b)=1$, then

    A. $a|c$

    B. $b|c$

    C. $a\nmid c$

    D. $a|(b+c)$

24. Let $a,b\in\mathbb{Z}-\{0\}$. Then a positive integer m is called ... of a and b if

    (i) $a|m$ and $b|m$

    (ii) If $a|n$ and $b|n$ then $m\leq n$.

    A. G. C. D

    B. L. C. M

    C. H. C. F

    D. Both B and C

    [We denote L. C. M of a and b as $<a,b>$, $[a,b]$ or $lcm(a,b)$.]

25. For any non zero integers a,b we have

    A. $<a,b>=ab(a,b)$

    B. $(a,b)=ab<a,b>$

    C. $a(a,b)=b<a,b>$

    D. $<a,b>(a,b)=ab$

26. If $a=bq+r$, then which of the following is true?

    A. $(a,b)=(b,r)$

    B. $(a,r)=(b,r)$

    C. $<a,b>=<b,r>$

    D. $<a,r>=<b,r>$

27. For any two non zero integers a,b, we have $(a,(a,b))=$

    A. b

    B. a

    C. ab

    D. $a+b$

28. Let a, b be non zero integers and $c \in \mathbb{Z}$, the equation $ax + by = c$ is called ... in two variables.

    A. Polynomial

    B. Linear Diophantine

    C. Linear Equation

    D. Quadratic

29. Let $d = (a, b)$. The Linear Diophantine equation $ax + by = c$ has a solution if and only if:

    A. $d|c$

    B. $c|d$

    C. $(c, d) = 1$

    D. $c|(a + b)$

30. If (x_o, y_o) is a solution of Linear Diophantine equation $ax + by = c$, then the solution set of equation is:

    A. $\{(x_o + \frac{b}{d}t, y_o + \frac{a}{d}t) : t \in \mathbb{Z}\}$

    B. $\{(x_o + \frac{b}{d}t, y_o - \frac{a}{d}t) : t \in \mathbb{Z}\}$

    C. $\{(x_o - \frac{b}{d}t, y_o + \frac{a}{d}t) : t \in \mathbb{Z}\}$

    D. $\{(x_o - \frac{b}{d}t, y_o - \frac{a}{d}t) : t \in \mathbb{Z}\}$

31. A point (x_o, y_o) with integral coordinates is called:

    A. Common point

    B. Lattice point

    C. Integral point

    D. None of these

32. A number n whose only positive divisors are 1 and n, is called:
    A. Prime
    B. Coprime
    C. Relatively prime
    D. All of these

33. The smallest prime number is:
    A. 1
    B. 2
    C. 3
    D. 5

34. An integer which is not a prime, nor composite is:
    A. 1
    B. 2
    C. 3
    D. 4

35. Every integer $n > 1$ has a:
    A. Prime divisor
    B. Composite divisor
    C. Common multiple
    D. Both A and C

36. If p is a prime and $p|ab$, then
    A. $p|a$ or $p|b$
    B. $p|a$ and $p|b$
    C. $p \nmid a$ and $p \nmid b$
    D. $p|a$ but $p \nmid b$

37. There are ... number of primes. (Euclid's theorem)
    A. Finite
    B. Infinite
    C. Countable
    D. None of these

38. Let $n > 1$ be a composite number, then there exists a prime p such that $p|n$ and
    - A. $p \leq \sqrt{n}$
    - B. $p \geq \sqrt{n}$
    - C. $p < \sqrt{n}$
    - D. $p > \sqrt{n}$

39. Every integer $n > 1$ can be represented uniquely as a product of:
    - A. Prime numbers
    - B. Composite numbers
    - C. Even numbers
    - D. Odd numbers

40. For $n > 0$, the numbers of the form $2^{2^n} + 1$ are called ... numbers.
    - A. Fermat
    - B. Mersenne
    - C. Perfect
    - D. None of these

41. Any two Fermat numbers are:
    - A. Prime
    - B. Coprime
    - C. Composite
    - D. None of these

42. For $n > 0$, the numbers of the form $M_n = 2^n - 1$ are called:
    - A. Fermat's
    - B. Mersenne
    - C. Perfect
    - D. None of these

43. If M_n is prime, then n is:
    - A. Prime
    - B. Composite
    - C. Not necessarily prime
    - D. Not necessarily composite

44. Given a positive integer n, $\tau(n)$ or $d(n)$ denotes the:
    A. Sum of positive divisors of n
    B. Number of positive divisors of n
    C. Number of coprime numbers of n
    D. None of these

45. Given a positive integer n, $\sigma(n)$ denotes the:
    A. Sum of positive divisors of n
    B. Number of positive divisors of n
    C. Number of coprime numbers of n
    D. None of these

46. $\tau(n)=$
    A. $\sum_{d|n} 1$
    B. $\sum_{d|n} d$
    C. Both of these
    D. None of these

47. $\sigma(n)=$
    A. $\sum_{d|n} 1$
    B. $\sum_{d|n} d$
    C. Both of these
    D. None of these

48. $\tau(10)=$
    A. 3
    B. 4
    C. 5
    D. 6

49. $\sigma(10)=$
    A. 5
    B. 9
    C. 10
    D. 18

50. If $n = p_1^{k_1} p_2^{k_2} ... p_r^{k_r}$, then $\tau(n)$=

A. $(k_1 + 1)(k_2 + 1)...(k_r + 1)$

B. $k_1 k_2 ... k_r$

C. $k_1(k_2 + 1)...(k_r + 1)$

D. $n(k_1 + 1)(k_2 + 1)...(k_r + 1)$

51. $\tau(180)$=

A. 18

B. 9

C. 180

D. 90

52. If n is a positive integer such that $\sigma(n) = 2n$, then n is called a ... number.

A. Mersenne

B. Fermat

C. Perfect

D. None of these

53. Let m be a fixed positive integer. Then an integer a is congruent to an integer b modulo m, written as $a \equiv b(mod\ m)$ if:

A. $a|(m + b)$

B. $m|(a - b)$

C. $m|(b - a)$

D. Both B and C

54. Congruence is ... relation on $\mathbb{Z}$.

A. Equivalence

B. Partial order

C. Anti symmetric

D. Anti reflexive

55. Let $a, b \in \mathbb{Z}$. Then $a \equiv b(mod\ m)$ if and only if a, b have the same ... after division by m.

A. Quotient

B. Remainder

C. Both A and B

D. None of these

56. If $a \equiv b(mod\ m)$ and $c \equiv d(mod\ m)$, then which of the following is false?
    A. $a + c \equiv b + d(mod\ m)$
    B. $ac \equiv bd(mod\ m)$
    C. $na \equiv nb(mod\ m)$, where $n \in \mathbb{Z}$
    D. None of these

57. Which of the following is true?
    A. If $a \equiv b(mod\ m)$, then $a^n \equiv b^n(mod\ m)$
    B. If $na \equiv nb(mod\ m)$ and $(m, n) = d$, then $a \equiv b(mod\ \frac{m}{d})$
    C. If $na \equiv nb(mod\ m)$ and $(m, n) = 1$, then $a \equiv b(mod\ m)$
    D. All of these

58. $\phi(n) = n - 1$ if and only if n is:
    A. Prime
    B. Odd prime
    C. Odd
    D. Even

59. $(p - 1)! \equiv -1(mod\ p)$ if and only if
    A. p is a prime
    B. p is an odd prime
    C. p is an odd integer
    D. None of these

60. For $a, m \in \mathbb{Z}$, $a^{\phi(m)} \equiv 1(mod\ m)$ if
    A. $(a, m) \neq 1$
    B. $(a, m) = 1$
    C. $< a, m > \neq 1$
    D. $< a, m > = 1$

61. Which of the following is true?
    A. If $(m, n = 1)$, then $\phi(mn) = \phi(m)\phi(n)$
    B. If $m = p_1^{k_1} p_2^{k_2} ... p_r^{k_r}$, then $\phi(m) = m(1 - \frac{1}{p_1})(1 - \frac{1}{p_2})...(1 - \frac{1}{p_r})$
    C. $\phi(372) = 120$
    D. All of these

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1. An ordinary differential equation is a differential equation containing one or more dependent variables of ... independent variable(s).
   - A. One
   - B. Two
   - C. More than one
   - D. More than two

2. Number of independent variables in partial differential equation are
   - A. Two
   - B. Three
   - C. More then two
   - D. More then one

3. The order of $[1+(\frac{dy}{dx})^2]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ is:
   - A. 1
   - B. 2
   - C. 3
   - D. None of these

4. The degree of $[1+(\frac{dy}{dx})^2]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ is:
   - A. 1
   - B. 2
   - C. 3
   - D. None of these

5. The order of the differential equation $(\frac{d^4y}{dx^4})^{\frac{2}{5}} + 5\frac{d^3y}{dx^3} + 5\frac{dy}{dx} - 6 = 0$ is:
   - A. 1
   - B. 2
   - C. 3
   - D. 4

6. The degree of the differential equation $(\frac{d^4y}{dx^4})^{\frac{2}{5}} + 5\frac{d^3y}{dx^3} + 5\frac{dy}{dx} - 6 = 0$ is:
   A. 1
   B. 2
   C. 3
   D. 4

7. The order of differential equation $x = \frac{dy}{dx} + (\frac{dy}{dx})^2 + (\frac{dy}{dx})^3 + ...$ is:
   A. 1
   B. 2
   C. 3
   D. 4

8. The degree of differential equation $x = \frac{dy}{dx} + (\frac{dy}{dx})^2 + (\frac{dy}{dx})^3 + ...$ is:
   A. 1
   B. 2
   C. 3
   D. 4

9. An ordinary differential equation of order n,

$$F(x, y, \frac{dy}{dx}, ..., \frac{d^ny}{dx^n}) = 0$$

   is said to be ... if F is a linear function of the variables $y, \frac{dy}{dx}, ..., \frac{d^ny}{dx^n}$.
   A. Linear
   B. Non Linear
   C. Quadratic
   D. None of these

10. Which of the following equations is linear?
   A. $\frac{d^3y}{dx^3} + x^2\frac{dy}{dx} - y = 0$
   B. $\frac{d^3y}{dx^3} + x^2\frac{dy}{dx} - \sin y = 0$
   C. $\frac{d^3y}{dx^3} + x^2y\frac{dy}{dx} - y = 0$
   D. $\frac{d^3y}{dx^3} + x^2(\frac{dy}{dx})^2 - y = 0$

11. The graph of a particular solution (integral) is called ... of the differential equation.
    A. Locus
    B. Differential curve
    C. Integral curve
    D. All of these

12. A solution of the differential equation $(\frac{dy}{dx})^2 - x\frac{dy}{dx} + y = 0$ is:
    A. $y = 2$
    B. $y = 2x$
    C. $y = 2x - 4$
    D. $y = 2x + 4$

13. Solution of the differential equation $\frac{dy}{dx} = 2x$ subject to the condition $y(1) = 4$ is:
    A. $y = x^2$
    B. $y = x + 3$
    C. $y = x^2 + 3$
    D. $y = 2x^3$

14. If $y = A\sin x + B\cos x$, then what is $y(0)$
    A. 0
    B. 1
    C. A
    D. B

15. The differential equation $x^2dy + y^2dx = 0$ has order
    A. 1
    B. 2
    C. 3
    D. 4

16. A differential equation $F(x)G(y)dx + f(x)g(y)dy = 0$ is called ... if it can be written as $\frac{F(x)}{f(x)}dy + \frac{g(y)}{G(y)}dx = 0.$
    A. Separable
    B. Exact
    C. Homogeneous
    D. Linear

17. $\dfrac{dy}{dx} = \dfrac{x^2}{y}$ has solution.

A. $y^2 + x^2 = c$

B. $3y = 2x^2 + c$

C. $3y^2 = 2x^3 + c$

D. $3y^2 = x + c$

18. $\dfrac{dy}{dx} = \frac{1}{x \tan y}$ has solution

A. $x \cos y = e^c$

B. $x \sin y = e^c$

C. $x \sin y = c$

D. None of these

19. $\dfrac{dy}{dx} = -y^2 \sin x$ has solution

A. $y + \cos x = c$

B. $y + \sin x = c$

C. $\frac{1}{y} + \cos x = c$

D. $\frac{1}{y} + \sin x = c$

20. The differential equation $(1 + x)dy - ydx = 0$ has the general solution

A. $y = c(1 - x)$

B. $y = c + x$

C. $y = cx$

D. $y = c + cx$

21. The differential equation with solution $y = A\sin x + B\cos x$ is:

    A. $\frac{dy}{dx} + y = 0$

    B. $\frac{dy}{dx} - y = 0$

    C. $\frac{d^2y}{dx^2} + y = 0$

    D. $\frac{d^2y}{dx^2} - y = 0$

22. The differential equation of all parabolas whose axis is parallel to the y-axis is:

    A. $\frac{dy}{dx} = x^2 + b$

    B. $\frac{d^2y}{dx^2} = 2x$

    C. $\frac{d^3y}{dx^3} = 0$

    D. All of these

23. A function $f : \mathbb{R}^2 \to \mathbb{R}$ is called ... of degree n if

$$f(tx, ty) = t^n f(x, y)$$

    where t is a nonzero real number.

    A. Linear

    B. Exact

    C. Homogeneous

    D. Separable

24. Which of the following is homogeneous?

    A. $\sqrt{xy}$

    B. $\cos(\frac{y}{x})$

    C. $ln(e^{xy})$

    D. All of these

25. The degree homogeneous function $\dfrac{x^{10} + y^{10}}{x^2 + y^2}$ is

    A. 7

    B. 9

    C. 8

    D. 2

26. A homogeneous equation $\frac{dy}{dx} = g(\frac{y}{x})$ can be transformed into a separable equation by substitution.

    A. $y = vx^2$

    B. $y = vx$

    C. $y = x^2$

    D. $y = x$

27. $\frac{dy}{dx} = g(\frac{y}{x})$ is homogeneous, its separable form is

    A. $[v - g(v)]dx + dv = 0$

    B. $[v - g(v)]dx + xdv = 0$

    C. $[x - g(x)]dx + dv = 0$

    D. $[x - g(x)]dx + vdv = 0$

28. The differential equation $(a_1x + b_1y + c_1)dx + (a_2x + b_2y + c_2) = 0$ is:

    A. Exact

    B. Not exact

    C. Homogeneous

    D. Non linear

29. By substitution ... differential equation $(a_1x + b_1y + c_1)dx + (a_2x + b_2y + c_2) = 0$ will be reduced to homogeneous equation.

    A. $x = X + h$, $y = Y + k$

    B. $x = X - h$, $y = Y - k$

    C. $z = a_1x + b_1y$

    D. None of these

30. The differential equation $(a_1x + b_1y + c_1)dx + (a_2x + b_2y + c_2) = 0$ is not exact, but if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the substitution ... will work to reduce it to homogeneous form?

    A. $x = X + h$, $y = Y + k$

    B. $x = X - h$, $y = Y - k$

    C. $z = a_1x + b_1y$

    D. None of these

31. The differential equation $(a_1x + b_1y + c_1)dx + (a_2x + b_2y + c_2) = 0$ is not exact, but if $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, then the substitution ... will work to reduce it to homogeneous form?

    A. $x = X + h$, $y = Y + k$

    B. $x = X - h$, $y = Y - k$

    C. $z = a_1x + b_1y$

    D. None of these

32. $(2x + y + 1)dx + (4x + 2y - 1)dy = 0$ has solution.

    A. $-x - 2y - ln \mid 2x + y - 1 \mid = c$

    B. $-x - 2y = ln \mid 2x + y - 1 \mid +c$

    C. $x + 2y + ln \mid 2x - y - 1 \mid = c$

    D. $x - 2y = ln \mid 2x + y + 1 \mid +c$

33. $(y^2 + 2xy)dx + x^2dy = 0$ has solution

    A. $\mid yx \mid = c \mid y + x \mid$

    B. $\mid yx^3 \mid = c \mid y + 3x^3 \mid$

    C. $\mid yx^3 \mid = c \mid y + 3x^3 \mid$

    D. $\mid yx^3 \mid = c \mid y + 3x \mid$

34. $(x^2 - 3y^2)dx + 2xydy = 0$ has solution

    A. $\mid sin(\frac{y}{x}) \mid = cx$

    B. $\mid sin(\frac{y}{x}) \mid^3 = cx^3$

    C. $\mid sin(\frac{y}{x}) \mid^3 = cx^2$

    D. $\mid sin(\frac{x}{y}) \mid = cx^2$

35. An expression $M(x, y)dx + N(x, y)dy$ is called a (an) ... if there exista a function $f(x, y)$ such that $df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$.

    A. Exact differential

    B. Non exact differential

    C. Homogeneous differential

    D. Linear differential

36. The differential equation $M(x,y)dx + N(x,y)dy = 0$ is an exact differential equation if and only if:
    - A. $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$
    - B. $\frac{\partial M}{\partial x} = -\frac{\partial N}{\partial y}$
    - C. $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$
    - D. $\frac{\partial M}{\partial y} = -\frac{\partial N}{\partial x}$

37. If the differential equation $M(x,y)dx+N(x,y)dy = 0$ is not exact but $\mu(x,y)M(x,y)dx+\mu(x,y)N(x,y)dy = 0$ is exact, then $\mu(x,y)$ is called ... of the differential equation.
    - A. Differential factor
    - B. Integrating multiple
    - C. Integrating factor
    - D. Differential operator

38. $\frac{xdy-ydx}{x^2}=$
    - A. $d(\frac{x}{y})$
    - B. $d(\frac{y}{x})$
    - C. $d(ln\frac{y}{x})$
    - D. $d(ln\frac{x}{y})$

39. $\frac{ydx-xdy}{x^2+y^2}=$
    - A. $d(\frac{x}{y})$
    - B. $d(\frac{y}{x})$
    - C. $d(ln\frac{y}{x})$
    - D. $d(\arctan\frac{y}{x})$

40. Solution of the differential equation $ydx + (x^2y - x)dy = 0$ is:
    - A. $xy^2 - 2y = cx$
    - B. $xy^2 - 2y = c$
    - C. $xy^2 - 2y^2 = cx$
    - D. $x^2y - 2y = cx$

41. The integrating factor of the differential equation $dx + (\frac{x}{y} - \sin y)dy = 0$ is:

    A. x

    B. y

    C. $\frac{1}{x}$

    D. $\frac{1}{y}$

42. A first order differential equation of the form $\frac{dy}{dx} + P(x)y = Q(x)$ is called:

    A. Linear

    B. Quadratic

    C. Exact

    D. Homogeneous

43. Integrating factor of the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ is:

    A. $\exp[\int P(x)dx]$

    B. $\exp[\int Q(x)dx]$

    C. $-\exp[\int P(x)dx]$

    D. $-\exp[\int Q(x)dx]$

44. Integrating factor of the differential equation $(x-1)^3\frac{dy}{dx} + 4(x-1)^2y = x+1$ is:

    A. $(x-1)^2$

    B. $(x-1)^4$

    C. $x+1$

    D. $\frac{1}{x+1}$

45. An equation of the form $\frac{dy}{dx} + P(x)y = Q(x)y^n$ is called:

    A. Bernoulli equation

    B. Ricatti equation

    C. Bessel equation

    D. None of these

46. The two curves are said to be ... of their tangents at the point of intersection are perpendicular to each other.

    A. Perpendicular

    B. Orthogonal

    C. Parallel

    D. Both A and B

47. The orhogonal trajectories of the family of circles $x^2 + y^2 = c^2$ are:
    A. $y = k + x$
    B. $y = kx$
    C. $x = k + y$
    D. None of these

48. The orhogonal trajectories of the family of of curves $y = ce^{-\frac{x}{4}}$ are:
    A. $y^2 = 8x + k$
    B. $y = 8x^2 + k$
    C. $y^2 = 8x^2 + k$
    D. All of these

49. A family of curves whose family of orthogonal trajectories is the same as the given family is called:
    A. Self orthogonal
    B. Orthogonal
    C. Perpendicular
    D. None of these

50. Which of the following is (are) self orthogonal?
    A. $y^2 = 4cx + 4c^2$
    B. $\frac{x^2}{c^2} + \frac{y^2}{c^2-1} = 1$
    C. Both A and B
    D. None of these

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1. An infinite sequence in a non empty set X is a function:
   - A. $f : X \to \mathbb{N}$
   - B. $f : \mathbb{N} \to X$
   - C. Both A and B
   - D. None of these

2. A sequence $\{a_n\}$ is said to ... if given every $\epsilon > 0$, there exists $n_o \in \mathbb{N}$ such that

$$|a_n - l| < \epsilon, \quad \forall n \geq n_o.$$

   - A. Converge
   - B. Diverge
   - C. Converge absolutely
   - D. Converge conditionally

3. The sequence $\{\frac{1}{n}\}$:
   - A. Converges to 0
   - B. Diverges to 0
   - C. Converges to 1
   - D. Diverges to 1

4. For $\epsilon = 0.01$, we have smallest $n_o =$... such that $\{\frac{1}{n}\}$ converges to 0.
   - A. 100
   - B. 101
   - C. 1000
   - D. 1001

5. A sequence $\{a_n\}$ of real numbers is said to be ... if there is a positive number M such that $|a_n| \geq M$, $\forall n$.
   - A. Convergent
   - B. Divergent
   - C. Bounded
   - D. Unbounded

6. A convergent sequence is:
   A. Bounded
   B. Not necessarily bounded
   C. Unbounded
   D. Can be unbounded

7. A bounded sequence is:
   A. Convergent
   B. Not necessarily convergent
   C. Divergent
   D. Always divergent

8. An unbounded sequence:
   A. Is convergent
   B. Is divergent
   C. May converge
   D. None of these

9. A divergent sequence:
   A. Is bounded
   B. Is unbounded
   C. Is not necessarily bounded
   D. Is not necessarily unbounded

10. If $\lim_{n\to\infty} a_n \neq 0$, then the sequence $\{a_n\}$:
   A. Converges
   B. Diverges
   C. Diverges absolutely
   D. None of these

11. A sequence $\{a_n\}$ is said to be non-decreasing if, $\forall n$,
   A. $a_n \leq a_{n+1}$
   B. $a_n \geq a_{n+1}$
   C. $a_n < a_{n+1}$
   D. $a_n > a_{n+1}$

12. A bounded and monotonic sequence:
    - A. Converges
    - B. Diverges
    - C. May converge
    - D. May diverge

13. $\lim_{n\to\infty} \frac{\ln n}{n} =$
    - A. 0
    - B. 1
    - C. -1
    - D. ∞

14. $\lim_{n\to\infty} n^{\frac{1}{n}} =$
    - A. 0
    - B. 1
    - C. -1
    - D. ∞

15. For $x > 0$, $\lim_{n\to\infty} x^{\frac{1}{n}} =$
    - A. 0
    - B. 1
    - C. -1
    - D. ∞

16. For $|x| < 1$, $\lim_{n\to\infty} x_n =$
    - A. 0
    - B. 1
    - C. -1
    - D. ∞

17. For all real x, $\lim_{n\to\infty} \frac{x^n}{n!} =$
    - A. 0
    - B. 1
    - C. -1
    - D. ∞

18. The sequence $\{\tan^{-1} n\}$:
    - A. Converges to $\frac{\pi}{2}$
    - B. Diverges
    - C. Is unbounded
    - D. Is bounded but diverges

19. The sum of the series

$$\sum_{n=1}^{\infty} a_n = \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + ...$$

is:
    - A. $\frac{1}{3}$
    - B. $\frac{2}{3}$
    - C. $\frac{1}{9}$
    - D. ∞

20. The geometric series $\sum_1^{\infty} ar^n$ is convergent if:
    - A. $r < 1$
    - B. $0 < r < 1$
    - C. $|r| < 1$
    - D. $|r| \leq 1$

21. The series $\sum_1^{\infty} \frac{1}{n^2}$ is named as:
    - A. Geometric series
    - B. Harmonic series
    - C. Euler series
    - D. None of these

22. $\sum_1^{\infty} \frac{1}{n^2}$ is:
    - A. Bounded
    - B. Convergent
    - C. Monotonically increasing
    - D. All of these

23. The harmonic series $\sum_1^\infty \frac{1}{n}$ is:
    - A. Convergent
    - B. Divergent
    - C. Absolutely convergent
    - D. Conditionally convergent

24. If $\sum_1^\infty a_n$ converges, then
    - A. $\lim_{n\to\infty} a_n = 0$
    - B. $\{a_n\}$ converges
    - C. $\{a_n\}$ diverges
    - D. Both A and B

25. If $\sum_1^\infty a_n$ diverges, then
    - A. $\lim_{n\to\infty} a_n = 0$
    - B. $\{a_n\}$ converges
    - C. $\{a_n\}$ diverges
    - D. Both A and B

26. If $\lim_{n\to\infty} a_n = 0$, then $\sum_1^\infty a_n$
    - A. Converges
    - B. Diverges
    - C. Not necessarily converges
    - D. None of these

27. If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_1^\infty a_n$
    - A. Converges
    - B. Diverges
    - C. Not necessarily converges
    - D. None of these

28. Sum of two convergent series:
    - A. Converges
    - B. Diverges
    - C. Not necessarily converges
    - D. None of these

29. If $\sum_1^\infty a_n$ converges and $\sum_1^\infty b_n$ diverges, then $\sum_1^\infty (a_n + b_n)$
    - A. Converges
    - B. Diverges
    - C. Not necessarily converges
    - D. Not necessarily diverges

30. $\sum_1^\infty (\frac{1}{n^2} + \frac{1}{n})$ is
    - A. Convergent
    - B. Divergent
    - C. Harmonic series
    - D. None of these

31. If $\sum_1^\infty a_n$ and $\sum_1^\infty b_n$ diverges, then $\sum_1^\infty (a_n + b_n)$
    - A. Converges
    - B. Diverges
    - C. May converge
    - D. None of these

32. The sum of the series $\sum_1^\infty \frac{1}{n(n+1)}$ is:
    - A. 0
    - B. 1
    - C. ∞
    - D. The series diverges

33. Let $\sum_1^\infty a_n$, $\sum_1^\infty b_n$ be series of positive terms with $a_n \leq b_n$, $\forall n > n_o$ for some integer n_o. If $\sum_1^\infty b_n$ converges, then $\sum a_n$:
    - A. Converges
    - B. Diverges
    - C. Can converge
    - D. Can diverge

34. Let $\sum_1^\infty a_n$, $\sum_1^\infty b_n$ be series of positive terms with $a_n \leq b_n$, $\forall n > n_o$ for some integer n_o. If $\sum_1^\infty a_n$ diverges, then $\sum b_n$:
    - A. Converges
    - B. Diverges
    - C. Can converge
    - D. Can diverge

35. Let $\sum_1^\infty a_n$, $\sum_1^\infty b_n$ be series of positive terms with $a_n \leq b_n$, $\forall n > n_o$ for some integer n_o. If $\sum_1^\infty b_n$ diverges, then $\sum a_n$:

   A. Converges

   B. Diverges

   C. Always diverge

   D. None of these

36. Let $\sum_1^\infty a_n$, $\sum_1^\infty b_n$ be series of positive terms with $a_n \leq b_n$, $\forall n > n_o$ for some integer n_o. If $\sum_1^\infty a_n$ converges, then $\sum b_n$:

   A. Converges

   B. Diverges

   C. Always converge

   D. None of these

37. The series $\sum_1^\infty \frac{1}{1+n^2}$:

   A. Converges

   B. Diverges

   C. Is unbounded

   D. None of these

38. The series $\sum_1^\infty \frac{2}{1+n^2}$:

   A. Converges

   B. Diverges

   C. Is bounded

   D. Both A and C

39. The series $\sum_1^\infty \frac{n+16}{n^2+3}$:

   A. Converges

   B. Diverges

   C. Is bounded

   D. Both A and C

40. Let $\sum_1^\infty a_n$ and $\sum_1^\infty b_n$ be series of positive terms. If $\lim_{n\to\infty} \frac{b_n}{a_n}$ is a nonzero finite number and $\sum_1^\infty b_n$ diverges, then $\sum_1^\infty a_n$

 A. Converges

 B. Diverges

 C. More information is needed

 D. None of these

41. The series $\sum_1^\infty \frac{2n+1}{3n^2+2}$

 A. Converges

 B. Diverges

 C. Converges but unbounded

 D. None of these

42. Let $\sum_1^\infty a_n$ be a positive term series. If f is continuous and non increasing function on $[1,\infty)$ such that $f(n) = a_n$ for all positive integers n, then $\sum_1^\infty a_n$ and $\int_1^\infty f(x)dx$ behave similarly. This is called:

 A. Limit Comparison Test

 B. Root Test

 C. Ratio Test

 D. Integral Test

43. The p-series (or Hyperharmonic series) $\sum_1^\infty \frac{1}{n^p}$ converges if:

 A. $p > 1$

 B. $p \geq 1$

 C. $p < 1$

 D. $p \leq 1$

44. The series $\sum_1^\infty k^{-\frac{1}{6}}$:

 A. Converges

 B. Diverges

 C. Diverges and bounded

 D. None of these

45. Let $\sum_1^\infty a_n$ be a series of positive terms and suppose that $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L$. If $0 \le L < 1$, then the series $\sum_1^\infty a_n$

A. Converges

B. Diverges

C. Test fails

D. None of these

46. Let $\sum_1^\infty$ be a series of positive terms and suppose that $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L$. If $L > 1$, then the series $\sum_1^\infty a_n$

A. Converges

B. Diverges

C. Test fails

D. None of these

47. Let $\sum_1^\infty$ be a series of positive terms and suppose that $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L$. If $L = 1$, then the series $\sum_1^\infty a_n$

A. Converges

B. Diverges

C. Test fails

D. None of these

48. The series $\sum_1^\infty \frac{n!}{n^2}$

A. Converges

B. Diverges

C. The behavior of this series can not be determined by Ratio Test

D. None of these

49. The series $\sum_1^\infty \frac{n^2}{n!}$

A. Converges

B. Diverges

C. The behavior of this series can not be determined by Ratio Test

D. None of these

50. Let $\sum_1^\infty$ be a series of positive terms and suppose that $\lim_{n\to\infty}(a_n)^{\frac{1}{n}} = L$. If $0 \leq L < 1$, then the series $\sum_1^\infty a_n$

 A. Converges

 B. Diverges

 C. Test fails

 D. None of these

51. Let $\sum_1^\infty$ be a series of positive terms and suppose that $\lim_{n\to\infty}(a_n)^{\frac{1}{n}} = L$. If $L > 1$, then the series $\sum_1^\infty a_n$

 A. Converges

 B. Diverges

 C. Test fails

 D. None of these

52. Let $\sum_1^\infty$ be a series of positive terms and suppose that $\lim_{n\to\infty}(a_n)^{\frac{1}{n}} = L$. If $L = 1$, then the series $\sum_1^\infty a_n$

 A. Converges

 B. Diverges

 C. Test fails

 D. None of these

53. Which of the following series diverges?

 A. $\sum_1^\infty (\frac{n}{2n+1})^n$

 B. $\sum_1^\infty (n)^n$

 C. $\sum_1^\infty \frac{n^3}{2^n}$

 D. $\sum_1^\infty \frac{1}{n^n}$

54. An infinite series having both positive and negative terms is called:

 A. Alternating series

 B. Convergent series

 C. Mixed series

 D. Divergent series

55. If $a_n > 0$ for all n, then the series $\sum_1^\infty (-1)^n a_n$ is called:
    A. Alternating series
    B. Harmonic series
    C. Telescopic sum
    D. Euler's series

56. The alternating series $\sum_1^\infty a_n$ converges if:
    A. $\{a_n\}$ is a non increasing sequence
    B. $\lim_{n\to\infty} a_n = 0$
    C. Both A and B must be satisfied
    D. None of these

57. The alternating series $\sum_1^\infty (-1)^{n+1} \frac{1}{n}$ is:
    A. Convergent
    B. Divergent
    C. Oscillating
    D. None of these

58. The alternating series $\sum_1^\infty (-1)^n \frac{n+3}{2n-3}$ is:
    A. Convergent
    B. Divergent
    C. Oscillating
    D. None of these

59. A series $\sum_1^\infty a_n$ is said to converge ... if the series $\sum_1^\infty |a_n|$ converges.
    A. Absolutely
    B. Conditionally
    C. Only
    D. None of these

60. If $\sum_1^\infty |a_n|$ converges, then
    A. $\sum_1^\infty a_n$ also converge
    B. $\sum_1^\infty a_n$ not necessarily converge
    C. $\sum_1^\infty a_n$ diverge
    D. $\sum_1^\infty a_n$ can diverge

61. If $\sum_1^\infty a_n$ converges, then $\sum_1^\infty |a_n|$:

A. Converge

B. Diverge

C. Not necessarily converge

D. None of these

62. A series $\sum_1^\infty a_n$ is said to converge conditionally if:

A. $\sum_1^\infty a_n$ and $\sum_1^\infty |a_n|$ both converge

B. $\sum_1^\infty a_n$ and $\sum_1^\infty |a_n|$ both diverge

C. $\sum_1^\infty a_n$ converges but $\sum_1^\infty |a_n|$ diverges

D. $\sum_1^\infty |a_n|$ converges but $\sum_1^\infty a_n$ diverges

63. If a series converges absolutely then series itself

A. Converges

B. Diverges

C. Can converge

D. Can diverge

64. An infinite series of the form $\sum_{n=0}^\infty c_n(x-a)^n$ is called a ... series in $x-a$, where a is a constant, called center of this series.

A. Taylor

B. Power

C. Alternating

D. Maclaurin

65. The values of x for which the power series $\sum_1^\infty \frac{x^n}{(2n)!}$ converges are

A. $(-1, 1)$

B. $\mathbb{R}$

C. $(-\frac{3}{2}, 1)$

D. None of these

66. If a power series $\sum_1^\infty c_n x^n$ converges for $x = x_1$, then it converges absolutely for all x such that

A. $|x| < |x_1|$

B. $|x| \leq |x_1|$

C. $|x| > |x_1|$

D. $|x| \geq |x_1|$

67. The set I of all numbers x for which a power series $\sum_1^\infty c_n(x-a)^n$ converges, is called:

A. Radius of convergence

B. Interval of convergence

C. Diameter of convergence

D. All of these

68. If $(a-R, a+R)$ is the interval of convergence of a power series $\sum_1^\infty c_n(x-a)^n$, then R is called

A. Diameter of convergence

B. Radius of convergence

C. Length of convergence

D. None of these

Radius of Convergence of a Power Series
Let $\sum_1^\infty c_n(x-a)^n$ be a power series with radius of convergence R. Suppose that $\lim_{n\to\infty} |\frac{c_{n+1}}{c_n}| = L$, where L is either a nonzero number or $+\infty$
(i). If L is a positive real number, then $R = \frac{1}{L}$.
(ii). If $L = 0$, then $R = +\infty$.
(iii). If $L = +\infty$, then $R = 0$.

69. The radius of convergence of power series $\sum_1^\infty \frac{x^n}{n}$ is:

A. 0

B. 1

C. 5

D. ∞

70. The radius of convergence of power series $\sum_1^\infty \frac{(x+3)^n}{3^n}$ is:

A. 0

B. 1

C. 2

D. 3

71. The radius of convergence of power series $\sum_1^\infty (-1)^n \frac{(x-17)^n}{n!}$ is:

A. 0

B. 1

C. 4

D. ∞

72. The radius of convergence of power series $\sum_1^\infty (nx)^n$ is:

A. 0

B. 1

C. 2

D. ∞

Differentiation and Integration of a Power Series
1. Suppose that $f(x) = \sum_0^\infty c_n(x-a)^n$,
where the power series has radius of convergence R. Then
(i). For $|x-a| < R$, $f'(x) = \sum_1^\infty nc_n(x-a)^{n-1}$,
(ii). For $|x-a| < R$, $\int f(x)dx = \sum_0^\infty \frac{c_n}{n+1}(x-a)^{k+1} + C$.
2. The three power series $\sum_0^\infty c_n(x-a)^n$, $\sum_1^\infty nc_n(x-a)^{n-1}$, $\sum_0^\infty \frac{c_n}{n+1}(x-a)^{k+1} + C$ all have the same radius of convergence.

73. The power series representation of $\frac{1}{1-x}$ is:

A. $1 + x - x^2 + ...$

B. $1 - x + x^2 - ...$

C. $1 + x + x^2 + ...$

D. $1 + x + \frac{x^2}{2!} + ...$

74. The power series representation of e^x is:

A. $1 + x - x^2 + ...$

B. $1 - x + x^2 - ...$

C. $1 + x + x^2 + ...$

D. $1 + x + \frac{x^2}{2!} + ...$

75. For $|x| \le 1$, the series $\tan^{-1} x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...$ is called:

A. Maclaurin Series

B. Taylor Series

C. Gregory's Series

D. All of these

76. $\int_0^1 e^{x^2} dx$ equals:

A. $e - 1$

B. $1 + \frac{1}{3} + \frac{1}{5.2!} + \frac{1}{7.3!} + ...$

C. $1 - \frac{1}{3} + \frac{1}{5.2!} - \frac{1}{7.3!} + ...$

D. Can not be evaluated

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Summary of Convergence and Divergence Tests for Series

Test	Series	Convergence or Divergence	Comments
Divergence	$\sum_{n=1}^{\infty} a_n$	Diverges if $\lim_{n\to\infty} a_n \neq 0$	Inconclusive if $\lim_{n\to\infty} a_n = 0$
Geometric series	$\sum_{n=0}^{\infty} ar^n$	(1) Converges to $S = \frac{a}{1-r}$ if $\|r\| < 1$; (2) Diverges if $\|r\| \geqslant 1$	Useful for comparison tests if the n^{th}-term a_n of a series is similar to ar^n.
p-series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	(1) Converges if $p > 1$; (2) Diverges if $p \leqslant 1$	Useful for comparison tests if the n^{th}-term a_n of a series is similar to $\frac{1}{n^p}$.
Integral	$\sum_{n=1}^{\infty} a_n$ $a_n = f(n)$	(1) Converges if $\int_1^{\infty} f(x)\,dx$ converges; (2) Diverges if $\int_1^{\infty} f(x)\,dx$ diverges	The function f obtained from $a_n = f(n)$ must be continuous, positive, decreasing, and readily integrable.
Comparison	$\sum_{n=1}^{\infty} a_n$, $\sum_{n=1}^{\infty} b_n$ $a_n > 0$, $b_n > 0$	(1) If $\sum_{n=1}^{\infty} b_n$ converges and $a_n \leqslant b_n$ for every n, then $\sum_{n=1}^{\infty} a_n$ converges; (2) If $\sum_{n=1}^{\infty} b_n$ diverges and $a_n \geqslant b_n$ for every n, then $\sum_{n=1}^{\infty} a_n$ diverges; (3) If $\lim_{n\to\infty} \frac{a_n}{b_n} > 0$ (not ∞), then both series converge or both diverge.	The comparison series $\sum_{n=1}^{\infty} b_n$ is often a geometric series or a p-series. To find b_n in (3), consider only the terms of a_n that have the greatest effect on the magnitude.
Ratio	$\sum_{n=1}^{\infty} a_n$	If $\lim_{n\to\infty} \left\| \frac{a_{n+1}}{a_n} \right\| = L$ (or ∞), the series (1) converges (absolutely) if $L < 1$; (2) diverges if $L > 1$ (or ∞)	Inconclusive if $L = 1$. Useful if a_n involves factorials or n^{th} powers. If $a_n > 0$ for every n, the absolute value sign may be disregarded.
Root	$\sum_{n=1}^{\infty} a_n$	If $\lim_{n\to\infty} \sqrt[n]{\|a_n\|} = L$ (or ∞), the series (1) converges (absolutely) if $L < 1$; (2) diverges if $L > 1$ (or ∞)	Inconclusive if $L = 1$. Useful if a_n involves n^{th} powers. If $a_n > 0$ for every n, the absolute value sign may be disregarded.
Alternating Series	$\sum_{n=1}^{\infty} (-1)^n a_n$ $a_n > 0$	Converges if $a_k \geqslant a_{k+1}$ for every k and $\lim_{n\to\infty} a_n = 0$.	Applicable only to an alternating series.