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Support α , β and Break of Line.**



Final Year Project Report
Presented by

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In Partial Fulfillment
of the Requirement for the Degree of
Master in Mathematics

DEPARTMENT OF MATHEMATICS
COMSATS INSTITUTE OF INFORMATION TECHNOLOGY
Attock Campus
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**COMSATS INSTITUTE OF INFORMATION TECHNOLOGY
ATTOCK CAMPUS**

FINAL APPROVAL

This project titled
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of Line.**

submitted to the Department of Mathematics
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DECLARATION OF THE STUDENT

I, **Umer Din Khan**, Registration Number **CIIT/FA14-RMT-001/ATK**, hereby solemnly declare that I have produced the work presented in this project, during the scheduled period of study.

Date: **June 14, 2016**

.....
Signature of the Student

dedicated to my beloved parents

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Abstract

A function is convex if the line segment joining two points on the graph lies above the graph. These functions have important properties and applications in mathematics. Specially, they are very important in optimization and minimization problems. Also these functions are used in statistic and functional analysis. A positive function f is logarithmic convex if $\log f$ is convex. It would seem that log convex functions unremarkable because they are so simply related to convex functions. But they have some surprising properties.

In the first chapter we generalize results for logarithmic convexity of Giaccardi's difference for classes of functions with the help of divided difference.

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Student Name

Notations

The notation and concepts used in this monograph are more or less specified. The reader is assumed to be familiar with the elements of Mathematical Analysis, as well as General Algebra, Matrix Theory and Topology, and since the standard notation and concepts were used, it was believed unnecessary to define all of them.

We give some of the Notation used in the Monograph.

\mathbb{Z}	the set of integer
\mathbb{N}	the set of positive integer
\mathbb{Q}	the set of rational numbers
\mathbb{R}	the set of real numbers

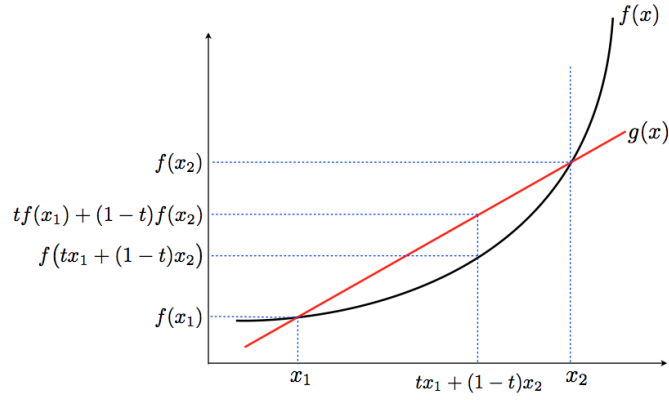


Figure 1.1: Graph of convex function

1.2 Convex functions

The fundamental work of Jensen in the years 1905, 1906 is the starting point of the systematic study of convex functions. Even before Jensen, the literature shows results which refer to convex functions. In fact the roots of such functions can be found in the work of Hölder in 1889 and J. Hadamard in 1893, although these roots were not explicitly specified in their works. The general theory of convex functions is the origin of powerful tools for the study of problems in analysis. Inequalities involving convex functions are the most efficient tools in the development of several branches of mathematics and has been given considerable attention in the literature.

Chapter 2

Inequalities of Hadamard's Type for Lipschitzian Mappings.

In this chapter, we give some inequalities of Hadamard's type for M -Lipschitzian function. Some application which are connected with \log functions, exponential functions etc., for two positive numbers are also given.

Definition 2.0.1. [2] A function $f : I \rightarrow \mathbb{R}$ defined on a closed interval $I = [a, b]$ is said to satisfy a Lipschitz condition if for any constant M and for points $x, y \in [a, b]$

$$|f(x) - f(y)| \leq M |x - y|.$$

2.1 Hadamard's type inequalities

We will start with the following theorem containing two inequalities of Hadamard's type for M -Lipschitzian mapping. We need the following Lemma to prove the theorem.

Lemma 2.1.1.

$$\int_0^1 |2t - 1| dt = \frac{1}{2}. \quad (2.1.1)$$

Proof. As

$$|2t - 1| = \begin{cases} +(2t - 1) & \text{if } t \geq \frac{1}{2} \\ -(2t - 1) & \text{if } t < \frac{1}{2} \end{cases}$$

so

$$\int_0^1 |2t - 1| dt = \int_0^{\frac{1}{2}} -(2t - 1) dt + \int_{\frac{1}{2}}^1 (2t - 1) dt.$$

Now as

$$\int_0^{\frac{1}{2}} (-(2t-1))dt = \frac{1}{4} \text{ and } \int_{\frac{1}{2}}^1 (2t-1)dt = \frac{1}{4}.$$

Therefore

$$\int_0^1 |2t-1| dt = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}. \quad \square$$

Theorem 2.1.2. *Let $f : I \subseteq \mathbb{R} \rightarrow \mathbb{R}$ be an M -Lipschitzian mapping on I and $a, b \in I$ with $a < b$. Then we have the inequalities*

$$\left| f\left(\frac{a+b}{2}\right) - \frac{1}{b-a} \int_a^b f(x)dx \right| \leq \frac{M}{4}(b-a) \quad (2.1.2)$$

and

$$\left| \frac{f(a)+f(b)}{2} - \frac{1}{b-a} \int_a^b f(x)dx \right| \leq \frac{M}{3}(b-a). \quad (2.1.3)$$

Proof. Let $t \in [0, 1]$. Then we have, for all $a, b \in I$,

$$\begin{aligned} & |tf(a) + (1-t)f(b) - f(ta + (1-t)b)| \\ &= |t(f(a) - f(ta + (1-t)b)) + (1-t)(f(b) - f(ta + (1-t)b))| \\ &\leq t|f(a) - f(ta + (1-t)b)| + (1-t)|f(b) - f(ta + (1-t)b)| \\ &\leq tM|a - (ta + (1-t)b)| + (1-t)M|b - (ta + (1-t)b)| \\ &= 2t(1-t)M|b-a|. \end{aligned} \quad (2.1.4)$$

If we choose

$$t = \frac{1}{2},$$

we have

$$\left| \frac{f(a)+f(b)}{2} - f\left(\frac{a+b}{2}\right) \right| \leq \frac{M}{2}|b-a|. \quad (2.1.5)$$

If we put $ta + (1-t)b$ instead of a and $(1-t)a + tb$ instead of b in (2.1.5) respectively, then we have

$$\left| \frac{f(ta + (1-t)b) + f((1-t)a + tb)}{2} - f\left(\frac{a+b}{2}\right) \right| \leq \frac{M|2t-1|}{2}|b-a|. \quad (2.1.6)$$

for all $t \in [0, 1]$.

If we integrate the inequality (2.1.6) on $[0, 1]$, we have

$$\left| \frac{1}{2} \left[\int_0^1 f(ta + (1-t)b)dt + \int_0^1 f((1-t)a + tb)dt \right] - f\left(\frac{a+b}{2}\right) \right| \leq \frac{M|b-a|}{2} \int_0^1 |2t-1| dt. \quad (2.1.7)$$

Also we have

$$\int_0^1 f(ta + (1-t)b)dt = \int_0^1 f((1-t)a + tb)dt = \frac{1}{b-a} \int_a^b f(x)dx. \quad (2.1.8)$$

Now using equation (2.1.8) and from Lemma 2.1.1 in equation (2.1.7), we have

$$\left| \frac{1}{b-a} \int_a^b f(x)dx - f\left(\frac{a+b}{2}\right) \right| \leq \frac{M|b-a|}{4} \quad (2.1.9)$$

$$\left| f\left(\frac{a+b}{2}\right) - \frac{1}{b-a} \int_a^b f(x)dx \right| \leq \frac{M|b-a|}{4}, \quad (2.1.10)$$

which is required inequality.

From equation (2.1.4), we have

$$|tf(a) + (1-t)f(b) - f(ta + (1-t)b)| \leq 2t(1-t)M(b-a)$$

for all $t \in [0, 1]$ and $a, b \in I$ with $a < b$. Integrating on $[0, 1]$, we have

$$\left| f(a) \int_0^1 tdt + f(b) \int_0^1 (1-t)dt - \int_0^1 f(ta + (1-t)b)dt \right| \leq 2M(b-a) \int_0^1 t(1-t)dt$$

Hence, from

$$\int_0^1 tdt = \int_0^1 (1-t)dt = \frac{1}{2}$$

and

$$\int_0^1 f(ta + (1-t)b)dt = \frac{1}{b-a} \int_a^b f(x)dx,$$

we have

$$\left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(x)dx \right| \leq \frac{M(b-a)}{3},$$

□

which is our required equation.

Bibliography

- [1] S. S. Dragomir, Y. J. Cho and S.S. Kim, Inequalities of Hadamard's Type for Lipschitzian Mapping and Their Applications, J. Math. Anal. Appl. **245** (2000), 489-501.
- [2] S. S. Dragomir, Two refinements of Hadamard's inequalities, Coll. of Sci. Pap. of the Fac. of Sci., Kragujevac (Yugoslavia) **11** (1990), 23-26.
- [3] H. Jeffreys and B. Jeffreys, *Methods of Mathematical Physics*, 3rd edition, Cambridge University Press, 1956.
- [4] S. Abramovich, G. Farid, and J. E. Pečarić, *More About Hermite-Hadamard inequalities, Cauchy's means, and superquadracity*, J. Inequal. App., **2010**, Article ID 102467, 14 pages.
- [5] M. Alomari and M. Darus, *Co-ordinated s -convex function in the first sense with some Hadamard-type inequalities*, Int. J. Contemp. Math. Sci. **3**(32) (2008), 1557-1567.
- [6] M. Alomari and M. Darus, *On the Hadamards inequality for log-convex functions on the coordinates*, J. Inequal. Appl. **2009**, Article ID 283147, 13 pages.
- [7] G. H. Toader, *Means and convexity, On generalization of convexity*, Mathematica **30** (53) (1988), 83-87.
- [8] G. H. Toader, *Some generalizations of convexity*, Proc. Colloq. Approx. Optim, Cluj Napoca (Romania), (1984), 329-338.