SHORT QUESTIONS

Q.1- Define a set and write some well-known sets of numbers.

Ans:

Set: A collection of well defined distinct objects is called a "Set". For example a collection of students of 9th class, members of a cricket team etc.

Sets of Numbers:
- Set of Natural Numbers = \( N = \{1, 2, 3, \ldots\} \)
- Set of Whole Numbers = \( W = \{0, 1, 2, 3, \ldots\} \)
  - Set of Integers = \( Z = \{-3, -2, -1, 0, 1, 2, 3, \ldots\} \)
- Set of Even Numbers = \( E = \{-4, -2, 0, 2, 4, \ldots\} \)
- Set of Odd Numbers = \( O = \{-3, -1, 1, 3, 5, \ldots\} \)
- Set of Prime Numbers = \( P = \{2, 3, 5, 7, 11, 13, 17, \ldots\} \)

Q.2- If \( A = \{2, 3, 5, 7, 11\} \)

\( B = \{1, 3, 5, 7, 9\} \)

Find \( A \cup B \) and \( A \cap B \)

Solution:

\[ A \cup B = \{2, 3, 5, 7, 11\} \cup \{1, 3, 5, 7, 9\} = \{1, 2, 3, 5, 7, 9, 11\} \]

\[ A \cap B = \{2, 3, 5, 7, 11\} \cap \{1, 3, 5, 7, 9\} = \{3, 5, 7\} \]
Q.3-  If \( A = \{2,3,4,5\}, B = \{2,4,6,8\} \). Then find \( A - B \) and \( B - A \).

Solution:-
\[
A - B = \{2,3,4,5\} - \{2,4,6,8\} = \{3,5\} \\
B - A = \{2,4,6,8\} - \{2,3,4,5\} = \{6,8\}
\]

Q.4-  If \( \mathbb{U} = \{1,2,3,4,5,6,7\}, A = \{3,4,5\}, B = \{1,3,5,7\} \) Find \( (A \cup B)' \) and \( (A \cap B)' \).

Solution:-
\[
A \cup B = \{3,4,5\} \cup \{1,3,5,7\} = \{1,3,4,5,7\} \\
(A \cup B)' = \mathbb{U} - (A \cup B) = \{1,2,3,4,5,6,7\} - \{1,3,4,5,7\} = \{2,6\} \\
A \cap B = \{3,4,5\} \cap \{1,3,5,7\} = \{3,5\} \\
(A \cap B)' = \mathbb{U} - (A \cap B) = \{1,2,3,4,5,6,7\} - \{3,5\} = \{1,2,4,6,7\}
\]

Q.5-  Show two sets \( A \) and \( B \) by Venn Diagram When.

(i)  They are disjoint

(ii)  They are overlapping

Solution:-

(i)  The figure shows that \( \hat{A} \) and \( B \) are disjoint.

\[\text{Diagram:} \quad \mathbb{U} \quad \hat{A} \quad B\]
(ii) The figure given below shows that $A$ and $B$ are overlapping.

Q.6- **State De-Morgan's Laws.**

Ans. These laws state that

(i) $$(A \cup B)^c = A^c \cap B^c$$

(ii) $$(A \cap B)^c = A^c \cup B^c$$

Q.7- If $A = \{3,5,6\}$, $B = \{1,3\}$ then find $A \times B$ and $B \times A$.

Ans. $A \times B = \{3,5,6\} \times \{1,3\} = \{(3,1),(3,3),(5,1),(5,3),(6,1),(6,3)\}$

$B \times A = \{1,3\} \times \{3,5,6\} = \{(1,3),(1,5),(1,6),(3,3),(3,5),(3,6)\}$

Q.8- **Define a binary relation from a set A to set B.**

Ans. If $A$ and $B$ are two non empty sets then any subset of $A \times B$ is called a binary relation from $A$ to $B$.

Q.9- If $A = \{1,2,3\}$, $B = \{3,4\}$. Find any two binary relations from $A$ to $B$.

Ans. $A \times B = \{(1,3),(1,4),(2,3),(2,4),(3,3),(3,4)\}$

$R_1 = \{(1,3),(2,4),(3,3)\}$

$R_2 = \{(1,4),(3,4)\}$

Q.10- **Define Domain and Range of a binary relation.**

Ans. It $R$ is a binary relation. Then Domain of $R$ is the set of all first elements of ordered pairs in $R$. The set of all second elements of ordered pairs in $R$ is called Range of $R$.

**Example:**

$R = \{(1,3),(2,4),(3,5),(4,6)\}$
Dom  \( R = \{1, 2, 3, 4, \} \)

Rng  \( R = \{3, 4, 5, 6, \} \)

**Q.11-** Define a function from a set \( A \) to the set \( B. \)

**Ans.** Let \( A \) and \( B \) are two non empty sets and \( f \) is a binary relation from \( A \) to \( B \) such that

(i)  Domain.  \( f = A \)

(ii) There is no repetition in the first elements of ordered pairs in \( f \). Then \( f \) is said to be a function from \( A \) to \( B \). It is expressed as \( f: A \rightarrow B \)

**Q.12-** Let \( A = \{l, m, n\}, B = \{3, 5, 7\} \)

Show that \( f = \{(l, 3), (m, 3), (n, 3)\} \) is a function from \( A \) to \( B. \)

**Solution:-**

(i) Domain  \( f = \{l, m, n\} = A \)

First condition is satisfied.

(ii) All the three ordered pairs in \( f \) have different first elements and there is no repetition of first elements.

So 2nd condition is also satisfied.

Thus \( f \) is a function from \( A \) to \( B \)

**Q.13-** Define an into function?

**Solution:-**

Let \( f \) be a function from \( A \) to \( B \) then \( f \) is called a function from \( A \) into \( B \) if

Range of  \( f \neq B \)

**Example:**

If \( A = \{a, b, c\}, B = \{x, y\} \)

Then \( f = \{(a, x), (b, x), (c, x)\} \) is an into function

(from \( A \) into \( B \))

**Q.14-** Define an Onto function.

**Ans.** Let \( f \) be a function from \( A \) to \( B \) such that

Range : \( f = B. \)

Then \( f \) is called a function from \( A \) onto \( B. \)
Example:
Let \( A = \{p, q, r\}, \ B = \{x, y, z\}\)
Then \( f = \{(p, x), (q, y), (r, z)\}\) is a function from \( A \) onto \( B \)
Because, Range \( f = \{x, y, z\} = B \)

Q.15- Define a one-one function.
Ans. Let \( f : A \rightarrow B \) is a function such that second element of each ordered pairs in \( f \) is also not repeated.

Example:
\( f = \{(a, x), (b, y), (c, z)\}\)
It is a one-one function.

Q.16- Let \( X = \{7, 8, 9\}, \ Y = \{d, e, f\}\)
and \( h = \{(7, e), (8, d), (9, f)\}\)
Show that \( h \) is a one-one function from \( A \) onto \( B \).

Solution:-
(i) Domain \( h = \{7, 8, 9\} = X \)
(ii) No first element is repeated in \( h \). So \( h \) is a function from \( x \) to \( y \).
(iii) Range : \( h = \{d, e, f\} = Y \)
So \( h \) is an onto function.
Now again non of the second elements is repeated.
So this function is one-one function.

**SOLVED EXERCISES**

**EXERCISE 8.1**

Q.1- If \( A = \{1, 4, 7, 8\}, \ B = \{4, 6, 8, 9\}\)
and \( C = \{3, 4, 5, 7\} \) Find:
(i) \( A \cup B \) (ii) \( B \cup C \) (iii) \( A \cap C \) (iv) \( A \cap (B \cap C) \)
(v) \( A \cup (B \cup C) \) (vi) \( A \cap (B \cap C) \)

Solution:-
(i) \( A \cup B = \{1, 4, 7, 8\} \cup \{4, 6, 8, 9\} = \{1, 4, 6, 7, 8, 9\} \) Ans.
(ii) \[ B \cup C = \{4,6,8,9\} \cup \{3,4,5,7\} = \{3,4,5,6,7,8,9\} \text{ Ans.} \]

(iii) \[ A \cap C = \{1,4,7,8\} \cap \{3,4,5,7\} = \{4,7\} \text{ Ans.} \]

(iv) \[ A \cap (B \cap C) = ? \]
\[ (B \cap C) = \{4,6,8,9\} \cap \{3,4,5,7\} = \{4\} \]
\[ \text{Now} \quad A \cap (B \cap C) = \{1,4,7,8\} \cap \{4\} = \{4\} \text{ Ans.} \]

(v) \[ (A \cup B) \cup C = ? \]
\[ (A \cup B) = \{1,4,7,8\} \cup \{4,6,8,9\} = \{1,4,6,7,8,9\} \]
\[ \text{Now} \quad (A \cup B) \cup C = \{1,4,6,7,8,9\} \cup \{3,4,5,7\} \]
\[ = \{1,3,4,5,6,7,8,9\} \text{ Ans.} \]

(vi) \[ (A \cap B) \cap C = ? \]
\[ A \cap B = \{1,4,7,8\} \cap \{4,6,8,9\} = \{4,8\} \]
\[ \text{Now} \quad (A \cap B) \cap C = \{4,8\} \cap \{3,4,5,7\} = \{4\} \text{ Ans.} \]

Q.2- If \( A = \{1,7,11,15,17,21\} \), \( B = \{11,17,19,23\} \) and \( C = \{2,3,5\} \).
Verify that: \( (A \cap B) \cap C = A \cap (B \cap C) \)

Solution:-
\[ A \cap B = \{1,7,11,15,17,21\} \cap \{11,17,19,23\} \]
\[ A \cap B = \{11,17\} \]
\[ \text{Now} \quad (A \cap B) \cap C = \{11,17\} \cap \{2,3,5\} \]
\[ (A \cap B) \cap C = \{\} = \emptyset \ldots (1) \]
\[ \text{Now} \quad B \cap C = \{11,17,19,23\} \cap \{2,3,5\} = \{\} = \emptyset \]
\[ A \cap (B \cap C) = \{1,7,11,15,17,21\} \cap \emptyset \]
\[ A \cap (B \cap C) = \emptyset \ldots (2) \]

Results (1) and (2) show that \( (A \cap B) \cap C = A \cap (B \cap C) \).

Q.3- If \( A = \{2,4,6\} \), \( B = \{3,6,9,12\} \) and \( C = \{4,6,8,10\} \)
verify that: \( A \cup (B \cup C) = (A \cup B) \cup C \)

Solution:-
\[ A = \{2,4,6\} \), \( B = \{3,6,9,12\} \)
\[ C = \{4, 6, 8, 10\} \]

We have to show that \[ A \cup (B \cup C) = (A \cup B) \cup C \]

To solve the L.H.S.
\[ B \cup C = \{3, 6, 9, 12\} \cup \{4, 6, 8, 10\} \]
\[ = \{3, 4, 6, 8, 9, 10, 12\} \]
\[ A \cup (B \cup C) = \{2, 4, 6\} \cup \{3, 4, 6, 8, 9, 10, 12\} \]
\[ A \cup (B \cup C) = \{2, 3, 4, 6, 8, 9, 10, 12\} \quad (1) \]

Now to solve the R.H.S. Consider
\[ A \cup B = \{2, 4, 6\} \cup \{3, 6, 9, 12\} \]
\[ A \cup B = \{2, 3, 4, 6, 9, 12\} \]
\[ (A \cup B) \cup C = \{2, 3, 4, 6, 9, 12\} \cup \{4, 6, 8, 10\} \]
\[ (A \cup B) \cup C = \{2, 3, 4, 6, 8, 9, 10, 12\} \quad (2) \]

Results (1) and (2) show that
\[ A \cup (B \cup C) = (A \cup B) \cup C \]

Q.4- If \[ A = \{2, 3, 5, 7, 9\}, \quad B = \{1, 3, 5, 7\} \]
and \[ C = \{2, 3, 4, 5, 6\} \]
verify that: \[ (A \cap B) \cap C = A \cap (B \cap C) \]

Solution:- We are given that
\[ A = \{2, 3, 5, 7, 9\}, \quad B = \{1, 3, 5, 7\} \]
\[ C = \{2, 3, 4, 5, 6\} \]

We have to prove that
\[ (A \cap B) \cap C = A \cap (B \cap C) \]

First we will solve L.H.S. Consider
\[ A \cap B = \{2, 3, 5, 7, 9\} \cap \{1, 3, 5, 7\} = \{3, 5, 7\} \]
\[ (A \cap B) \cap C = \{3, 5\} \quad (1) \]

Now we will solve R.H.S. Consider
\[ B \cap C = \{1, 3, 5, 7\} \cap \{2, 3, 4, 5, 6\} \]
\[ B \cap C = \{3, 5\} \]

Now \[ A \cap (B \cap C) = \{2, 3, 5, 7, 9\} \cap \{3, 5\} \]
\[ A \cap (B \cap C) = \{3, 5\} \quad (2) \]
Results (1) and (2) show that 
\((A \cap B) \cap C = A \cap (B \cap C)\)

**Q.5-** If \(U = \{7, 8, 9, 10, 11, 12, 13, 14\}\)
\[A = \{7, 10, 13, 14\}\]
and \(B = \{7, 8, 11, 12\}\) then verify \((A \cap B)^0 = A^0 \cup B^0\)

**Solution:** We are given that
\[U = \{7, 8, 9, 10, 11, 13, 14\}\]
\[A = \{7, 10, 13, 14\}\]
\[B = \{7, 8, 11, 12\}\]

We are to verify \((A \cap B)^C = (A^C \cup B^C)\).

To solve L.H.S.
\[A \cap B = \{7, 10, 13, 14\} \cap \{7, 8, 11, 12\} = \{7\}\]
\[(A \cap B)^C = U - (A \cap B)\]
\[= \{7, 8, 9, 10, 11, 12, 13, 14\} - \{7\}\]
\[(A \cap B)^C = \{8, 9, 10, 11, 12, 13, 14\}\] ... (1)

Now to solve R.H.S.
\[A^C = U - A = \{7, 8, 9, \ldots, 14\} - \{7, 10, 13, 14\}\]
\[= \{8, 9, 11, 12\}\]
\[B^C = U - B = \{7, 8, 9, \ldots, 14\} - \{7, 8, 11, 12\}\]
\[= \{9, 10, 13, 14\}\]
\[A^C \cup B^C = \{8, 9, 11, 12\} \cup \{9, 10, 13, 14\}\]
\[A^C \cup B^C = \{8, 9, 10, 11, 12, 13, 14\}\] ... (2)

Results (1) and (2) show that 
\[(A \cap B)^C = A^C \cup B^C\]

**Q.6-** If \(U = \{4, 6, 8, 9, 10\}\) \(A = \{4, 6\}\) \(B = \{6, 8, 9\}\)

We are to verify De Morgans Laws 
\((A \cup B)^C = A^C \cap B^C\) and \((A \cap B)^C = A^C \cup B^C\)

**Solution:**

First Consider \((A \cup B)^C = A^C \cap B^C\)

To solve L.H.S.
\[ A \cup B = \{4,6\} \cup \{6,8,9\} \]
\[ A \cup B = \{4,6,8,9\} \]
\[ (A \cup B)^c = U - (A \cup B) \]
\[ = \{4,6,8,9,10\} - \{4,6,8,9\} \]
\[ (A \cup B)^c = \{10\} \quad ... (1) \]

Now to solve R.H.S.
\[ A^c = U - A = \{4,6,8,9,10\} - \{4,6\} \]
\[ = \{8,9,10\} \]
\[ B^c = U - B = \{4,6,8,9,10\} - \{6,8,9\} \]
\[ = \{4,10\} \]

Now \[ A^c \cup B^c = \{8,9,10\} \cup \{4,10\} \]
\[ = \{10\} \quad ... (2) \]

Results (1) and (2) show that
\[ (A \cup B)^c = A^c \cap B^c \]

Now take De. Morgans 2nd law
\[ (A \cap B)^c = A^c \cup B^c \]

To solve the L.H.S.
\[ A \cap B = \{4,6\} \cap \{6,8,9\} = \{6\} \]
\[ (A \cap B)^c = U - (A \cap B) \]
\[ = \{4,6,8,9,10\} - \{6\} \]
\[ (A \cap B)^c = \{4,8,9,10\} \quad ... (1) \]

Now
\[ A^c = U - A = \{4,6,8,9,10\} - \{4,6\} \]
\[ = \{8,9,10\} \]
\[ B^c = U - B = \{4,6,8,9,10\} - \{6,8,9\} \]
\[ B^c = \{4,10\} \]
\[ A^c \cup B^c = \{8,9,10\} \cup \{4,10\} \]
\[ = \{4,8,9,10\} \quad ... (2) \]

Results (1) and (2) show that
\[ (A \cap B)^c = A^c \cup B^c \]
Q.7- If \[ U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \]
\[ A = \{2, 3, 6, 9\} \]
and \[ B = \{1, 3, 6, 7, 8\} \] then
verify \( (A \cup B)^c = A^c \cap B^c \)

Solution: We are to prove that
\( (A \cup B)^c = A^c \cap B^c \)

To solve L.H.S.
\[ A \cup B = \{2, 3, 6, 9\} \cup \{1, 3, 6, 7, 8\} \]
\[ = \{1, 2, 3, 6, 7, 8, 9\} \]

\( (A \cup B)^c = U - (A \cup B) \)
\[ = \{1, 2, 3, ... 9\} - \{1, 2, 3, 6, 7, 8, 9\} \]

\( (A \cup B)^c = \{4, 5\} \) ...(1)

Now to solve R.H.S.
\[ A^c = U - A = \{1, 2, 3, ... 9\} - \{2, 3, 6, 9\} \]
\[ = \{1, 4, 5, 7, 8\} \]

\[ B^c = U - B = \{1, 2, 3, ... 9\} - \{1, 3, 6, 7, 8\} \]
\[ = \{2, 4, 5, 9\} \]

\[ A^c \cap B^c = \{1, 4, 5, 7, 8\} \cap \{2, 4, 5, 9\} \]
\[ A^c \cap B^c = \{4, 5\} \) ...(2)

From (1) and (2). We get.
\( (A \cup B)^c = A^c \cap B^c \)

Q.8- Fill in the blanks:

(i) \( A \cup A = \) ________

(ii) \( A \cap A = \) ________

(iii) \( A \cup \Phi = \) ________

(iv) \( A \cap \Phi = \) ________

(v) \( \Phi \cup \Phi = \) ________

(vi) \( (A \cap B)' = \) ________

(vii) \( (A \cap B)' = \) ________

(viii) \( (A)' = \) ________

(ix) \( \Phi \cap \Phi = \) ________

(x) \( A \cap A' = \) ________

Solution: -

(i) \( A \cup A = A \)

(ii) \( A \cap A = A \)

(iii) \( A \cup \Phi = A \)

(iv) \( A \cap \Phi = \Phi \)

(v) \( \Phi \cap \Phi = \Phi \)

(vi) \( (A \cap B)' = A' \cup B' \)
(vii) \((A \cup B)' = A' \cap B'\)  \hspace{1cm} (viii) \((A')' = A\)
(ix) \(\Phi \cap \Phi' = \Phi\)  \hspace{1cm} (x) \(A \cap A' = \Phi\)

**EXERCISE 8.2**

**Q.1-** If \(A = \{3, 5, 6\}\), \(A = \{1, 3\}\), Find \(A \times B\) and \(B \times A\) also the domains and ranges of the two binary relations established at your own for each case.

Solution:-

\[A = \{3, 5, 6\}\], \(B = \{1, 3\}\)

\(A \times B = \{(3,1), (3,3), (5,1), (5,3), (6,1), (6,3)\}\)

\(B \times A = \{(1,3), (1,5), (1,6), (3,3), (3,5), (3,6)\}\)

Two binary relations in \(A \times B\) are

\(R_1 = \{(3,1), (5,3), (5,1)\}\)

\(R_2 = \{(3,1), (3,3), (5,3), (6,3)\}\).

\(\text{Dom } R_1 = \{3, 5\}\), \(\text{Range } R_1 = \{1, 3\}\)

\(\text{Dom } R_2 = \{3, 5, 6\}\), \(\text{Range } R_2 = \{1, 3\}\)

Two binary relations in \(B \times A\) are

\(R_3 = \{(1,3), (1,6), (3,3)\}\)

\(R_4 = \{(1,5), (3,5)\}\).

\(\text{Dom } R_3 = \{1, 3\}\), Range \(R_3 = \{3, 6\}\)

\(\text{Dom } R_4 = \{1, 3\}\), Range \(R_4 = \{5\}\)

**Q.2-** If \(A = \{-2, 1, 4\}\), then write two binary relations in \(A\) also write their domains and ranges.

Solution:-

\(A = \{-2, 1, 4\}\)

\(A \times A = \{-2, 1, 4\} \times \{-2, 1, 4\}\)

\[\{( -2, -2), (-2, 1), (-2, 4), (1, -2), (1, 1), (1, 4), (4, -2), (4, 1), (4, 4)\}\]

Now any subset of \(A \times A\) is a binary relation in \(A\).

Thus two binary relations are

\(R_1 = \{( -2, -2), (1, -2), (4, 1)\}\)
\[ R_2 = \{(-2, 1), (1, 1), (4, 1)\} \]

Dom \( R_i \) = set of first elements of ordered pairs in \( R_i \)

= \{-2, 1, 4\}

Rang \( R_i \) = set of 2nd elements of ordered pairs in \( R_i \)

= \{-2, 1\}

Similarly.

Dom \( R_2 = \{-2, 1, 4\} \), Rang \( R_2 = \{1\} \)

Q.3- Write the number of binary relations possible in each of following cases.

(i) In \( C \times C \) when the number of elements in \( C \) is 3.

(ii) In \( A \times B \) if the number of elements in set \( A \) is 3 and in set \( B \) is 4.

Solution:

(i) Numbers of elements in \( C = 3 \)

Numbers of elements in \( C \times C = 3 \times 3 = 9 \)

So, number of binary relations in \( C \times C \)

= Number of all subsets of \( C \times C \)

= \( 2^9 \) Ans.

(ii) Numbers of elements in \( A = 3 \)

Numbers of elements in \( B = 4 \)

Thus Numbers of elements in \( A \times B = 3 \times 4 = 12 \)

So, Number of all subsets of \( A \times B = 2^{12} \)

and number of all possible binary relations in \( A \times B = 2^{12} \) Ans.

Q.4- If \( L = \{1, 2, 3\} \), and \( M = \{2, 3, 4\} \), then write a binary relation \( R \) such that

\[ R = \{(x, y) \mid x \in L, y \in M \land y \leq x\} \]

Also write Dom(\( R \)) and Range(\( R \)).

Solution:

\( L = \{1, 2, 3\}, \ M = \{2, 3, 4\} \)

\( L \times M = \{(1, 2), (1, 3), (1, 4), (2, 2), (2, 3)\} \)
Now \( R = \{(x, y) \mid x \in L, y \in M \wedge y \leq x\} \)
\[ R = \{(2, 2), (3, 2), (3, 3)\} \]

\( \text{Dom}(R) = \{2, 3\}, \quad \text{Rng}(R) = \{2, 3\} \)

**Q.5-** If \( X = \{0, 3, 5\} \) and \( Y = \{2, 4, 8\} \), then establish any four binary relations in \( X \times Y \).

**Solution:**
\[ X \times Y = \{(0, 2), (0, 4), (0, 8), (3, 2), (3, 4), (3, 5), (5, 2), (5, 4), (5, 8)\} \]

Binary relation in \( X \times Y \) is any subset of \( X \times Y \). So four binary relations in \( X \times Y \) are:

\[ R_1 = \{(0, 2), (3, 2), (5, 2)\} \]
\[ R_2 = \{(0, 4), (0, 8), (3, 2), (5, 8)\} \]
\[ R_3 = \{(0, 8), (3, 4), (5, 2)\} \]
\[ R_4 = \{(5, 2), (5, 4), (5, 8)\} \]

**Q.6-** If \( A = \{a, b, c\} \) and \( B = \{2, 4, 6\} \) and
\[ f = \{(a, 4), (b, 4), (c, 4)\} \] is a binary relation from \( A \times B \) then show that "f" is a function from \( A \) into \( B \).

**Solution:**
\[ f = \{(a, 4), (b, 4), (c, 4)\} \]
\[ \text{Dom } f = \{a, b, c\} = A \]

Now we see that non of the 1st elements of ordered pairs in \( f \) is repeated. So \( f \) is a function from \( A \) to \( B \).

Now \( \text{Range}(f) = \{4\} \neq B \)

It means \( f \) is a function from \( A \) into \( B \).

**Q.7-** If \( A = \{l, m, n\} \) and \( B = \{2, 4, 6\} \)
and \( g = \{(l, 3), (m, 1), (n, 1)\} \) is a binary relation in \( A \times B \), then show that "g" is \( A \) into \( B \) function.

**Solution:**
\[ g = \{(l, 3), (m, 1), (n, 1)\} \]
\[ \text{Dom } (g) = \{l, m, n\} = A \]
We see that none of the first elements ordered pairs in $g$ is repeated.
So $g$ is a function from $A$ to $B$.
Now $\text{Rng}(g) = \{1, 3\} \neq B$
It shows that $g$ is a function from $A$ into $B$.

Q.8- If $A = \{1, 3, 5\}$ and $B = \{x, y, z\}$ and $g = \{(1, x), (3, y), (5, z)\}$ is a binary relation from $A \times B$, then show that "g" is a onto $B$ function.

Solution:-
$g = \{(1, x), (3, y), (5, z)\}$
$\text{Dom}(g) = \{1, 3, 5\}$
Also none of the elements of ordered pairs in $g$ is repeated. So $g$ is a function from $A$ to $B$.
Now $\text{Rng}(g) = \{x, y, z\} = B$.
It shows that $g$ is a function from $A$ onto $B$.

**Review Exercise 8**

Q.1- Encircle the correct answer.
(i) If $A$ and $B$ are two non-empty sets, then $A \cup B = ?$
   
   (a) $\Phi$  
   (b) $B \cup A$  
   (c) $A \cap B$  
   (d) $B \cap A$

(ii) If $A$ and $B$ are two non-empty overlapping sets, then $A \cap B = ?$
    
    (a) $\Phi$  
    (b) $B \cap A$  
    (c) $A \cap B$  
    (d) $B \cup A$

(iii) For any two sets $A$ and $B$, $A \cup B = B \cup A$ is called:
       
       (a) Commutative law  
       (b) Associative law

(iv) $A \cup (B \cup C) = \overline{(A \cup B) \cup C}$ is called
    
    (a) Commutative law  
    (b) Associative law

(v) If $U = \{1, 2, 3, 4\}$, $A = \{4\}$, then $A' = ?$
   
   (a) $\{1, 2, 3\}$  
   (b) $\Phi$  
   (c) $\{1\}$  
   (d) $\{1, 2, 3, 4\}$
(vi) If $U = \{1, 2, 3\}, A = \{1\}$, then $U - A = \phi$.
   
   (a) $\{2, 3\}$  
   (b) $\{1, 2\}$  
   (c) $\{1, 3\}$  
   (d) $\phi$

(vii) $(A \cup B)' = \phi$
   
   (a) $A' \cup B'$  
   (b) $A' \cap B'$
   (c) $(A \cap B)'$
   (d) $\phi$

(viii) $(A \cap B)' = \phi$
   
   (a) $A' \cap B'$  
   (b) $A' \cup B'$
   (c) $A \cap B$
   (d) $A \cup B$

(ix) If $R = \{(4, 5), (5, 4), (5, 6), (6, 4)\}$ then domain of $R$.
   
   (a) $\{4, 6\}$  
   (b) $\{4, 5\}$  
   (c) $\{4, 5, 6\}$  
   (d) $\{5, 6\}$

(x) If $R = \{(4, 5), (5, 4), (5, 6), (6, 4)\}$ then range of $R$ is:
   
   (a) $\{4\}$  
   (b) $\{5\}$  
   (c) $\{6\}$  
   (d) $\{4, 5, 6\}$

Ans.

<table>
<thead>
<tr>
<th>(i)</th>
<th>b</th>
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<th>(iii)</th>
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<tr>
<td>(vi)</td>
<td>a</td>
<td>(vii)</td>
<td>b</td>
<td>(viii)</td>
<td>b</td>
<td>(ix)</td>
<td>c</td>
<td>(x)</td>
<td>$\phi$</td>
</tr>
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</table>

Q.2- Fill in the blanks.

(i) $(A \cup B)' = \phi$.

(ii) $(A \cap B)' = \phi$.

(iii) $A \cup (B \cup C)$ = \phi.

(iv) $A \cap (B \cap C)$ = \phi.

(v) If $A$ and $B$ be the two non-empty sets, then $A \cup B = B \cup A$ is called the.

(vi) If $A$ and $B$ be the two non-empty sets, then $A \cap B = B \cap A$ is called.

(vii) Any sub-set of a cartesian product is called a.

(viii) If $R_i = \{(1, 2), (3, 4), (5, 6)\}$ then domain of $R_i$ is.

(xi) If $R_i = \{(1, 2), (3, 4), (5, 6)\}$ then range of $R_i$ is.

(x) If $f : A \rightarrow B$ then every element of a set $A$ has its image in.
Q.3- If \( A = \{1,2,3,4,5,6\} \), \( B = \{2,3,4,6\} \) and \( C = \{2,3,4,7,8,9\} \). Verify that : \( (A \cap B)C = A \cap (B \cap C) \)

Solution:-
\[
A = \{1,2,3,4,5,6\}, \quad B = \{2,3,4,6\} \\
C = \{2,3,4,7,8,9\} \\
\]
We have to prove that
\[
(A \cap B) \cap C = A \cap (B \cap C) \\
\]
To solve L.H.S
\[
A \cap B = \{1,2,3,4,5,6\} \cap \{2,3,4,6\} = \{2,3,4,6\} \\
(A \cap B) \cap C = \{2,3,4,6\} \cap \{2,3,4,7,8,9\} = \{2,3,4\} \ldots (1) \\
\]
Now to solve R.H.S
\[
B \cap C = \{2,3,4,6\} \cap \{2,3,4,7,8,9\} = \{2,3,4\} \\
A \cap (B \cap C) = \{1,2,3,4,5,6\} \cap \{2,3,4\} = \{2,3,4\} \ldots (2) \\
\]
Results (1) and (2) show that \( (A \cap B) \cap C = A \cap (B \cap C) \)

Q.4- If \( A = \{2,3,4\} \), \( B = \{3,6,9,12\} \) and \( C = \{4,6,8,10\} \). Verify that : \( A \cup (B \cup C) = (A \cup B) \cup C \)

Solution:-
\[
A = \{2,3,4\}, \quad B = \{3,6,9,12\} \\
C = \{4,6,8,10\} \\
\]
We have to prove that
\[
A \cup (B \cup C) = (A \cup B) \cup C \\
\]
To solve L.H.S
\[
B \cup C = \{3,6,9,12\} \cup \{4,6,8,10\} = \{3,4,6,8,9,10,12\} \\
A \cup (B \cup C) = \{2,3,4\} \cup \{3,4,6,8,9,10,12\} = \{2,3,4,6,8,9,10,12\} \ldots (1) \\
\]
Now to solve R.H.S

\((A \cup B) = \{2,3,4\} \cup \{3,6,9,12\}\)

\[=\{2,3,4,6,9,12\}\]

\((A \cup B) \cup C = \{2,3,4,6,9,12\} \cup \{4,6,8,10\}\)

\[=\{2,3,4,6,8,9,10,12\}\] ... (2)

Results (1) and (2) show that

\[A \cup (B \cup C) = (A \cup B) \cup C\]

Q.5- If \(A = \{2,3,4\}\) and \(B = \{1,3\}\). Find \(A \times B\) and \(B \times A\). Also establish two binary relations each from these cartesian products.

Solution:-

\(A = \{2,3,4\}, B = \{1,3\}\)

\(A \times B = \{2,3,4\} \times \{1,3\}\)

\[=\{(2,1),(2,3),(3,1),(3,3),(4,1),(4,3)\}\]

Two binary relations in \(A \times B\) are

\(R_1 = \{(2,1),(3,1),(4,1)\}\)

\(R_2 = \{(2,3),(3,1),(3,3),(4,1)\}\)

Now \(B \times A = \{1,3\} \times \{2,3,4\}\)

\[=\{(1,2),(1,3),(1,4),(3,2),(3,3),(3,4)\}\]

Two binary relations in \(B \times A\) are

\(R_3 = \{(1,2),(1,4),(3,3)\}\)

\(R_4 = \{(1,3),(1,4),(3,4),(3,2)\}\)

Q.6- Write the number of binary relations possible in each of the following cases.

(i) In \(C \times C\), when the number of elements in \(C\) are 4.

(ii) In \(A \times B\), if number of elements in \(A\) are 2 and in \(B\) are 3.

Solution:-

(i) Number of elements in \(C = 4\)

Number of elements in \(C \times C = 4 \times 4 = 16\)
Thus Number of all subsets of \( C \times C = 2^{16} \)
So Number of all Binary relations = \( 2^{16} \)

\( (ii) \)
Number of elements of \( A = 2 \)
Number of elements of \( B = 3 \)
Number of elements of \( A \times B = 2 \times 3 = 6 \)
Thus Number of all subsets of \( A \times B = 2^6 = 64 \)
So Number of all binary relation in \( A \times B = 64 \)

Q.7- If \( R = \{(a,b) \mid a, b \in W, 3a + 2b = 16\} \). Find its domain and range \( R \).

Solution:-
\( R = \{(a,b) \mid a, b \in W, 3a + 2b = 16\} \)
Consider the equation
\( 3a + 2b = 16 \)
Put \( a = 0, 2 \) and \( 4 \)
For \( a = 0 \Rightarrow b = 8 \Rightarrow (0,8) \in R \)
For \( a = 2 \Rightarrow b = 5 \Rightarrow (2,5) \in R \)
For \( a = 4 \Rightarrow b = 2 \Rightarrow (4,2) \in R \)
Now \( R = \{(0,8),(2,5),(4,2)\} \)
Thus
\( \text{Dom} (R) = \{0, 2, 4\} \)
\( \text{Rang} (R) = \{2, 5, 8\} \)

\[ \textbf{Multiple Choice Question} \]

Q.1- The set \( \left[ \frac{p}{q} : p, q \in \mathbb{Z} \land q \neq 0 \right] \) is the set of
\( (a) \) Real Numbers \hspace{1cm} (b) Rational Numbers
\( (c) \) Irrational Numbers \hspace{1cm} (d) Prime Numbers

Q.2- Zero = 0 , is
\( (a) \) An even number \hspace{1cm} (b) Odd numbers
\( (c) \) Imaginary numbers \hspace{1cm} (d) Irrational numbers
Q.3- \( A \cup B = \)
(a) \( \{x \mid x \in A \lor x \in B\} \)
(b) \( \{x \mid x \in A \land x \in B\} \)
(c) \( \{x \mid x \in A \land x \notin B\} \)
(d) \( \{x \mid x \notin A \land x \in B\} \)

Q.4- The set \( \{x \mid x \in U \land x \notin A\} \) is equal to
(a) \( A \)
(b) \( A' \)
(c) \( A' \)
(d) \( A - B \)

Q.5- The set \( \{x \mid x \in A \land x \notin B\} \) is equal to
(a) \( A' \)
(b) \( B' \)
(c) \( A - B \)
(d) \( B - A \)

Q.6- \( A \cup (B \cup C) = (A \cup B) \cup C \) is the law
(a) De Morgan
(b) Commutative
(c) Associative
(d) Distributive

Q.7- In the venn diagram two sets A and B are such that
(a) \( A \subseteq B \)
(b) \( B \subseteq A \)
(c) Overlapping
(d) Disjoint

Q.8- The statement \( (A \cup B)^c = A^c \cap B^c \) is of
(a) Distributive law
(b) Associative law
(c) De-Morgans law
(d) Commutative law

Q.9- If \( A = \{1,2,3,4,5,6\} \) and \( U = \{1,2,3,...10\} \)
Then \( A^c \) is equal to
(a) \( \{2,4,6,8,10\} \)
(b) \( \{1,3,5,7,9\} \)
(c) \( \{7,8,9,10\} \)
(d) \( \{1,2,3,4\} \)

Q.10- If \( A = \{1,2,3\} \), \( B = \{y,z\} \), then all the binary relations in \( A \times B \) are
(a) 6 \( (b) 9 \) \( (c) 32 \) \( (d) 64 \)

Q.11- \( R = \{(1,2),(1,3),(2,5),(3,10)\} \) is a binary relations.
Its Domain is
(a) \( \{1,1,2,3\} \)
(b) \( \{1,2,3\} \)
(c) \( \{2,3,5,10\} \)
(d) \( \{1,2,3,5,10\} \)

Q.12- If \( A = \{a,b\} \), \( B = \{x,y\} \), Then the function from \( A \) onto \( B \) is
(a) \( \{(a,x),(b,x)\} \)
(b) \( \{(b,x), (a,y)\} \)
(c) \( \{(a,x), (a,y)\} \)
(d) \( \{(b,x),(b,y)\} \)
Q.13- If \( f \) is a function from \( A \) to \( B \) such that \( \text{Rang} \ F = B \) Then it is a function

(a) Into  \hspace{1cm} (b) Onto
(c) One-One  \hspace{1cm} (d) Corresponding

Q.14- A one-one and onto function is called

(a) Injective  \hspace{1cm} (b) Surjective
(c) Bijective  \hspace{1cm} (d) Objective

Q.15- If \( A \) and \( B \) are disjoint sets then

(a) \( A \cap B = \Phi \)  \hspace{1cm} (b) \( A \cup B = \Phi \)
(c) \( A^c = B \)  \hspace{1cm} (d) \( B^c = A \)

**MODEL CLASS TEST**

Time : 40 mins  \hspace{1cm} Max Marks : 25

Q.1- Tich the best choice.

(i) The law \( A \cup B = B \cup A \) is called

(a) De-Morgan  \hspace{1cm} (b) Associative
(c) Commutative  \hspace{1cm} (d) Distributive

(ii) If \( R = \{(1,3),(1,4),(2,3)\} \) Then \( \text{Dom}(R) = \)

(a) \( \{1,1,2\} \)  \hspace{1cm} (b) \( \{1,2\} \)
(c) \( \{3,3,4\} \)  \hspace{1cm} (d) \( \{3,4\} \)

(iii) If "f" is a function, such that non of 2nd element of ordered pairs in \( f \) is repeated. Then \( f \) is called

(a) Onto  \hspace{1cm} (b) into  \hspace{1cm} (c) One-One  \hspace{1cm} (d) Bijective.

(iv) Complement of universal set is equal to

(a) Universal set  \hspace{1cm} (b) Empty set
(c) Sub set  \hspace{1cm} (d) Super set

(v) \( (A \cup B)^c \) is equal to

(a) \( A^c \cup B^c \)  \hspace{1cm} (b) \( (A \cap B)^c \)
(c) \( A^c \cap B^c \)  \hspace{1cm} (d) \( \Phi \)

(vi) \( A \cup \Phi \) is equal to

(a) \( A \)  \hspace{1cm} (b) \( \Phi \)  \hspace{1cm} (c) \( A \cap \Phi \)  \hspace{1cm} (d) \( A^c \)
(vii) \( \{2, 4\} \cap \{1, 3, 5\} \) is equal to
(a) \( \{3\} \) (b) \( \{1, 2, 4\} \) (c) \( \Phi \) (d) \( \{1, 2, 3, 4, 5\} \)

Q.2- Attempt any five of the following short questions.
(i) If \( A = \{a, b, c\} \) and \( B = \{a, e, i, o, u\} \)
Then find \( A \cup B \) and \( A \cap B \)
(ii) If \( U = \{1, 2, 3, \ldots, 10\} \) and \( A = \{1, 2, 3, 4\} \)
Then find \( A^c \)
(iii) If \( U = \{1, 2, 3, \ldots, 10\} \) and \( B = \{1, 2, 3, 4\} \)
Then find \( B \cup B^c \)
(iv) If \( R = \{(1, 5), (2, 6), (2, 7), (3, 7)\} \)
Then find Dom\( (R) \) and Rng\( (R) \)
(v) If \( A = \{5, 6, 7\} \) , \( B = \{1, 2\} \) Then find the function from \( A \) onto \( B \)
(vi) If \( A = \{a, b, c, d\} \) and \( B = \{1, 3\} \)
Write a binary relation from \( A \) to \( B \) which is not a function.

Attempt any two of the following questions.

Q.3- If \( U = \{1, 2, 3, \ldots, 9\} \) , \( A = \{2, 3, 6, 9\} \) , \( B = \{1, 3, 6, 7, 8\} \)
Then verify \( (A \cup B)^c = A^c \cap B^c \)

Q.4- If \( A = \{2, 3, 5, 7, 9\} \) , \( B = \{1, 3, 5, 7\} \) , \( C = \{2, 3, 4, 5, 6\} \)
Then verify \( (A \cap B) \cap C = A \cap (B \cap C) \)

Q.5- If \( A = \{1, 3, 5\} \) , \( B = \{2, 4, 6\} \) Then find \( A \times B \) and a b injective function from \( A \) to \( B \).