Chapter 5

Factorization:

Writing an algebraic expression as the product of two or more algebraic expressions is called factorization

Example 1: i). Factorize $15+10x-5x^2$

Solution: Given $15+10x-5x^2$

$$=5\times 3+5\times 2x-5x^2$$

Taking 5 as a common factors

$$=5\left(3+2x-x^2\right)$$

Example 1: Ii). Factorize $12x^2y^2-20x^3y$

Solution: Given $12x^2y^2-20x^3y$

Taking $4x^2y$ as a common factors

$$=4x^2y\left(3y-5x\right)$$

Example 2: Factorize $a^2-ab-3a+3b$

Solution: Given $a^2-ab-3a+3b$

$$=a\left(a-b\right)-3\left(a-b\right)$$

$$=\left(a-b\right)\left(a-3\right)$$

Example 3: i). Factorize $x^2+8x+16$

Solution: Given $x^2+8x+16$

$$=\left(x\right)^2+2\left(x\right)\left(4\right)+\left(4\right)^2$$

$$=\left(x+4\right)^2$$

Example 3: ii). Factorize $25y^2-30y+9$

Solution: Given $25y^2-30y+9$

$$=\left(5y\right)^2-2\left(5y\right)\left(3\right)+\left(3\right)^2$$

$$=\left(5y-3\right)^2$$

Example 4: a). Factorize x^2-16

Solution: Given x^2-16

$$=x^2-4^2$$

$$=\left(x-4\right)\left(x+4\right)$$

Example 4: b). Factorize $9a^2-25$

Solution: Given $9a^2-25$

$$=\left(3a\right)^2-\left(5\right)^2$$

$$=\left(3a-5\right)\left(3a+5\right)$$

Example 4: c). Factorize $6x^4-6y^4$

Solution: Given $6x^4-6y^4$

$$=6\left[x^4-y^4\right]$$

$$=6\left[\left(x^2\right)^2-\left(y^2\right)^2\right]$$

$$=6\left(x^2+y^2\right)\left[x^2-y^2\right]$$

$$=6\left(x^2+y^2\right)\left(x+y\right)\left(x-y\right)$$

Example 5: Factorize $a^2+4ab+4b^2-c^2$

Solution: Given $a^2+4ab+4b^2-c^2$

$$=\left(a\right)^2+2\left(a\right)\left(2b\right)+\left(2b\right)^2-c^2$$

$$=\left(a+2b\right)^2-c^2$$

$$=\left(a+2b+c\right)\left(a+2b-c\right)$$

Example 6: Factorize a^2+b^2-2b-1

Solution: Given a^2+b^2-2b-1

$$=a^2-\left[b^2-2b+1\right]$$

$$=\left(a\right)^2-\left[\left(b\right)^2-2\left(b\right)\left(1\right)+\left(1\right)^2\right]$$

$$=\left(a\right)^2-\left(b-1\right)^2$$

$$=\left[a+\left(b-1\right)\right]\left[a-\left(b-1\right)\right]$$

$$=\left(a+b-1\right)\left(a-b+1\right)$$

Rule 1 Common

Rule 2: Pair Common

Rule 3: $a^2\pm 2ab+b^2\rightarrow\left(a\pm b\right)^2$

Rule 4: $a^2-b^2\rightarrow\left(a+b\right)\left(a-b\right)$

Rule 5: $\left(a\pm b\right)^2-\left(c\right)^2\rightarrow\left(a\pm b+c\right)\left(a\pm b-c\right)$

Exercise 5.1

Q1. Factorize $9s^3t+15s^2t^3-3s^2t^2$

Solution: Given $9s^3t+15s^2t^3-3s^2t^2$

$$=3s^2t\left(3s+5t^2-t\right)$$

Q2. Factorize $10a^2b^3c^4-15a^3b^2c^2+30a^4b^3c^2$

Sol: Given $10a^2b^3c^4-15a^3b^2c^2+30a^4b^3c^2$

$$=5a^2b^2c^2\left(2bc^2-3a+6a^2b\right)$$

Q3. Factorize $ax-a-x+1$

Solution: Given $ax-a-x+1$

$$=a\left(x-1\right)-1\left(x-1\right)$$

$$=\left(a-1\right)\left(x-1\right)$$

Q4. Factorize $x^2-2y^3-2xy^2+xy$

Solution: Given $x^2-2y^3-2xy^2+xy$

Rearranging $=x^2+xy-2xy^2-2y^3$

$$=x\left(x+y\right)-2y^2\left(x+y\right)$$

$$=\left(x+y\right)\left(x-2y^2\right)$$

Q5. Factorize $4x^2+4+\frac{1}{x^2}$

Solution: Given $4x^2+4+\frac{1}{x^2}$

$$=\left(2x\right)^2+2\left(2x\right)\left(\frac{1}{x}\right)+\left(\frac{1}{x}\right)^2$$

$$=\left(2x+\frac{1}{x}\right)^2$$

Q6. Factorize $4\left(x+y\right)^2-20\left(x+y\right)z+25z^2$

Solution: Given $4\left(x+y\right)^2-20\left(x+y\right)z+25z^2$

$$=\left\{2\left(x+y\right)\right\}^2-2\left\{2\left(x+y\right)\right\}\left(5z\right)+\left(5z\right)^2$$

$$=\left\{2\left(x+y\right)-5z\right\}^2$$

Q7. Factorize $\frac{x^4}{y^4}-\frac{y^4}{x^4}$

Solution: Given $\frac{x^4}{y^4}-\frac{y^4}{x^4}$

$$=\left(\frac{x^2}{y^2}\right)^2-\left(\frac{y^2}{x^2}\right)^2$$

$$=\left(\frac{x^2}{y^2}+\frac{y^2}{x^2}\right)\left(\frac{x^2}{y^2}-\frac{y^2}{x^2}\right)$$

$$=\left(\frac{x^2}{y^2}+\frac{y^2}{x^2}\right)\left\{\left(\frac{x}{y}\right)^2-\left(\frac{y}{x}\right)^2\right\}$$

$$=\left(\frac{x^2}{y^2}+\frac{y^2}{x^2}\right)\left(\frac{x}{y}+\frac{y}{x}\right)\left(\frac{x}{y}+\frac{y}{x}\right)$$

Q8. Factorize $2x^2-288$

Solution: Given $2x^2-288$

$$=2\{x^2-144\}$$

$$==2\{(x)^2-(12)^2\}$$

$$=2(x+12)(x-12)$$

Q9. Factorize $1+2uv-u^2-v^2$

Solution: Given $1+2uv-u^2-v^2$

Rearranging $=1-u^2-v^2+2uv$

$$=1-(u^2+v^2-2uv)$$

$$=(1)^2-(u-v)^2$$

$$=\{1+(u-v)\}\{1-(u-v)\}$$

$$=(1+u-v)(1-u+v)$$

Q10. Factorize $25a^2b^2-20abc+4c^2-16d^2$

Solution: Given $25a^2b^2-20abc+4c^2-16d^2$

$$=(5ab)^2-2(5ab)(2c)+(2c)^2-(4d)^2$$

$$=(5ab-2c)^2-(4d)^2$$

$$=(5ab-2c+4d)(5ab-2c-4d)$$

Example 7: Factorize a^4+4b^4

Sol: Given a^4+4b^4

$$=(a^2)^2+(2b^2)^2+2(a^2)(2b^2)-2(a^2)(2b^2)$$

$$=(a^2+2b^2)^2-4a^2b^2$$

$$=(a^2+2b^2)^2-(2ab)^2$$

$$=(a^2+2b^2+2ab)(a^2+2b^2-2ab)$$

Example 8: Factorize $a^4+a^2b^2+b^4$

Sol: Given $a^4+a^2b^2+b^4$

$$=a^4+b^4+a^2b^2$$

$$=(a^2)^2+(b^2)^2+2(a^2)(b^2)-2(a^2)+(b^2)+a^2b^2$$

$$=(a^2+b^2)^2-2a^2b^2+a^2b^2$$

$$=(a^2+b^2)^2-a^2b^2$$

$$=(a^2+b^2)^2-(ab)^2$$

$$=(a^2+b^2+ab)(a^2+b^2-ab)$$

Example 9: Factorize x^4-7x^2+1

Sol: Given x^4-7x^2+1

$$=x^4+1-7x^2$$

$$=(x^2)^2+(1)^2+2(x^2)(1)-2(x^2)(1)-7x^2$$

$$=(x^2+1)^2-2x^2-7x^2$$

$$=(x^2+1)^2-9x^2$$

$$=(x^2+1)^2-(3x)^2$$

$$=(x^2+1+3x)(x^2+1-3x)$$

Rule 6

$$a^4+b^4\to(a^2)^2+(b^2)^2+2a^2b^2-2a^2b^2$$

$$\to(a^2+b^2)^2-(c)^2 \quad \because c=2a^2b^2$$

$$\to(a^2+b^2+c)(a^2+b^2-c)$$

Exercise 5.2

Q1. Factorize x^4+64

Solution: Given x^4+64

$$=(x^2)^2+(8)^2$$

$$=(x^2)^2+(8)^2+2(x^2)(8)-2(x^2)(8)$$

Using formula $a^2+2ab+b^2=(a+b)^2$

$$=(x^2+8)^2-2(x^2)(8)$$

$$=(x^2+8)^2-16x^2$$

$$=(x^2+8)^2-(4x)^2$$

Using $a^2-b^2=(a-b)(a+b)$

$$=(x^2+8+4x)(x^2+8-4x)$$

$$=(x^2+4x+8)(x^2-4x+8)$$

Q2. Factorize $4x^4+81$

Solution: Given $4x^4+81$

$$=(2x^2)^2+(9)^2$$

$$=(2x^2)^2+(9)^2+2(2x^2)(9)-2(2x^2)(9)$$

Using formula $a^2+2ab+b^2=(a+b)^2$

$$=(2x^2+9)^2-2(2x^2)(9)$$

$$=(2x^2+9)^2-36x^2$$

$$=(2x^2+9)^2-(6x)^2$$

$$=(2x^2+9+6x)(2x^2+9-6x)$$

$$=(2x^2+6x+9)(2x^2-6x+9)$$

Q3. Factorize $a^4+a^2b^2+b^4$

Sol: Given $a^4+a^2b^2+b^4$

$$=a^4+b^4+a^2b^2$$

$$=(a^2)^2+(b^2)^2+2(a^2)(b^2)-2(a^2)+(b^2)+a^2b^2$$

$$=(a^2+b^2)^2-2a^2b^2+a^2b^2$$

$$=(a^2+b^2)^2-a^2b^2$$

$=\left(a^2+b^2\right)^2-\left(ab\right)^2$

$=\left(a^2+b^2+ab\right)\left(a^2+b^2-ab\right)$

Q4. Factorize x^4+x^2+1

Solution: Given x^4+x^2+1

$=x^4+1+x^2$

$=\left(x^2\right)^2+\left(1\right)^2+x^2$

$=\left(x^2\right)^2+\left(1\right)^2+2\left(x^2\right)\left(1\right)-2\left(x^2\right)\left(1\right)+x^2$

Using formula $a^2+2ab+b^2=\left(a+b\right)^2$

$=\left(x^2+1\right)^2-2\left(x^2\right)\left(1\right)+x^2$

$=\left(x^2+1\right)^2-2x^2+x^2$

$=\left(x^2+1\right)^2-x^2$

$=\left(x^2+1\right)^2-\left(x\right)^2$

Using $a^2-b^2=\left(a-b\right)\left(a+b\right)$

$=\left(x^2+1+x\right)\left(x^2+1-x\right)$

$=\left(x^2+x+1\right)\left(x^2-x+1\right)$

Q5: x^8+x^4+1

Sol: Given x^8+x^4+1

$=x^8+1+x^4$

$=\left(x^4\right)^2+\left(1\right)^2+x^4$

$=\left(x^4\right)^2+\left(1\right)^2+2\left(x^4\right)\left(1\right)-2\left(x^4\right)\left(1\right)+x^4$

Using formula $a^2+2ab+b^2=\left(a+b\right)^2$

$=\left(x^4+1\right)^2-2\left(x^4\right)\left(1\right)+x^4$

$=\left(x^4+1\right)^2-2x^4+x^4$

$=\left(x^4+1\right)^2-x^4$

$=\left(x^4+1\right)^2-\left(x^2\right)^2$

Using $a^2-b^2=\left(a-b\right)\left(a+b\right)$

$=\left(x^4+1-x^2\right)\left(x^4+1+x^2\right)$

$=\left(x^4-x^2+1\right)\left\{\left(x^2\right)^2+\left(1\right)^2+x^2\right\}$

$=\left(x^4-x^2+1\right)\left\{\left(x^2\right)^2+\left(1\right)^2+2\left(x^2\right)\left(1\right)-2\left(x^2\right)\left(1\right)+x^2\right\}$

Using $a^2+2ab+b^2=\left(a+b\right)^2$

$=\left(x^4-x^2+1\right)\left\{\left(x^2+1\right)^2-2\left(x^2\right)\left(1\right)+x^2\right\}$

$=\left(x^4-x^2+1\right)\left\{\left(x^2+1\right)^2-2x^2+x^2\right\}$

$=\left(x^4-x^2+1\right)\left\{\left(x^2+1\right)^2-x^2\right\}$

$=\left(x^4-x^2+1\right)\left\{\left(x^2+1\right)^2-\left(x\right)^2\right\}$

Using $a^2-b^2=\left(a-b\right)\left(a+b\right)$

$=\left(x^4-x^2+1\right)\left(x^2+1+x\right)\left(x^2+1-x\right)$

$=\left(x^4-x^2+1\right)\left(x^2+x+1\right)\left(x^2-x+1\right)$

Q6. Factorize $x^4+\frac{1}{x^4}-7$

Solution: Given $x^4+\frac{1}{x^4}-7$

$=\left(x^2\right)^2+\left(\frac{1}{x^2}\right)^2-7$

$=\left(x^2\right)^2+\left(\frac{1}{x^2}\right)^2+2\left(x^2\right)\left(\frac{1}{x^2}\right)-2\left(x^2\right)\left(\frac{1}{x^2}\right)-7$

Using formula $a^2+2ab+b^2=\left(a+b\right)^2$

$=\left(x^2+\frac{1}{x^2}\right)^2-2\left(x^2\right)\left(\frac{1}{x^2}\right)-7$

$=\left(x^2+\frac{1}{x^2}\right)^2-2-7$

$=\left(x^2+\frac{1}{x^2}\right)^2-9$

$=\left(x^2+\frac{1}{x^2}\right)^2-\left(3\right)^2$

$=\left(x^2+\frac{1}{x^2}+3\right)\left(x^2+\frac{1}{x^2}-3\right)$

Q7. Factorize $81x^4+\frac{1}{81x^4}-14$

Solution: Given $81x^4+\frac{1}{81x^4}-14$

$=\left(9x^2\right)^2+\left(\frac{1}{9x^2}\right)^2-14$

$=\left(9x^2\right)^2+\left(\frac{1}{9x^2}\right)^2+2\left(9x^2\right)\left(\frac{1}{9x^2}\right)-2\left(9x^2\right)\left(\frac{1}{9x^2}\right)-14$

Using $a^2+2ab+b^2=\left(a+b\right)^2$

$=\left(9x^2+\frac{1}{9x^2}\right)^2-2\left(9x^2\right)\left(\frac{1}{9x^2}\right)-14$

$=\left(9x^2+\frac{1}{9x^2}\right)^2-2-14$

$=\left(9x^2+\frac{1}{9x^2}\right)^2-16$

$=\left(9x^2+\frac{1}{9x^2}\right)^2-\left(4\right)^2$

Using $a^2-b^2=\left(a-b\right)\left(a+b\right)$

$=\left(9x^2+\frac{1}{9x^2}+4\right)\left(9x^2+\frac{1}{9x^2}-4\right)$

Q8. Factorize $4x^4-4x^2y^2+64y^4$

Solution: Given $4x^4-4x^2y^2+64y^4$

$=4x^4+64y^4-4x^2y^2$

$=4\left\{x^4+16y^4-x^2y^2\right\}$

$=4\left\{\left(x^2\right)^2+\left(4y^2\right)^2-x^2y^2\right\}$

$=4\left\{\left(x^2\right)^2+\left(4y^2\right)^2+2\left(x^2\right)\left(4y^2\right)-2\left(x^2\right)\left(4y^2\right)-x^2y^2\right\}$

Using formula $a^2+2ab+b^2=\left(a+b\right)^2$

$=4\left\{\left(x^2+4y^2\right)^2-2\left(x^2\right)\left(4y^2\right)-x^2y^2\right\}$

$=4\left\{\left(x^2+4y^2\right)^2-8x^2-x^2\right\}$

$=4\left\{\left(x^2+4y^2\right)^2-9x^2y^2\right\}$

$=4\left\{\left(x^2+4y^2\right)^2-\left(3xy\right)^2\right\}$

Using $a^2-b^2=(a-b)(a+b)$

$=4\left(x^2+4y^2+3xy\right)\left(x^2+4y^2-3xy\right)$

$=4\left(x^2+3xy+4y^2\right)\left(x^2-3xy+4y^2\right)$

Q9. Factorize $16m^4+4m^2n^2+n^4$

Solution: Given $16m^4+4m^2n^2+n^4$

$=16m^4+n^4+4m^2n^2$

$=\left(4m^2\right)^2+\left(n^2\right)^2+4m^2n^2$

$=\left(4m^2\right)^2+\left(n^2\right)^2+2\left(4m^2\right)\left(n^2\right)-2\left(4m^2\right)\left(n^2\right)+4m^2n^2$

Using formula $a^2+2ab+b^2=(a+b)^2$

$=\left(4m^2+n^2\right)^2-2\left(4m^2\right)\left(n^2\right)+4m^2n^2$

$=\left(4m^2+n^2\right)^2-8m^2n^2+4m^2n^2$

$=\left(4m^2+n^2\right)^2-4m^2n^2$

$=\left(4m^2+n^2\right)^2-(2mn)^2$

Using $a^2-b^2=(a-b)(a+b)$

$=\left(4m^2+n^2+2mn\right)\left(4m^2+n^2-2mn\right)$

$=\left(4m^2+2mn+n^2\right)\left(4m^2-2mn+n^2\right)$

Q10. Factorize $4x^5y+11x^3y^3+9xy^5$

Solution: Given $4x^5y+11x^3y^3+9xy^5$

$=xy\left\{4x^4+11x^2y^2+9y^4\right\}$

$=xy\left\{4x^4+9y^4+11x^2y^2\right\}$

$=xy\left\{\left(2x^2\right)^2+\left(3y^2\right)^2+11x^2y^2\right\}$

$=xy\left\{\left(2x^2\right)^2+\left(3y^2\right)^2+2\left(2x^2\right)\left(3y^2\right)-2\left(2x^2\right)\left(3y^2\right)+11x^2y^2\right\}$

$=xy\left\{\left(2x^2+3y^2\right)^2-2\left(2x^2\right)\left(3y^2\right)+11x^2y^2\right\}$

$=xy\left\{\left(2x^2+3y^2\right)^2-12x^2y^2+11x^2y^2\right\}$

$=xy\left\{\left(2x^2+3y^2\right)^2-x^2y^2\right\}$

$=xy\left\{\left(2x^2+3y^2\right)^2-(xy)^2\right\}$

Using $a^2-b^2=(a-b)(a+b)$

$=xy\left(2x^2+3y^2+xy\right)\left(2x^2+3y^2-xy\right)$

Ex 10: Factorize $y^2-7y+12$

Sol: Given $y^2-7y+12$

$\therefore$ 12 is not a perfect square

$=y^2-4y-3y+12$

$=y(y-4)-3(y-4)$

$=(y-4)(y-3)$

	+	12
-	×	-
6	×	1
5	×	2
4	×	3

Ex 11: Factorize $x^2+10x+21$

Sol: Given $x^2+10x+21$

$\therefore$ 21 is not a perfect square

$=x^2+7x+3x+21$

$=x(x+7)+3(x+7)$

$=(x+3)(x+7)$

	+	21
+	×	+
9	×	1
8	×	2
7	×	3

Example 12: Find an expression of the perimeter of a rectangle whose area is given by $x^2+13x-90$

Solution: Area = L x w

Area $=x^2+13x-90$

$\therefore$ 90 is not a perfect square

$=x^2+18x-5x-90$

$=x(x+18)-5(x+18)$

	-	90
+	×	-
14	×	1
15	×	2
16	×	3
17	×	4
18	×	5

Area of rectangle $=(x+18)(x-5)$

Here L= $x+18$ and B= $x-5$

Perimeter of Rectangle = 2L+2B

Putting the values

Perimeter = $2(x+18)+2(x-5)$

$=2x+36+2x-10$

$=4x+26$

Example 13: Factorize $2x^2-7x-4$

Sol: Given $2x^2-7x-4=2x^2-8x+x-4$

$=2x(x-4)+1(x-4)$

$=(2x+1)(x-4)$

Rule 7: $a^2\pm2ab+b^2$ any term is missing

Consider ax^2+bx+c

Multiply with signs + Ac

Sign of b × Answer

If sign of ac is +ve × If sign of ac is -ve

Reduce 1 from b × Increase 1 in b

Exercise 5.3

Q1. Factorize $x^2-7x+12$

Solution: Given $x^2-7x+12$

$\therefore$ 12 is not a perfect square

$=x^2-4x-3x+12$

$=x(x-4)-3(x-4)$

$=(x-3)(x-4)$

	+	12
-	×	-
6	×	1
5	×	2
4	×	3

Q2. Factorize x^2+x-12

Solution: Given x^2+x-12

$\therefore$ 12 is not a perfect square

$=x^2+4x-3x-12$

$=x(x+4)-3(x+4)$

$=(x-3)(x+4)$

	-	12
+	×	-
2	×	1
3	×	2
4	×	3

Q3. Factorize $20-x-x^2$

Solution: Given $20-x-x^2$

20 is not a perfect square

$=20-5x+4x-x^2$

$=5(4-x)+x(4-x)$

$=(5+x)(4-x)$

	-	20
-	×	+
2	×	1
3	×	2
4	×	3
5	X	4

Q4. Factorize $2y^2-7y+3$

Solution: Given $2y^2-7y+3$

$\therefore$ 3 is not a perfect square

$= 2y^2 - 6y - 1y + 3$

$= 2y(y-3) - 1(y-3)$

$= (y-3)(2y-1)$

+		6
-	×	-
6	×	1

Q5. Factorize $4x^2+8x+3$

Solution: Given $4x^2+8x+3$

$\therefore$ 3 is not a perfect square

$= 4x^2+6x+2x+3$

$= 2x(2x+3)+1(2x+3)$

$= (2x+3)(2x+1)$

+		12
+	×	+
7	×	1
6	×	2

Q6: Factorize $10y^2-3y-1$

Solution: Given $10y^2-3y-1$

$= 10y^2-5y+2y-1$

$= 5y(2y-1)+1(2y-1)$

$= (5y+1)(2y-1)$

R. Work

-		10
-	×	+
4	×	1
5	×	2

Q7 Factorize $6x^3-15x^2-9x$

Sol: Given $6x^3-15x^2-9x$

$= 6x^3-15x^2-9x$

$= 3x\{2x^2-5x-3\}$

$= 3x\{2x^2-6x+1x-3\}$

$= 3x\{2x(x-3)+1(x-3)\}$

$= 3x(2x+1)(x-3)$

R. Work

-		6
-	×	+
6	×	1

Q8: Factorize $2xy^2+8xy-24x$

So: Given $2xy^2+8xy-24x$

$= 2x\{y^2+4y-12\}$

$= 2x\{y^2+6y-2y-12\}$

$= 2x\{y(y+6)-2(y+6)\}$

$= 2x(y-2)(y+6)$

R. Work

-		12
+	×	-
5	×	1
6	×	2

Q9: $2+5t-12t^2$

Sol: Given $2+5t-12t^2$

$= 2+8t-3t-12t^2$

$= 2(1+4t)-3t(1+4t)$

$= (2-3t)(1+4t)$

R. Work

-		24
+	×	-
6	×	1
7	×	2
8	×	3

Q10: $-16x^3y-20x^2y^2-6xy^3$

Sol: $-16x^3y-20x^2y^2-6xy^3$

$= -2xy\{8x^2+10xy+3y^2\}$

$= -2xy\{8x^2+6xy+4xy+3y^2\}$

$= -2xy\{2x(4x+3y)+y(4x+3y)\}$

$= -2xy(2x+y)(4x+3y)$

R. Work

+		24
+	×	+
9	×	1
8	×	2
7	×	3
6	×	4

Q11. Factorize $(x+1)^2+3(x+1)+2$

Solution: Given $(x+1)^2+3(x+1)+2$

2 is not a perfect square

$= (x+1)^2+2(x+1)+1(x+1)+2$

$= (x+1)\{x+1+2\}+1\{x+1+2\}$

$= \{(x+1)+1\}\{x+1+2\}$

$= (x+2)(x+3)$

+		2
+	×	+
2	×	1

Q12. Factorize $4x^8y^{10}-40x^5y^7+84x^2y^4$

Solution: Given $4x^8y^{10}-40x^5y^7+84x^2y^4$

$= 4x^2y^4\left[x^6y^6-10x^3y^3+21\right]$

21 is not a perfect square

$= 4x^2y^4\left[x^6y^6-7x^3y^3-3x^3y^3+21\right]$

$= 4x^2y^4\left[x^3y^3(x^3y^3-7)-3(x^3y^3-7)\right]$

$= 4x^2y^4(x^3y^3-7)(x^3y^3-3)$

Q13. Factorize Find an expression for perimeter of a rectangle with area given by $x^2+24x-81$

Solution: Area = L x w

Area $= x^2+24x-81$

$\therefore$ 81 is not a perfect square

$= x^2+27x-3x-81$

$= x(x+27)-3(x+27)$

-		81
+	×	-
25	×	1
26	×	2
27	×	3

Area of rectangle $=(x+27)(x-3)$

Here L= $x+27$ and B= $x-3$

Perimeter of Rectangle = 2L+2B

Putting the values

Perimeter = $2(x+27)+2(x-3)$

$= 2x+57+2x-6$

$= 4x+51$

Ex 14: Factorize $(x^2+7x+10)(x^2+7x+12)+1$

Sol: Given $(x^2+7x+10)(x^2+7x+12)+1$

Here two terms are same Let $y=x^2+7x$ so,

$=(y+10)(y+12)+1$

$= y^2+12y+10y+120+1$

$= y^2+22y+121$

$= y^2+2.y.11+11^2$

$=(y+11)^2$

Putting back value of $y=x^2+7x$

$= (x^2+7x+11)^2$

Exp15: Factorize $(3x^2+11x+2)(3x^2+11x+3)-12$

Sol: Given $(3x^2+11x+2)(3x^2+11x+3)-12$

Here two terms are same Let $y=3x^2+11x$

$=(y+2)(y+3)-12$

$= y^2+3y+2y+6-12$

$= y^2+5y-6$

$= y^2+6y-1y-6$

$= y(y+6)-1(y+6)$

$=(y-1)(y+6)$

Putting back value of $y=3x^2+11x$

$=(3x^2+11x-1)(3x^2+11x+6)$

$=(3x^2+11x-1)(3x^2+9x+2x+6)$

$=(3x^2+11x-1)\left[3x(x+3)+2(x+3)\right]$

$=(3x^2+11x-1)(3x+2)(x+3)$

Example 16: $(x+1)(x+2)(x+3)(x+4)+1$

Sol: Given $(x+1)(x+2)(x+3)(x+4)+1$

Here $1+4=2+3$, so rearranging

$=(x+1)(x+4)(x+2)(x+3)+1$

$=(x^2+4x+1x+4)(x^2+3x+2x+6)+1$

$=(x^2+5x+4)(x^2+5x+6)+1$

Here two terms are same Let $y=\ x^2+5x$

$=(y+4)(y+6)+1$

$=y^2+6y+4y+24+1$

$=y^2+10y+25$

$=y^2+2.y.5+5^2$

$=(y+5)^2$

Putting back value of $y=\ x^2+5x$

$=(x^2+5x+5)^2$

Exp17: Factorize $(x^2-5x+6)(x^2+5x+6)-2x^2$

Sol: Given $(x^2-5x+6)(x^2+5x+6)-2x^2$

$=(x^2-3x-2x+6)(x^2+3x+2x+6)-2x^2$

$=(x(x-3)-2(x-3))(x(x+3)+2(x+3))-2x^2$

$=(x-2)(x-3)(x+2)(x+3)-2x^2$

$=(x-2)(x+2)(x-3)(x+3)-2x^2$

$=(x^2-4)(x^2-9)-2x^2$

$=x^4-9x^2-4x^2+36-2x^2$

$=x^4-15x^2+36$

$=x^4-12x^2-3x^2+36$

$=x^2(x^2-12)-3(x^2-12)$

$=(x^2-12)(x^2-3)$

Exp 18: Factorize $8a^3+36a^2b+54ab^2+27b^3$

Sol: Given $8a^3+36a^2b+54ab^2+27b^3$

$=(2a)^3+3(2a)^2(3b)+3(2a)(3b)^2+(3b)^3$

$=(2a+3b)^3$

Exp 19: Factorize $27x^3-27x^2y+9xy^2-y^3$

Sol: Given $27x^3-27x^2y+9xy^2-y^3$

$=(3x)^3-3(3x)^2(y)+3(3x)(y)^2-(y)^3$

$=(3x-y)^3$

Rule8: $(ax^2+bx+c)(ax^2+bx+d)+e$

Suppose $y=ax^2+bx$ same terms

Rule 9 $(x+a)(x+b)(x+c)(x+d)+e$

Multiply those factors which $a+b=c+d$

Rule 10: $a^3\pm 3a^2b+3ab^2\pm b^3\rightarrow(a\pm b)^3$

Exercise 5.4

Q1:Factorize $(4x^2-16x+7)(4x^2-16x+15)+16$

Sol: Given $(4x^2-16x+7)(4x^2-16x+15)+16$

Here two terms are same Let $y=4x^2-16x$

$=(y+7)(y+15)+16$

$=y^2+15y+7y+105+16$

$=y^2+22y+121$

$=y^2+2.y.11+11^2$

$=(y+11)^2$

$=(4x^2-16x+11)^2$ Since $y=4x^2-16x$

Q2:Factorize $(9x^2+9x-4)(9x^2+9x-10)-72$

Sol; Given $(9x^2+9x-4)(9x^2+9x-10)-72$

Here two terms are same Let $y=9x^2+9x$

$=(y-4)(y-10)-72$

$=y(y-10)-4(y-10)-72$

$=y^2-10y-4y+40-72$

$=y^2-14y-32$

$=y^2-16y+2y-32$

$=y(y-16)+2(y-16)$

$=(y+2)(y-16)$ $\therefore y=9x^2+9x$

$=(9x^2+9x+2)(9x^2+9x-16)$

Q3: $(x+2)(x+4)(x+6)(x+8)-9$

Sol: Given $(x+2)(x+4)(x+6)(x+8)-9$

Rearranging accordingly 4 + 6 = 2 + 8

$=(x+4)(x+6)(x+2)(x+8)-9$

$=\{x(x+6)+4(x+6)\}\{x(x+8)+2(x+8)\}-9$

$=\{x^2+6x+4x+24\}\{x^2+8x+2x+16\}-9$

$=(x^2+10x+24)(x^2+10x+16)-9$

Here two terms are same Let $y=x^2+10x$

$=(y+24)(y+16)-9$

$=y(y+16)+24(y+16)-9$

$=y^2+16y+24y+384-9$

$=y^2+40y+375$

$=y^2+25y+15y+375$

$=y(y+25)+15(y+25)$

$=(y+15)(y+25)$ $\therefore y=x^2+10x$

$=(x^2+10x+15)(x^2+10x+25)$

$=(x^2+10x+15)(x^2+2.x.5+5^2)$

$=(x^2+10x+15)(x+5)^2$

Q4: $x(x+1)(x+2)(x+3)+1$

Sol: Given $x(x+1)(x+2)(x+3)+1$

Rearranging accordingly 0 + 3 = 1 + 2

$=x(x+3)(x+1)(x+2)+1$

$=\{x(x+3)\}\{x(x+2)+1(x+2)\}+1$

$=\{x^2+3x\}\{x^2+2x+1x+2\}+1$

$=(x^2+3x)(x^2+3x+2)+1$

Here two terms are same Let $y=x^2+3x$

$=y(y+2)+1$

$=y^2+2y+1$

$=y^2+2.y.1+1^2$

$=(y+1)^2 \qquad \therefore y=x^2+3x$

$=(x^2+3x+1)^2$

Q5: $(x+1)(x+2)(x+3)(x+6)-3x^2$

Sol: Given $(x+1)(x+2)(x+3)(x+6)-3x^2$

Here $a+b\neq c+d$ but $1\times 6=2\times 3$ so

$=(x+1)(x+6)(x+2)(x+3)-3x^2$

$=(x^2+6x+1x+6)(x^2+3x+2x+6)-3x^2$

$=(x^2+7x+6)(x^2+5x+6)-3x^2$

$=(x^2+6+7x)(x^2+6+5x)-3x^2$

Here two terms are same Let $y=x^2+6$

$=(y+7x)(y+5x)-3x^2$

$=y^2+5xy+7xy+35x^2-3x^2$

$=y^2+12xy+32x^2$

$=y^2+8xy+4xy+32x^2$

$=y(y+8x)+4x(y+8x)$

$=(y+4x)(y+8x)$

Putting back value of $y=x^2+6$

$=(x^2+6+4x)(x^2+6+8x)$

$=(x^2+4x+6)(x^2+8x+6)$

Q6: $64x^3-144x^2y+108xy^2-27y^3$

Sol: Given $64x^3-144x^2y+108xy^2-27y^3$

Using $(a-b)^3=a^3-3a^2b+3ab^2-b^3$

$=(4x)^3-3(4x)^2(3y)+3(4x)(3y)^2-(3y)^3$

$=(4x-3y)^3$

Q7: $\frac{a^3}{8}-\frac{1}{4}a^2b+\frac{1}{6}ab^2-\frac{b^3}{27}$

Sol: Given $\frac{a^3}{8}-\frac{1}{4}a^2b+\frac{1}{6}ab^2-\frac{b^3}{27}$

Using $(a-b)^3=a^3-3a^2b+3ab^2-b^3$

$=\left(\frac{a}{2}\right)^3-3\left(\frac{a}{2}\right)^2\left(\frac{b}{3}\right)+3\left(\frac{a}{2}\right)\left(\frac{b}{3}\right)^2-\left(\frac{b}{3}\right)^3$

$=\left(\frac{a}{2}-\frac{b}{3}\right)^3$

Q8: $\frac{x^3}{a^3}+\frac{3x}{a}+\frac{3a}{x}+\frac{a^3}{x^3}$

Sol: Given $\frac{x^3}{a^3}+\frac{3x}{a}+\frac{3a}{x}+\frac{a^3}{x^3}$

Using $(a+b)^3=a^3+3a^2b+3ab^2+b^3$

$=\left(\frac{x}{a}\right)^3+3\left(\frac{x}{a}\right)^2\left(\frac{a}{x}\right)+3\left(\frac{x}{a}\right)\left(\frac{a}{x}\right)^2+\left(\frac{a}{x}\right)^3$

$=\left(\frac{x}{a}+\frac{a}{x}\right)^3$

Q9: $27a^3+189a^2b+441ab^2+343b^3$

Sol: Given $27a^3+189a^2b+441ab^2+343b^3$

Using $(a+b)^3=a^3+3a^2b+3ab^2+b^3$

$=(3a)^3+3(3a)^2(7b)+3(3a)(7b)^2+(7b)^3$

$=(3a+7b)^3$

Q10: $8x^3-4x+\frac{2}{3x}-\frac{1}{27x^3}$

Sol: Given $8x^3-4x+\frac{2}{3x}-\frac{1}{27x^3}$

$=(2x)^3-3(2x)^2\left(\frac{1}{3x}\right)+3(2x)\left(\frac{1}{3x}\right)^2-\left(\frac{1}{3x}\right)^3$

$=\left(2x-\frac{1}{3x}\right)^3$

Example 20: Factorize $2x^3+16$

Sol: Given $2x^3+16$

$=2(x^3+8)$

$=2(x^3+2^3)$

Using $a^3+b^3=(a+b)(a^2-ab+b^2)$

$=2(x+2)(x^2-x.2+2^2)$

$=2(x+2)(x^2-2x+4)$

Example 21: Factorize a^3-64b^3

Sol: Given a^3-64b^3

$=(a)^3-(4b)^3$

Using $a^3-b^3=(a-b)(a^2+ab+b^2)$

$=(a-4b)(a^2+a.4b+(4b)^2)$

$=(a-4b)(a^2+4ab+16b^2)$

Rule11: $a^3\pm b^3\rightarrow(a\pm b)(a^2\mp ab+b^2)$

Exercise 5.5

Q1. Factorize a^3-27

Solution: Given a^3-27

$=a^3-3^3$

Using $a^3-b^3=(a-b)(a^2+ab+b^2)$

$=(a-3)(a^2+a.3+3^2)$

$=(a-3)(a^2+3a+9)$

Q2. Factorize a^6+b^6

Solution: Given a^6+b^6

$=(a^2)^3+(b^2)^3$

Using $a^3+b^3=(a+b)(a^2-ab+b^2)$

$=(a^2+b^2)\{(a^2)^2-(a^2)(b^2)+(b^2)^2\}$

$=(a^2+b^2)(a^4-a^2b^2+b^4)$

Q3. Factorize $24x^3+3$

Solution: Given $24x^3+3$

$=3(8x^3+1)$

$=3\{(2x)^3+(1)^3\}$

Using $a^3+b^3=(a+b)(a^2-ab+b^2)$

$=3(2x+1)\{(2x)^2-(2x)(1)+(1)^2\}$

$=3(2x+1)\{4x^2-2x+1\}$

Q4. Factorize $1-27r^3$

Solution: Given $1-27r^3$

$=(1)^3-(3r)^3$

Using $a^3-b^3=(a-b)(a^2+ab+b^2)$

$=(1-3r)\{(1)^2-(1)(3r)+(3r)^2\}$

$=(1-3r)(1+3r+9r^2)$

Q5: $2x^3-128$

Solution: Given $2x^3-128$

$=2\{x^3-64\}$

$=2\{x^3-4^3\}$

Using $a^3-b^3=(a-b)(a^2+ab+b^2)$

$=2(x-4)\{x^2+x.4+4^2\}$

$=2(x-4)(x^2+4x+16)$

Q6: $4x^5-256x^2$

Solution: Given $4x^5-256x^2$

$=4x^2\{x^3-64\}$

$=4x^2\{(x)^3-(4)^3\}$

Using $a^3-b^3=(a-b)(a^2+ab+b^2)$

$=4x^2(x-4)\{(x)^2+(x)(4)+(4)^2\}$

$=4x^2(x-4)(x^2+4x+16)$

Q7: $18(x-y)^3-144(a-b)^3$

Solution: Given $18(x-y)^3-144(a-b)^3$

$=18\left[(x-y)^3-8(a-b)^3\right]$

$=18\left[(x-y)^3-\{2(a-b)\}^3\right]$

$=18\left[(x-y)^3-\{2a-2b\}^3\right]$

Using $a^3-b^3=(a-b)(a^2+ab+b^2)$

$=18\left[(x-y)-(2a-2b)\right]\left[(x-y)^2+(x-y)(2a-2b)-(2a-2b)^2\right]$

$=18\left[x-y-2a+2b\right]\left[(x-y)^2+(x-y)(2a-2b)-(2a-2b)^2\right]$

Q8. Factorize x^9+1

Solution: Given x^9+1

$=(x^3)^3+(1)^3$

Using $a^3+b^3=(a+b)(a^2-ab+b^2)$

$=(x^3+1)\{(x^3)^2-(x^3)(1)+(1)^2\}$

$=(x^3+1^3)\{x^6-x^3+1\}$

Again Using $a^3+b^3=(a+b)(a^2-ab+b^2)$

$=(x+1)(x^2-x.1+1^2)\{x^6-x^3+1\}$

$=(x+1)(x^2-x+1)\{x^6-x^3+1\}$

Q9. Factorize $a^3-(c+d)^3$

Solution: Given $a^3-(c+d)^3$

Using $a^3-b^3=(a-b)(a^2+ab+b^2)$

$=\{a+(c+d)\}\{(a)^2-(a)(c+d)+(c+d)^2\}$

$=(a+c+d)\{a^2-ac-ad+(c+d)^2\}$

Using $(a+b)^2=(a^2+2ab+b^2)$

$=(a+c+d)\{a^2-ac-ad+c^2+2cd+d^2\}$

$=(a+c+d)\{a^2+c^2+d^2-ac-ad+2cd\}$

Q10. Factorize $27x^3-y^3$

Solution: Given $27x^3-y^3$

$=(3x)^3-(y)^3$

Using $a^3-b^3=(a-b)(a^2+ab+b^2)$

$=(3x-y)\{(3x)^2+(3x)(y)+(y)^2\}$

$=(3x-y)(9x^2+3xy+y^2)$

Remainder Theorem:

When a polynomial $P(x)$ of degree $n\geq 1$ is divided by $x-r$ gives a constant remainder then $R=P(r)$

Dividend = Divisor x Quotient + Remainder

Or $P(x)=(x-r)Q(x)+R$

Where $P(x)=$ Dividend, $x-r$ = Divisor

$Q(x)=$ Quotient, $R=$ Remainder

Exp 22: Divide $3x^3-7x^2+6x-3$ by $x-2$ & verify your answer by remainder theorem.

Sol: Divide $P(x)=3x^3-7x^2+6x-3$ by $x-2$

$$\begin{array}{r} 3x^2 - x + 4 \\ x-2 \overline{\big) \; 3x^3 - 7x^2 + 6x - 3} \\ \underline{\pm 3x^3 \mp 6x^2} \qquad\qquad \\ -x^2 + 6x \qquad \\ \underline{\mp x^2 \pm 2x} \qquad \\ 4x - 3 \\ \underline{\pm 4x \mp 8} \\ R = 5 \end{array}$$

Now using remainder theorem

Take divisor $x-2=0 \quad \Rightarrow x=2$

Put in dividend

$P(2)=3(2)^3-7(2)^2+6(2)-3$

$P(2)=3(8)-7(4)+12-3$

$P(2)=24-28+9$

$P(2)=5=R$

Which is the same obtained by actual division

Example 23: Without performing division. Find the remainder when $2x^3-3x^2+x-2$ is divided by $x-3$

Sol: Here Dividend $P(x)=2x^3-3x^2+x-2$

And take divisor $x-3=0 \quad \Rightarrow x=3$ put

$P(3)=2(3)^3-3(3)^2+(3)-2$

$P(3)=2(27)-3(9)+1$

$P(3)=54-27+1$

$P(3)=28=R$

Hence Remainder = 28

Example 24: For what value of k $3x^3+9x^2-(3k-4)x+2$ will be exactly divisible by $x-1$

Solution: Dividend $P(x)=3x^3+9x^2-(3k-4)x+2$

And take divisor $x-1=0 \quad \Rightarrow x=1$ Given that exactly divisible means $P(1)=R=0$

$P(1)=3(1)^3+9(1)^2-(3k-4)(1)+2$

$P(1)=3+9-(3k-4)+2=0$

$12-3k+4+2=0$

$12+4+2=3k$

$3k=18$

$k=\dfrac{18}{3}=6$

Example 25: Find the zero of the polynomial $P(x)=x^2-4x+3$

Solution: Given $P(x)=x^2-4x+3$

Let $x=1$ then

$P(1)=(1)^2-4(1)+3$

$P(1)=1-4+3$

$P(1)=0$

Hence 1 is a zero of the given polynomial

Similarly when $x=3$, then

$P(3)=(3)^2-4(3)+3$

$P(3)=9-12+3$

$P(3)=0$

Therefore 3 is another zero of given polynomial

Factor Theorem:

Let $P(x)$ be a polynomial or dividend and a linear polynomial or divisor or factor of $P(x)$ if and only if k is a zero of polynomial $P(x)$ i.e. $P(k)=0$

Example 26: using factor theorem, prove that $x-3$ is a factor of x^3-x^2-5x-3 and find the other factors.

Sol: Given $P(x)=x^3-x^2-5x-3$ and

Factor $x-3=0 \quad \Rightarrow \quad x=3$ putting

$P(3)=(3)^3-(3)^2-5(3)-3$

$P(3)=27-9-15-3$

$P(3)=27-27=0$

Therefore by factor theorem $x-3$ is a factor of $P(x)$ to find the remaining factors

$$\begin{array}{r} x^2 + 2x + 1 \\ x-3 \overline{\big) \; x^3 - x^2 - 5x - 3} \\ \underline{\pm x^3 \mp 3x^2} \qquad\qquad \\ +2x^2 - 5x \qquad \\ \underline{\pm 2x^2 \mp 6x} \qquad \\ x - 3 \\ \underline{\pm x \mp 3} \\ R = 0 \end{array}$$

Here $Q(x)=x^2+2x+1$ and $R=0$

Since $P(x)=(x-k)Q(x)+R$

$x^3-x^2-5x-3=(x-3)(x^2+2x+1)+0$

$=(x-3)(x^2+2.x.1+1^2)$

$=(x-3)(x+1)^2$

Example 27: Prove that $x+2$ is a factor of x^3-7x-6 and hence find the other factors.

Sol: here $P(x)=x^3-7x-6$ and factor

$x+2=0 \quad \Rightarrow \quad x=-2$ putting

$P(-2)=(-2)^3-7(-2)-6$

$=-8+14-6$

$=-14+14$

$=0$

Therefore $x+2$ is a factor of given polynomial

To find other factors

$$\begin{array}{r|l} & x^2-2x-3 \\ x+2 & x^3 \qquad -7x-6 \\ & \underline{\pm x^3 \pm 2x^2} \\ & -2x^2-7x \\ & \underline{\mp 2x^2 \mp 4x} \\ & -3x-6 \\ & \underline{\mp 3x \mp 6} \\ & R=0 \end{array}$$

Here $Q(x)=x^2-2x-3$ and $R=0$

Since $P(x)=(x-k)Q(x)+R$

$x^3-7x-6=(x+2)(x^2-2x-3)+0$

$=(x+2)(x^2-3x+1x-3)$

$=(x+2)\{x(x-3)+1(x-3)\}$

$=(x+2)(x+1)(x-3)$

Rule 12: $P(x)=ax^3+bx^2+cx+d$ & $x-k$

Put $x-k=0 \quad \Rightarrow x=k$

Then $P(k)=a(k)^3+b(k)^2+c(k)+d=R$

Rule 13: Given $P(x)=ax^3+bx^2+cx+d$

choose $k=1,-1,2,-2,3,-3$

Put $P(k)=a(k)^3+b(k)^2+c(k)+d=0$

So
$$\begin{array}{r|l} & Q(x)=Ax^2+Bx+C \\ x-k & ax^3+bx^2+cx+d \end{array}$$

Then

$ax^3+bx^2+cx+d=(x-k)(Ax^2+Bx+C)$

Exercise 5.6

Q1 By using reminder theorem find reminder of following polynomials where

i). $x^3+6x^2+8x-11$ is divided by $x-1$

Sol: Given $P(x)=x^3+6x^2+8x-11\cdots(1)$

and take factor $x-r=x-1=0 \Rightarrow r=1$

Using reminder theorem i.e., $P(r)=R$

put the value of r in eq (1)

$P(r)=(1)^3+6(1)^2+8(1)-11$

$R=1+6+8-11$

$R=15-11$

$R=4$

Therefore the reminder = 4

ii). $2x^3+4x^2+7x-5$ is divided by $x+3$

Sol: Given $P(x)=2x^3+4x^2+7x-5\cdots(1)$ &

take factor $x-r=x+3=0 \quad \Rightarrow r=-3$

Using reminder theorem i.e., $P(r)=R$

put the value of r in eq (1)

$P(r)=2(-3)^3+4(-3)^2+7(-3)-5$

$P(-3)=2(-27)+4(9)-21-5$

$R=-54+36-26$

$R=-54+10$

$R=-44$

Therefore the reminder = - 44

iii). $3x^3+x-200$ is divided by $x-4$

Sol: Given $P(x)=3x^3+x-200\cdots(1)$ and

take factor $x-r=x-4=0 \quad \Rightarrow r=4$

Using reminder theorem i.e., $P(r)=R$

put the value of r in eq (1)

$P(4)=3(4)^3+(4)-200$

$P(r)=3(64)+4-200$

$R=192+4-200$

$R=196-200$

$R=-4$

Therefore the reminder = - 4

Q2. Without preforming division find value of a, when $2x^3-ax^2-2ax+3x+2$ is divided by $x+1$

Sol: Given $P(x)=2x^3-ax^2-2ax+3x+2$ &

the factor $x+1=0 \Rightarrow x=-1$ using factor theorem i.e. $P(r)=0 \quad \Rightarrow R=0$

$P(-1)=2(-1)^3-a(-1)^2-2a(-1)+3(-1)+2=0$

$\Rightarrow \quad 2(-1)-a(+1)+2a-3+2=0$

$\Rightarrow \quad -2-a+2a-1=0$

$\Rightarrow \quad a-3=0$

$\Rightarrow \quad a=3$

Q3. Without preforming division find value of b, when x^3-4x^2+bx-2 is divided by $x-1$

Sol: Given $P(x)=x^3-4x^2+bx-2$ & the factor $x-1=0 \Rightarrow x=1$ using factor theorem i.e., $P(r)=0 \quad \Rightarrow R=0$

$P(1)=(1)^3-4(1)^2+b(1)-2=0$

$\Rightarrow \quad 1-4+b-2=0$

$\Rightarrow \quad b-2-3=0$

$\Rightarrow \quad b-5=0$

$\Rightarrow \quad b=5$

Q4. Using factor theorem, factorize following polynomials

i). x^3-2x^2-5x+6

Sol: Given $P(x)=x^3-2x^2-5x+6$

Using factor theorem to find other factors we take $P(r)=0$, where

$r=\pm1,\pm2,\pm3,...$ Take r = 1

$P(1)=(1)^3-2(1)^2-5(1)+6$

$P(r)=1-2-5+6$

$R=7-7$

$R=0$

Since $P(1)=0$, therefore by factor theorem $x-1$ is a factor of $P(x)$. To find the other factors divide $P(x)$ by $x-1$ as

$$\begin{array}{r|l} & x^2-x-6 \\ \hline x-1 & x^3-2x^2-5x+6 \\ & \underline{\pm x^3 \mp x^2} \\ & \quad -x^2-5x \\ & \quad \underline{\mp x^2 \pm x} \\ & \qquad -6x+6 \\ & \qquad \underline{-6x+6} \\ & \qquad\quad 0 \end{array}$$

Here $Q(x)=x^2-x-6$ and $R=0$

Since $P(x)=(x-r)Q(x)+R$

Putting the values we get

$$\begin{aligned} x^3-2x^2-5x+6&=(x-1)\{x^2-x-6\} \\ &=(x-1)\{x^2-3x+2x-6\} \\ &=(x-1)\{x(x-3)+2(x-3)\} \\ &=(x-1)(x+2)(x-3) \end{aligned}$$

ii). x^3+x^2-4x-4

Sol: Given $P(x)=x^3+x^2-4x-4$

Using factor theorem to find other factors we take $P(r)=0$, where $r=\pm1,\pm2,\pm3,...$

Take r = 1

$P(1)=(1)^3+(1)^2-4(1)-4$

$P(r)=1+1-4-4$

$R=2-8$

$R=-6\neq0$

Now take r = - 1

$P(1)=(-1)^3+(-1)^2-4(-1)-4$

$P(r)=-1+1+4-4$

$R=-5+5$

$R=0$

Since $P(-1)=0$, therefore by factor Theorem $x-r=x-(-1)=x+1$ is a factor of $P(x)$.

To find other factors divide $P(x)$ by $x-1$ as under

$$\begin{array}{r|l} & x^2-4 \\ \hline x+1 & x^3+x^2-4x-4 \\ & \underline{\pm x^3 \pm x^2} \\ & \qquad -4x-4 \\ & \qquad \underline{\mp 4x \mp 4} \\ & \qquad\quad 0 \end{array}$$

Here $Q(x)=x^2-4$ and $R=0$

Since $P(x)=(x-r)Q(x)+R$

Putting the values we get

$$\begin{aligned} x^3+x^2-4x-4&=(x+1)\{x^2-4\} \\ &=(x+1)\{x^2-2^2\} \\ &=(x+1)(x+2)(x-2) \end{aligned}$$

iii). x^3-7x+6

Sol: Given $P(x)=x^3-7x+6$

Using factor theorem to find other factors we take $P(r)=0$, where

$r=\pm1,\pm2,\pm3,...$ Take r = 1

$P(1)=(1)^3-7(1)+6$

$P(r)=1-7+6$

$R=7-7$

$R=0$

$\therefore$ $P(1)=0$, therefore by factor theorem $x-1$ is a factor of $P(x)$. To find other factors divide $P(x)$ by $x-1$ as under

$$\begin{array}{r|l} & x^2+x-6 \\ \hline x-1 & x^3+0.x^2-7x+6 \\ & \underline{\pm x^3 \mp x^2} \\ & \qquad x^2-7x \\ & \qquad \underline{\pm x^2 \mp x} \\ & \qquad\quad -6x+6 \\ & \qquad\quad \underline{-6x+6} \\ & \qquad\qquad 0 \end{array}$$

Here $Q(x)=x^2+x-6$ and $R=0$

Since $P(x)=(x-r)Q(x)+R$

Putting the values we get

$$\begin{aligned} x^3-7x+6&=(x-1)\{x^2+x-6\} \\ &=(x-1)\{x^2+3x-2x-6\} \\ &=(x-1)\{x(x+3)-2(x+3)\} \\ &=(x-1)(x-2)(x+3) \end{aligned}$$

iv). $x^3-9x^2+23x-15$

Sol: Given $P(x)=x^3-9x^2+23x-15$

Using factor theorem to find the other factors we take $P(r)=0$, where

$r=\pm1,\pm2,\pm3,...$

Take r = 1

$P(1)=(1)^3-9(1)^2+23(1)-15$
$P(r)=1-9+23-15$
$R=1+23-9-15$
$R=24-24=0$

$\therefore$ $P(1)=0$, therefore by factor theorem $x-1$ is a factor of $P(x)$. To find the other factors divide $P(x)$ by $x-1$ as under

$$\begin{array}{r|l} & x^2-8x+15 \\ x-1 & x^3-9x^2+23x-15 \\ & \underline{\pm x^3 \mp x^2} \\ & \quad -8x^2+23x \\ & \quad \underline{\mp 8x^2 \pm 8x} \\ & \qquad\quad 15x-15 \\ & \qquad\quad \underline{\pm 15x \mp 15} \\ & \qquad\qquad 0 \end{array}$$

Here $Q(x)=x^2-x-6$ and $R=0$

Since $P(x)=(x-r)Q(x)+R$

Putting the values we get

$$\begin{aligned} x^3-9x^2+23x-15 &= (x-1)\{x^2-8x+15\} \\ &= (x-1)\{x^2-5x-3x+15\} \\ &= (x-1)\{x(x-5)-3(x-5)\} \\ &= (x-1)(x-3)(x-5) \end{aligned}$$

v). $x^3-4x^2-3x+18$

Sol: Given $P(x)=x^3-4x^2-3x+18$

Using factor theorem to find other factors

we take $P(r)=0$, where $r=\pm1,\pm2,\pm3,\ldots$

Take r = 1

$P(1)=(1)^3-4(1)^2-3(1)+18$
$P(r)=1-4-3+18$
$R=1+18-4-3$
$R=19-7=12\neq 0$

Take r = - 1

$P(1)=(-1)^3-4(-1)^2-3(-1)+18$
$P(r)=-1-4+3+18$
$R=-5+21=16\neq 0$

Take r = 2

$P(1)=(2)^3-4(2)^2-3(2)+18$
$P(r)=8-16-6+18$
$R=8+18-16-6$
$R=26-22=4\neq 0$

Take r = - 2

$P(1)=(-2)^3-4(-2)^2-3(-2)+18$
$P(r)=-8-16+6+18$
$R=-24+24=0$

Since $P(-2)=0$, therefore by factor theorem $x-r=x-(-2)=x+2$ is a factor of $P(x)$. To find the other factors divide $P(x)$ by $x+2$ as under

$$\begin{array}{r|l} & x^2-6x+9 \\ x+2 & x^3-4x^2-3x+18 \\ & \underline{\pm x^3 \pm 2x^2} \\ & \quad -6x^2-3x \\ & \quad \underline{\mp 6x^2 \mp 12x} \\ & \qquad\quad 9x+18 \\ & \qquad\quad \underline{\pm 9x \pm 18} \\ & \qquad\qquad 0 \end{array}$$

Here $Q(x)=x^2-6x+9$ and $R=0$

Since $P(x)=(x-r)Q(x)+R$

Putting the values we get

$$\begin{aligned} x^3-4x^2-3x+18 &= (x+2)\{x^2-6x+9\} \\ &= (x+2)\{x^2-2.x.3+3^2\} \\ &= (x+2)(x-3)^2 \end{aligned}$$

vi). $x^3+2x^2-19x-20$

Sol: Given $P(x)=x^3+2x^2-19x-20$

Using factor theorem to find other factors we take $P(r)=0$, where $r=\pm1,\pm2,\pm3,\ldots$

Take r = 1

$P(1)=(1)^3+2(1)^2-19(1)-20$
$P(r)=1+2-19-20$
$R=3-39$
$R=-36\neq 0$

Take r = - 1

$P(1)=(-1)^3+2(-1)^2-19(-1)-20$
$P(r)=-1+2+19-20$
$R=-21+21$
$R=0$

Since $P(-1)=0$, therefore by factor theorem $x+1$ is a factor of $P(x)$. To find other factors divide $P(x)$ by $x+1$ as

$$\begin{array}{r|l} & x^2+x-20 \\ x+1 & x^3+2x^2-19x-20 \\ & \underline{\pm x^3 \pm x^2} \\ & \quad x^2-19x \\ & \quad \underline{\pm x^2 \pm x} \\ & \qquad\quad -20x-20 \\ & \qquad\quad \underline{\mp 20x \mp 20} \\ & \qquad\qquad 0 \end{array}$$

Here $Q(x)=x^2+x-20$ and $R=0$

Since $P(x)=(x-r)Q(x)+R$

Putting the values we get

$$x^3+2x^2-19x-20=(x+1)\{x^2+x-20\}$$
$$=(x+1)\{x^2+5x-4x-20\}$$
$$=(x+1)\{x(x+5)-4(x+5)\}$$
$$=(x+1)(x-4)(x+5)$$

vii). $x^3-x^2-14x+24$

Sol: Given $P(x)=x^3-x^2-14x+24$

Using factor theorem to find other factors we take $P(r)=0$, where

$r=\pm1,\pm2,\pm3,\ldots$

Take r = 1

$P(1)=(1)^3-(1)^2-14(1)+24$

$P(r)=1-1-14+24$

$R=10\neq 0$

Take r = - 1

$P(1)=(-1)^3-(-1)^2-14(-1)+24$

$P(r)=-1-1+14+24$

$R=-2+38=36\neq 0$

Take r = 2

$P(2)=(2)^3-(2)^2-14(2)+24$

$P(r)=8-4-28+24$

$R=8+24-4-28$

$R=32-32=0$

Since $P(2)=0$, therefore by factor theorem $x-r=x-2$ is a factor of $P(x)$. To find the other factors divide $P(x)$ by $x-2$ as under

$$\begin{array}{r|l}
 & x^2+x-12 \\
\hline
x-2 & x^3-x^2-14x+24 \\
 & \underline{\pm x^3 \mp 2x^2} \\
 & \quad x^2-14x \\
 & \quad \underline{\pm x^2 \mp 2x} \\
 & \qquad -12x+24 \\
 & \qquad \underline{\mp 12x \pm 24} \\
 & \qquad\qquad 0
\end{array}$$

Here $Q(x)=x^2+x-12$ and $R=0$

Since $P(x)=(x-r)Q(x)+R$

Putting the values we get

$$x^3-x^2-14x+24=(x-2)\{x^2+x-12\}$$
$$=(x-2)\{x^2+4x-3x-12\}$$
$$=(x-2)\{x(x+4)-3(x+4)\}$$
$$=(x-2)(x-3)(x+4)$$

viii). x^3-6x^2+32

Sol: Given $P(x)=x^3-6x^2+32$

Using factor theorem to find the other factors we take $P(r)=0$, where

$r=\pm1,\pm2,\pm3,\ldots$

Take r = 1

$P(1)=(1)^3-6(1)^2+32$

$P(r)=1-6+32$

$R=-5+32=27\neq 0$

Take r = - 1

$P(1)=(-1)^3-6(-1)^2+32$

$P(r)=-1-6+32$

$R=-7+32=25\neq 0$

Take r = 2

$P(2)=(2)^3-6(2)^2+32$

$P(r)=8-24+32$

$R=8+8=16\neq 0$

Take r = - 2

$P(-2)=(-2)^3-6(-2)^2+32$

$P(r)=-8-24+32$

$R=-32+32=0$

Since $P(-2)=0$, therefore by factor theorem $x-r=x-(-2)=x+2$ is a factor of $P(x)$. To find the other factors divide $P(x)$ by $x+2$ as under

$$\begin{array}{r|l}
 & x^2-8x+16 \\
\hline
x+2 & x^3-6x^2+0.x+32 \\
 & \underline{\pm x^3 \pm 2x^2} \\
 & \quad -8x^2+0.x \\
 & \quad \underline{\mp 8x^2 \mp 16x} \\
 & \qquad 16x+32 \\
 & \qquad \underline{\pm 16x \pm 32} \\
 & \qquad\qquad 0
\end{array}$$

Here $Q(x)=x^2-8x+16$ and $R=0$

Since $P(x)=(x-r)Q(x)+R$

Putting the values we get

$$x^3-6x^2+32=(x+2)\{x^2-8x+16\}$$
$$=(x+2)\{x^2-2.x.4+4^2\}$$
$$=(x+2)(x-4)^2$$

Review Exercise 5

Q1. Ture and false questions.

Read the following sentences carefully

i).	$x^2-11x+30=(x-6)(x-5)$	T
ii).	$x^3+27=(x+3)(x^2-3x+9)$	T
iii).	$x^3-8=(x-2)(x^2+2x+4)$	T
iv).	$x^4-y^4=(x-y)(x+y)(x+y)^2$	F
v).	$x^6+y^6=(x^3+y^3)(x^3-y^3)$	F
vi).	$x^5+y^5=(x+y)^5$	F

Q2. Complete the following sentences

i). $9x^2-16y^4=(3x-4y^2)(\underline{3x+4y^2})$

ii). $x^3-27y^3=(x-3y)(\underline{x^2+3xy+9y^2})$

iii). $x^2-5x+6=(x-2)(\underline{x-3})$

iv). $a^2+b^2=(a-b)^2+\underline{2ab}$

v). $8b^3+c^3=(2b+c)\left(\underline{4b^2-2bc+c^2}\right)$

Q3. Choose the correct answer

i). Factors of $x^2+2x-24$ are

a). $x+4, x-6$ b). $x-4, x+6$

c). $x+3, x-8$ d). $x+8, x-3$

ii). Factorization of $x^2+2xy+y^2-z^2$ is

a). $(x-y+z)(x-y-z)$ b). $(x+y-z)(x-y-z)$

c). $(x+y+z)(x+y-z)$ d). $(x+y+z)(x-y-z)$

iii). Factorization of a^3+b^3 is

a). $(a-b)(a^2+ab+b^2)$ b). $(a+b)(a^2+ab+b^2)$

c). $(a+b)(a^2+ab-b^2)$ d). $(a+b)(a^2-ab+b^2)$

iv). Factors of $8y^3-z^3$ are

a). $2y-z, 4y^2+2yz+z^2$ b). $2y-z, 2y-z, 2y-z$

c). $2y-z, 4y^2-2yz+z^2$ d). $2y-z, 4y^2-2yz-z^2$

v). In simplified form $\frac{1}{a+b}+\frac{b}{a^2-b^2}=......$

a). $\frac{b+1}{a^2-b^2}$ b). $\frac{a}{a^2-b^2}$

c). $\frac{b}{a^2-b^2}$ d). $\frac{b+a}{a^2-b^2}$

vi). Factorization of $a^2-b^2+10b-25$ is

a). $(a-b+5)(a-b+5)$

b). $(a+b-5)(a-b+5)$

c). $(a+b-5)(a+b-5)$

d). $(a+b-5)(a-b-5)$

vii). Completely factorize $16x^4-81$

a). $(2x-3)(2x+3)(4x^2+9)$

b). $(4x^2-9)(4x^2+9)$

c). $(2x-3)(2x+3)(2x-3)$

d). $(4x^2-9)(2x-3)(2x+3)$

Q4: i). Factorize $3x^3-3x^2-18x$

Solution: Given $3x^3-3x^2-18x$

$$3x^3-3x^2-18x$$
$$=3x(x^2-x-6)$$
$$=3x(x^2-3x+2x-6)$$
$$=3x\{x(x-3)+2(x-3)\}$$
$$=3x(x+2)(x-3)$$

Q4: ii). Factorize $64x^3+27$

Solution: Given $64x^3+27$

$$=(4x)^3+(3)^3$$
$$=(4x+3)\left\{(4x)^2-(4x)(3)+(3)^2\right\}$$
$$=(4x+3)(16x^2-12x+9)$$

Q4: iii). Factorize x^6-y^6

Solution: Given x^6-y^6

$$=(x^3)^2-(y^3)^2$$
$$=(x^3+y^3)(x^3-y^3)$$
$$=(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$$
$$=(x+y)(x-y)(x^2-xy+y^2)(x^2+xy+y^2)$$

Q4: iv). Factorize $343a^3-125b^3+5b-7a$

Solution: Given $343a^3-125b^3+5b-7a$

$$=343a^3-125b^3-7a+5b$$
$$=(7a)^3-(5b)^3-1(7a-5b)$$

Using $a^3-b^3=(a-b)(a^2+ab+b^2)$

$$=(7a)^3-(5b)^3-1(7a-5b)$$
$$=(7a-5b)\left\{(7a)^2+(7a)(5b)+(5b)^2\right\}-1(7a-5b)$$
$$=(7a-5b)\left\{49a^2+35ab+25b^2\right\}-1(7a-5b)$$
$$=(7a-5b)\left\{49a^2+35ab+25b^2-1\right\}$$

Q5. Factorize $(x+1)(x+2)(x+3)(x+4)-120$

Sol: Given $(x+1)(x+2)(x+3)(x+4)-120$

Here $1+4=2+3$, so rearranging

$$=(x+1)(x+4)(x+2)(x+3)-120$$
$$=(x^2+4x+1x+4)(x^2+3x+2x+6)-120$$
$$=(x^2+5x+4)(x^2+5x+6)-120$$

Here two terms are same Let $y=x^2+5x$

$$=(y+4)(y+6)-120$$
$$=y^2+6y+4y+24-120$$
$$=y^2+10y-96$$
$$=y^2+16y-6y-96$$
$$=y(y+16)-6(y+16)$$
$$=(y-6)(y+16)$$

Putting back value of $y=x^2+5x$

$$=(x^2+5x-6)(x^2+5x+16)$$
$$=(x^2+6x-1x-6)(x^2+5x+16)$$
$$=\{x(x+6)-1(x+6)\}(x^2+5x+16)$$
$$=(x-1)(x+6)(x^2+5x+16)$$

Q6: i). Using factor theorem factorize $x^3-17x+26$

Solution: Given $P(x)=x^3-17x+26$

Put $x=2$
$$P(2)=(2)^3-17(2)+26$$
$$P(2)=8-34+26$$
$$P(2)=34-34$$
$$P(2)=0$$

Hence $x-2$ is a factor so

$$
\begin{array}{r|l}
 & x^2+2x-13 \\
x-2 & x^3 \quad -17x+26 \\
 & \underline{\pm x^3 \mp 2x^2} \\
 & 2x^2-17x \\
 & \underline{\pm 2x^2 \mp 4x} \\
 & -13x+26 \\
 & \underline{\mp 13x \pm 26} \\
 & R=0
\end{array}
$$

Here $Q(x)=x^2+2x-13$ and $R=0$

Since $P(x)=(x-k)Q(x)+R$

$x^3-17x+26=(x-2)(x^2+2x-13)+0$

$=(x+2)(x^2+2x-13)$

Q6: ii). Using factor theorem factorize
$x^3-39x^2-124x-84$

Solution: Given $x^3-39x^2-124x-84$

Using factor theorem to find the other factors we take $P(r)=0$, where $r=\pm1,\pm2,\pm3,\ldots$

Take r = - 1

$x^3-39x^2-124x-84$

$P(-1)=(-1)^3-39(-1)^2-124(-1)-84$

$P(r)=-1-39+124-84$

$R=-40+40=0$

Since $P(-1)=0$, therefore by factor theorem $x-r=x-(-1)=x+1$ is a factor of $P(x)$. To find the other factors divide $P(x)$ by $x+1$ as under

$$
\begin{array}{r|l}
 & x^2-40x-84 \\
x+1 & x^3-39x^2-124x-84 \\
 & \underline{\pm x^3 \pm \quad x^2} \\
 & -40x^2-124x \\
 & \underline{\mp 40x^2 \mp 40x} \\
 & -84x-84 \\
 & \underline{\mp 84x \mp 84} \\
 & 0
\end{array}
$$

Here $Q(x)=x^2-40x-84$ and $R=0$

Since $P(x)=(x-r)Q(x)+R$

Putting the values we get

$x^3-39x^2-124x-84=(x+1)\{x^2-40x-84\}$

$=(x+1)\{x^2-42x+2x-84\}$

$=(x+1)\{x(x-42)+2(x-42)\}$

$=(x+1)(x+2)(x-42)$

Q7. Factorize $a^4+a^2b^2+b^4$

Sol: Given $a^4+a^2b^2+b^4$

$=a^4+b^4+a^2b^2$

$=(a^2)^2+(b^2)^2+2(a^2)(b^2)-2(a^2)+(b^2)+a^2b^2$

$=(a^2+b^2)^2-2a^2b^2+a^2b^2$

$=(a^2+b^2)^2-a^2b^2$

$=(a^2+b^2)^2-(ab)^2$

$=(a^2+b^2+ab)(a^2+b^2-ab)$

Q8. By using long division method, find the quotient and the remainder when
$2x^3+4x^2-6$ is divided by $x+3$

Solution: Given dividend $2x^3+4x^2-6$

Divisor $x+3$

$$
\begin{array}{r|l}
 & 2x^2-2x+6 \\
x+3 & 2x^3+4x^2 \quad -6 \\
 & \underline{\pm 2x^3 \pm 6x^2} \\
 & -2x^2 \quad -6 \\
 & \underline{\mp 2x^2 \mp 6x} \\
 & +6x-6 \\
 & \underline{\pm 6x \pm 18} \\
 & R=-24
\end{array}
$$

Here Q(x) = $2x^2-2x+6$ and R = -24

Q9: i. Find the remainder when
x^3+2x^2-3x+1 is divided by $x-1$

Solution: Given $P(x)=x^3+2x^2-3x+1$

And divisor $x-1=0 \quad \Rightarrow x=1$

$P(1)=(1)^3+2(1)^2-3(1)+1$

$P(1)=1+2-3+1$

$P(1)=1$

Hence remainder R = 1

Q9: i. Find the remainder when
x^3+2x^2-3x+1 is divided by $x+2$

Solution: Given $P(x)=x^3+2x^2-3x+1$

And divisor $x+2=0 \quad \Rightarrow x=-2$

$P(-2)=(-2)^3+2(-2)^2-3(-2)+1$

$P(-2)=-8+8+6+1$

$P(-2)=7$

Hence remainder R = 7

Q10. By using remainder theorem find remainder when $x^{72}-7x^{48}+1$ is divided by $x+1$

Solution: Given $P(x)=x^{72}-7x^{48}+1$

And divisor $x+1=0 \quad \Rightarrow x=-1$

$P(-1)=(-1)^{72}-7(-1)^{48}+1$

$P(-1)=1-7+1$

$P(-1)=-5$

Hence remainder R = - 5