Chapter 4

Variable and Constant:
A **variable** is a symbol which represents the elements of a non-empty set, for example, in $A=\{x|x\in N, x\le 10\}$, x is a natural number from 1 to 10. As the value of x is subject to change, it is called a **variable**. The value of natural number remains unchanged, so is called **constant**.

Co-efficient and exponent:
A constant number when multiplied with only a variable term is called co-efficient of that variable. For example, in $5x^3$, 5 is called co-efficient of x^3, and 3 is called the exponent or index of the variable.

Variable: A symbol which represents the elements of a non-empty set.

Algebraic expression: An expression which consists of variable and constants connected by algebraic operations of addition, subtraction, multiplication, division, or root extraction or rising integral or fractional powers, is called an **algebraic expression**.

Degree of an Algebraic expression: The largest exponent of the variable involved in an expression is called Degree of the expression

Term: Different parts of an algebraic expressions joined by the operations of addition or subtraction is called **terms** of an algebraic expiration.

Compound Expression: An expression consisting of two or more terms is called compound expression.

Kinds of Algebraic Expressions:

Polynomial expression:
An expression of the form $P(x)=a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\cdots+a_{n-1}x^1+a_n$, where n is positive integer or zero, $a_0\neq 0$ and the co-efficients $a_0, a_1, a_2, \ldots a_{n-1}, a_n$ are real numbers, this is called a **polynomial of degree n**.

Rational expression: an expression of the form $\frac{P(x)}{Q(x)}$, where P(x) and Q(x) are polynomials and $Q(x)\neq 0$ is called **Rational expression**.

Irrational Expression.
Similarly, an expression which cannot be written in the form $\frac{P(x)}{Q(x)}$, where P(x) and Q(x) are polynomials and $Q(x)\neq 0$ is called **Irrational expression**.

Polynomial with respect to variable:
Polynomial in **one variable** is written as $P(x)=ax^2+bx+c$ or $P(y)=ay^2+by=c$.
Polynomial in **two variables** x and y is written as $P(x,y)=3x+4y+8$.
Polynomial in **three variables** x, y and z is written as $P(x,y,z)=4x+5y+7z$.

Algebraic Fraction: If A and B are two algebraic expressions, then $\frac{A}{B}$ where $B\neq 0$ is called an algebraic fraction.

Example: Examine whether the following rational expressions are in their lowest terms or not.

i). $\frac{9}{x-3}$

Answer: Given that $\frac{9}{x-3}$

In given fraction no same factor is common in the numerator and denominator. So given fraction is in its lowest term.

ii). $\frac{x^2-7x}{x+4}$

Answer: Given that $\frac{x^2-7x}{x+4}$

In given fraction no same factor is common in the numerator and denominator. So given fraction is in its lowest term.

iii). $\frac{x^2-1}{x-1}$

Answer: Given that $\frac{x^2-1}{x-1}$

In given fraction same factor is common in the numerator and denominator. So
$\frac{x^2-1}{x-1}=\frac{(x-1)(x+1)}{x-1}$
So given fraction is not its lowest term.

Example: Reduce the following rational expressions to their lowest terms.

i). $\frac{x+3}{(x+3)(x-2)}$

Solution: Given that $\frac{x+3}{(x+3)(x-2)}$

$=\frac{1}{x-2}$ by cancelling same factor

ii). $\frac{x^2-b^2}{(x+b)x^2}$

Solution: Given that $\frac{x^2-b^2}{(x+b)x^2}$

$=\frac{(x+b)(x-b)}{(x+b)x^2}$

$=\frac{x-b}{x^2}$ by cancelling Same factor

Example: Find the sum and difference of rational expressions.

i). Add $\frac{x+2}{x+3}$ & $\frac{x+5}{x+3}$

Solution: Given $\frac{x+2}{x+3}$ & $\frac{x+5}{x+3}$

$$\frac{x+2}{x+3}+\frac{x+5}{x+3}=\frac{x+2+x+5}{x+3}$$

$$=\frac{2x+7}{x+3}$$

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ii). Add $\frac{4}{x-3}$ & $\frac{6}{x+3}$

Solution: Given $\frac{4}{x-3}$ & $\frac{6}{x+3}$

$$\frac{4}{x-3}+\frac{6}{x+3}=\frac{x+3}{x+3}\times\frac{4}{x-3}+\frac{6}{x+3}\times\frac{x-3}{x-3}$$

$$=\frac{4(x+3)+6(x-3)}{(x+3)(x-3)}$$

$$=\frac{4x+12+6x-18}{x^2-3^2}$$

$$=\frac{10x-6}{x^2-9}$$

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iii). Subtract $\frac{2}{(x-2)(x-3)}$ From $\frac{x}{(x+3)(x-2)}$

Solution: Given to subtract

$$\frac{x}{(x+3)(x-2)}-\frac{2}{(x-2)(x-3)}$$

$$=\frac{x-3}{x-3}\times\frac{x}{(x+3)(x-2)}-\frac{2}{(x-2)(x-3)}\times\frac{x+3}{x+3}$$

$$=\frac{x(x-3)-2(x+3)}{(x-3)(x+3)(x-2)}$$

$$=\frac{x^2-3x-2x-6}{(x^2-3^2)(x-2)}$$

$$=\frac{x^2-5x-6}{(x^2-9)(x-2)}$$

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iv). $\frac{1}{x+y}+\frac{1}{x-y}-\frac{1}{(x+y)(x-y)}$

Solution: Given $\frac{1}{x+y}+\frac{1}{x-y}-\frac{1}{(x+y)(x-y)}$

$$=\frac{x-y}{x-y}\times\frac{1}{x+y}+\frac{1}{x-y}\times\frac{x+y}{x+y}-\frac{1}{(x+y)(x-y)}$$

$$=\frac{x-y+x+y-1}{(x+y)(x-y)}$$

$$=\frac{2x-1}{x^2-y^2}$$

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v). Simplify $\frac{x}{x+2}\cdot\frac{x+3}{x-3}$

Solution: Given $\frac{x}{x+2}\cdot\frac{x+3}{x-3}$

$$=\frac{x(x+3)}{(x+2)(x-3)}$$

$$=\frac{x^2+3x}{x(x-3)+2(x-3)}$$

$$=\frac{x^2+3x}{x^2-3x+2x-6}$$

$$=\frac{x^2+3x}{x^2-x-6}$$

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vi). Multiply $\frac{m-2}{3m+9}$ & $\frac{2m+6}{2m-4}$

Solution: given $\frac{m-2}{3m+9}\times\frac{2m+6}{2m-4}$

$$=\frac{m-2}{3(m+3)}\times\frac{2(m+3)}{2(m-2)}$$

$=\frac{1}{3}$ by cancelling same factor

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vii). Divide $\frac{x^2+x-2}{3x^2+9x+6}\div(x-1)$

Solution: Given $\frac{x^2+x-2}{3x^2+9x+6}\div(x-1)$

$$=\frac{x^2+2x-x-2}{3(x^2+3x+2)}\times\frac{1}{x-1}$$

$$=\frac{x(x+2)-1(x+2)}{3(x^2+2x+1x+2)}\times\frac{1}{x-1}$$

$$=\frac{(x-1)(x+2)}{3(x(x+2)+1(x+2))}\times\frac{1}{x-1}$$

$$=\frac{(x-1)(x+2)}{3(x+1)(x+2)}\times\frac{1}{x-1}$$

$=\frac{1}{3(x+1)}$ by cancelling same factor

Exercise 4.1

Q1. Find which of following expressions are polynomials?

i). $1-5y+8y^2+6y^3$

Answer: Polynomial

ii). $\frac{5}{x^2}+\frac{3}{4x+1}$

Answer: Not Polynomial but Rational

iii). $\frac{\sqrt{x}}{6x-1}$

Answer: Not a Polynomial but irrational expression

Q2. Which of the following expressions are in lowest terms;

i). $\frac{5y^2-5}{y-1}$

Answer: Not in lowest term

$$\frac{5y^2-5}{y-1}=\frac{5(y^2-1)}{y-1}$$
$$=\frac{5(y-1)(y+1)}{y-1}$$
$$=5(y+1)$$

ii). $\dfrac{x^2-9}{x-2}$

Answer: lowest term

$$\frac{x^2-9}{x-2}=\frac{x^2-3^2}{x-2}$$
$$=\frac{(x+3)(x-3)}{x-2}$$ not having same factors

iii). $\dfrac{x+y}{x^2-y^2}$

Answer: Not in lowest term

$$\frac{x+y}{x^2-y^2}=\frac{x+y}{(x+y)(x-y)}$$
$$=\frac{1}{x-y}$$ by cancelling same factor

Q3: Reduced the following rational expressions to their lowest term:

i). $\dfrac{x-5}{x^2-5x}$

Solution: Given $\dfrac{x-5}{x^2-5x}$

$$=\frac{x-5}{x(x-5)}$$
$$=\frac{1}{x}$$ by cancelling same factor

ii). $\dfrac{t^3(t-3)}{(t-3)(t+5)}$

Solution: Given $\dfrac{t^3(t-3)}{(t-3)(t+5)}$

$$=\frac{t^3}{t+5}$$ by cancelling same factor

iii). $\dfrac{x^4+\frac{1}{x^4}}{x^2-\frac{1}{x^2}}$

Solution: Given $\dfrac{x^4+\frac{1}{x^4}}{x^2-\frac{1}{x^2}}$

Can not be reduce further

iv). $\dfrac{2a+6}{a^2-9}$

Solution: Given $\dfrac{2a+6}{a^2-9}$

$$=\frac{2(a+3)}{a^2-3^2}$$
$$=\frac{2(a+3)}{(a+3)(a-3)}$$
$$=\frac{2}{a-3}$$ by cancelling same factor

Q4. Add following rational expressions.

i). $4x^2-5x-10, 2x^2+5x+10$

Sol: Given $(4x^2-5x-10)+(2x^2+5x+10)$

$$=4x^2+2x^2-5x+5x-10+10$$
$$=6x^2$$

ii). $\dfrac{y+9}{y^2+3}, \dfrac{-7y+7}{y^2+3}$

Sol: Given $\dfrac{y+9}{y^2+3}+\dfrac{-7y+7}{y^2+3}$

$$=\frac{y+9-7y+7}{y^2+3}$$
$$=\frac{y-7y+9+4}{y^2+3}$$
$$=\frac{-6y+13}{y^2+3}$$

iii). $\dfrac{y}{y+4}, \dfrac{2y}{y-4}$

Solution: Given $\dfrac{y}{y+4}+\dfrac{2y}{y-4}$

$$=\frac{y-4}{y-4}\times\frac{y}{y+4}+\frac{2y}{y-4}\times\frac{y+4}{y+4}$$
$$=\frac{y(y-4)+2y(y+4)}{y^2-4^2}$$
$$=\frac{y^2-4y+2y^2+8y}{y^2-16}$$
$$=\frac{3y^2+4y}{y^2-16}$$

iv). $\dfrac{t}{t^2-25}, \dfrac{3t}{t+5}$

Solution: Given $\dfrac{t}{t^2-25}+\dfrac{3t}{t+5}$

$$=\frac{t}{t^2-25}+\frac{3t}{t+5}\times\frac{t-5}{t-5}$$
$$=\frac{t+3t(t-5)}{t^2-25}$$
$$=\frac{t+3t^2-15t}{t^2-25}$$
$$=\frac{3t^2-14t}{t^2-25}$$

Q5. Subtract the first expression from the second in the following:

i). $y^2+4y-15, 8y^2+2$

Solution: Given $(8y^2+2)-(y^2+4y-15)$

$$=8y^2+2-y^2-4y+15$$
$$=8y^2-y^2-4y+15+2$$
$$=7y^2-4y+17$$

ii). $\dfrac{8x^2-7}{x^2+1},\dfrac{8x^2+7}{x^2+1}$

Solution: Given $\dfrac{8x^2+7}{x^2+1}-\dfrac{8x^2-7}{x^2+1}$

$$=\frac{8x^2+7-8x^2+7}{x^2+1}$$
$$=\frac{14}{x^2+1}$$

iii). $\dfrac{1}{a-3},\dfrac{2a}{a^2-9}$

Solution: Given $\dfrac{2a}{a^2-9}-\dfrac{1}{a-3}$

$$=\frac{2a}{a^2-3^3}-\frac{1}{a-3}\times\frac{a+3}{a+3}$$
$$=\frac{2a-(a+3)}{a^2-3^2}$$
$$=\frac{2a-a-3}{a^2-3^2}$$
$$=\frac{a-3}{(a-3)(a+3)}$$
$$=\frac{1}{a+3}$$

iv). $\dfrac{x}{3x-6},\dfrac{x+2}{x-2}$

Solution: Given $\dfrac{x+2}{x-2}-\dfrac{x}{3x-6}$

$$=\frac{x+2}{x-2}-\frac{x}{3(x-2)}$$
$$=\frac{3(x+2)-x}{3(x-2)}$$
$$=\frac{3x+6-x}{3(x-2)}$$
$$=\frac{2x+6}{3(x-2)}$$
$$=\frac{2(x+3)}{3(x-2)}$$

Q6. Simplify

i). $\dfrac{2x}{6x-9}\cdot\dfrac{4x-6}{x^2+x}$

Solution: Given $\dfrac{2x}{6x-9}\cdot\dfrac{4x-6}{x^2+x}$

$$=\frac{2x}{3(2x-3)}\cdot\frac{2(2x-3)}{x(x+1)}$$
$$=\frac{4}{3(x+1)}$$

ii). $\dfrac{x+4}{3-x}\cdot\dfrac{x^2-9}{x^2-16}$

Solution: Given $\dfrac{x+4}{3-x}\cdot\dfrac{x^2-9}{x^2-16}$

$$=\frac{x+4}{-x+3}\cdot\frac{x^2-3^2}{x^2-4^2}$$
$$=\frac{x+4}{-(x-3)}\cdot\frac{(x-3)(x+3)}{(x+4)(x-4)}$$
$$=\frac{x+3}{-(x-4)}=\frac{x+3}{-x+4}$$
$$=\frac{x+3}{4-x}$$

iii). $\dfrac{3x-15}{2x+6}\times\dfrac{x^2-9}{x^2-25}$

Solution: Given $\dfrac{3x-15}{2x+6}\times\dfrac{x^2-9}{x^2-25}$

$$=\frac{3(x-5)}{2(x+3)}\times\frac{x^2-3^2}{x^2-5^2}$$
$$=\frac{3(x-5)}{2(x+3)}\times\frac{(x+3)(x-3)}{(x+5)(x-5)}$$
$$=\frac{3(x-3)}{2(x+5)}$$

Q7. Simplify the following

i). $\dfrac{2y-10}{3y}\div(y-5)$

Solution: Given $\dfrac{2y-10}{3y}\div(y-5)$

$$=\frac{2(y-5)}{3y}\times\frac{1}{y-5}$$
$$=\frac{2}{3y}$$

ii). $\dfrac{p}{q}\div\dfrac{r}{q}\cdot\dfrac{p}{q}$

Solution: Given $\dfrac{p}{q}\div\dfrac{r}{q}\cdot\dfrac{p}{q}$

$$=\frac{p}{q}\times\frac{q}{r}\cdot\frac{p}{q}$$
$$=\frac{p^2}{qr}$$

iii). $\dfrac{a^2-9}{(a-6)(a+4)}\div\dfrac{a-3}{a-6}$

Solution: Given $\dfrac{a^2-9}{(a-6)(a+4)}\div\dfrac{a-3}{a-6}$

$$=\frac{a^2-3^2}{(a-6)(a+4)}\times\frac{a-6}{a-3}$$
$$=\frac{(a-3)(a+3)}{(a-6)(a+4)}\times\frac{a-6}{a-3}$$
$$=\frac{a+3}{a+4}$$

Value of an algebraic expression at particular real number

The value of the polynomial $P(x)$ for any real number can be obtained by just replacing x of $P(x)$ by that real number.

Example: When $x=-3$ find the value of

i). $2x^2-3x$

Sol: Given $2x^2-3x$

Putting the value of $x=-3$ we get

$$=2(-3)^2-3(-3)$$
$$=2(9)+9$$
$$=18+9$$
$$=27$$

ii). $(3x)^2-x^2$

Sol: Given $(3x)^2-x^2$

Putting the value of $x=-3$ we get

$$=[3(-3)]^2-(-3)^2$$
$$=[-9]^2-9$$
$$=81-9$$
$$=72$$

Example: when $x=2, y=-3$ find the of

i). x^2-y^2

Sol: Given x^2-y^2

Putting the value of $x=2, y=-3$ we get

$$=(2)^2-(-3)^2$$
$$=4-9$$
$$=-5$$

ii). $(2x)^2-(3y)^2$

Sol: Given $(2x)^2-(3y)^2$

Putting the value of $x=2, y=-3$ we get

$$=[2(2)]^2-[3(-3)]^2$$
$$=(4)^2-(-9)^2$$
$$=16-81$$
$$=-65$$

Exercise No 4.2

Q1. Evaluate the following when a=3,b=-1,c=2

i). 5a-10

Solution: 5a-10= 5(5)-10
=15-10
=5

ii). 3b+5c

Solution: 3b+5c = 3(-1)+5(2)
= - 3+10
= 7

iii). 2a-3b+2c

Solution: 2a-3b+2c=2(3)-3(-1)+2(2)
= 6 + 3 + 4
= 13

Q2. Evaluate following when x=-5, y=2

i). 7-3xy

Solution: 7-3xy = 7-3(-5)(2)
=7 + 30
=37

ii). x^2+xy+y^2

Solution: x^2+xy+y^2

$$=(-5)^2+(-5)(2)+(2)^2$$
$$==25-10+4$$
$$=19$$

iii). $[3x]^2-[4y]^2$

Solution: Given $[3x]^2-[4y]^2$

$$=[3(-5)]^2-[4(2)]^2$$
$$=[-15]^2-[8]^2$$
$$=225-64$$
$$=161$$

Q3. Evaluate the following when $k=-2, l=3, m=4$

i). $k^2(2l-3m)$

Solution: $k^2(2l-3m)=(-2)^2(2(3)-3(4))$

$$=4(6-12)$$
$$=4(-6)$$
$$=-24$$

ii). $5m\sqrt{k^2+l^2}$

Solution: $5m\sqrt{k^2+l^2}=5(4)\sqrt{(-2)^2+(3)^2}$

$$=20\sqrt{4+9}=20\sqrt{13}$$

iii). $\dfrac{k+l+m}{k^2+l^2+m^2}$

Solution: $\dfrac{k+l+m}{k^2+l^2+m^2}=\dfrac{(-2)+(3)+(4)}{(-2)^2+(3)^2+(4)^2}$

$$=\frac{-2+3+4}{4+9+16}$$
$$=\frac{5}{29}$$

Q4i). Evaluate $\dfrac{a+1}{4a^2+1}$ when $a=\dfrac{1}{2}$

Sol: Given $\dfrac{a+1}{4a^2+1}$ putting $a=\dfrac{1}{2}$ we get

$$=\left(\frac{1}{2}+1\right)\div\left(4\left(\frac{1}{2}\right)^2+1\right)$$
$$=\frac{1+2}{2}\div\left(4\times\frac{1}{4}+1\right)$$
$$=\frac{3}{2}\div(1+1)=\frac{3}{2}\div 2$$
$$=\frac{3}{2}\times\frac{1}{2}=\frac{3}{4}$$

Q4ii). Evaluate $\dfrac{a+1}{4a^2+1}$ when $a=\dfrac{-1}{2}$

Sol: Given $\dfrac{a+1}{4a^2+1}$ putting $a=\dfrac{-1}{2}$ we get

$$=\left(\frac{-1}{2}+1\right)\div\left(4\left(\frac{-1}{2}\right)^2+1\right)$$
$$=\frac{-1+2}{2}\div\left(4\times\frac{1}{4}+1\right)$$
$$=\frac{1}{2}\div(1+1)=\frac{1}{2}\div 2$$
$$=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$$

Q5. If $a=9, b=12, c=15$ and $s=\dfrac{a+b+c}{2}$

Find the value of $\sqrt{s(s-a)(s-b)(s-c)}$

Solution: Given $a=9, b=12, c=15$ Find

$s=\dfrac{a+b+c}{2}$ and $\sqrt{s(s-a)(s-b)(s-c)}$

First to find $s=\dfrac{a+b+c}{2}$ putting values

$$s=\frac{9+12+15}{2}=\frac{36}{2}=18$$

Now to find $\sqrt{s(s-a)(s-b)(s-c)}$ putting

$$=\sqrt{18(18-9)(18-12)(18-15)}$$
$$=\sqrt{18(9)(6)(3)}$$
$$=\sqrt{2916}$$
$$=54$$

Formula 1: $(a+b)^2=a^2+2ab+b^2$

Taking LHS

$$(a+b)^2=(a+b)(a+b)$$
$$=a(a+b)+b(a+b)$$
$$=a^2+ab+ba+b^2$$
$$=a^2+ab+ab+b^2 \qquad \therefore ab=ba$$
$$=a^2+2ab+b^2$$

Formula 2: $(a-b)^2=a^2-2ab+b^2$

Taking LHS

$$(a-b)^2=(a-b)(a-b)$$
$$=a(a-b)+b(a-b)$$
$$=a^2-ab-ba+b^2$$
$$=a^2-ab-ab+b^2 \qquad \therefore ab=ba$$
$$=a^2-2ab+b^2$$

Formula 3: $(a-b)(a+b)=a^2-b^2$

Taking LHS

$$(a-b)(a+b)=a(a+b)-b(a+b)$$
$$=a^2+ab-ba-b^2$$
$$=a^2+ab-ab-b^2 \qquad \therefore ab=ba$$
$$=a^2-b^2$$

Formula 4:

$$(a+b)^2+(a-b)^2=2(a^2+b^2)$$

Taking LHS

$$(a+b)^2+(a-b)^2$$
$$=a^2+2ab+b^2+a^2-2ab+b^2$$
$$=a^2+a^2+b^2+b^2$$
$$=2a^2+2b^2$$
$$=2(a^2+b^2)$$

Formula 5:

$$(a+b)^2+(a-b)^2=4ab$$

Taking LHS

$$(a+b)^2-(a-b)^2$$
$$=(a^2+2ab+b^2)-(a^2-2ab+b^2)$$
$$=a^2+2ab+b^2-a^2+2ab-b^2$$
$$=2ab+2ab$$
$$=4ab$$

Formulae:

1.	$(a+b)^2=a^2+2ab+b^2$
2.	$(a-b)^2=a^2-2ab+b^2$
3.	$(a-b)(a+b)=a^2-b^2$
4.	$(a+b)^2=(a-b)^2+4ab$
5.	$(a+b)^2+(a-b)^2=2(a^2+b^2)$

Example: Find the values of a^2+b^2 and ab when $a+b=5$ and $a-b=2$

Sol: Given $a+b=5$ & $a-b=2$ $a^2+b^2=?$

Using formula $2(a^2+b^2)=(a+b)^2+(a-b)^2$

Putting values $2(a^2+b^2)=(5)^2+(2)^2$

$$2(a^2+b^2)=25+4$$
$$a^2+b^2=\frac{29}{2}$$

Now using $4ab=(a+b)^2-(a-b)^2$

Putting values $4ab=(5)^2-(2)^2$

$$4ab=25-4$$
$$ab=\frac{21}{4}$$

Exercise No 4.3

Q1. Find values of x^2+y^2 and xy, when

i). x+y = 8, x-y = 3

Sol: Using $2(x^2+y^2)=(x+y)^2+(x-y)^2$

Putting the values

$$2(x^2+y^2)=(8)^2+(3)^2$$
$$2(x^2+y^2)=64+9$$
$$x^2+y^2=\frac{73}{2}$$

Now Using $4xy=(x+y)^2-(x-y)^2$

Putting values

$$4xy=(8)^2-(3)^2$$
$$4xy=64-9$$
$$xy=\frac{55}{4}$$

ii). x+y = 10, x-y = 7

Sol: Given x+y = 10, x-y = 7

Using $2(x^2+y^2)=(x+y)^2+(x-y)^2$

Putting the values

$$2(x^2+y^2)=(10)^2+(7)^2$$
$$2(x^2+y^2)=100+49$$
$$x^2+y^2=\frac{149}{2}$$

Now Using $4xy=(x+y)^2-(x-y)^2$

Putting the values

$$4xy=(10)^2-(7)^2$$
$$4xy=100-49$$
$$xy=\frac{51}{4}$$

iii). x+y = 11, x-y = 5

Sol: x+y = 11, x-y = 5

Using $2(x^2+y^2)=(x+y)^2+(x-y)^2$

Putting the values

$$2(x^2+y^2)=(11)^2+(5)^2$$
$$2(x^2+y^2)=121+25$$
$$x^2+y^2=\frac{146}{2}=73$$

Now Using $4xy=(x+y)^2-(x-y)^2$

Putting the values

$$4xy=(11)^2-(5)^2$$
$$4xy=121-25$$
$$xy=\frac{96}{4}=24$$

iv). x+y = 7, x-y = 4

Sol: Given x+y = 7, x-y = 4

Using $2(x^2+y^2)=(x+y)^2+(x-y)^2$

Putting the values

$$2(x^2+y^2)=7^2+4^2$$
$$2(x^2+y^2)=49+16$$
$$2(x^2+y^2)=65$$
$$x^2+y^2=\frac{65}{2}=32.5$$

Now Using $4xy=(x+y)^2-(x-y)^2$

Putting the values

$$4xy=7^2-4^2$$
$$4xy=49-16$$
$$4xy=33$$
$$xy=\frac{33}{4}=8.25$$

Q2.i) Find the values of a^2+b^2, and ab when a+b = 7, a - b = 3

Sol: Given a+b = 7, a - b = 3

Using $2(a^2+b^2)=(a+b)^2+(a-b)^2$

Putting the values

$$2(a^2+b^2)=(7)^2+(3)^2$$
$$2(a^2+b^2)=49+9$$
$$a^2+b^2=\frac{58}{2}=29$$

Now Using $4ab=(a+b)^2-(a-b)^2$

Putting the values

$$4ab=(7)^2-(3)^2$$
$$4ab=49-9$$
$$ab=\frac{40}{4}=10$$

Q2.ii) Find the values of a^2+b^2, and ab when a+b = 9, a - b = 1

Sol: Given a+b = 9, a - b = 1

Using $2(a^2+b^2)=(a+b)^2+(a-b)^2$

Putting the values

$$2(a^2+b^2)=(9)^2+(1)^2$$
$$2(a^2+b^2)=81+1$$
$$a^2+b^2=\frac{82}{2}=41$$

Now Using $4ab=(a+b)^2-(a-b)^2$

Putting the values

$$4ab=(9)^2-(1)^2$$
$$4ab=81-1$$
$$ab=\frac{80}{4}=20$$

Q3i). If $a+b=$ 10 and $a-b=$ 6 then find the value of a^2+b^2

Sol: Given $a+b=$ 10 & $a-b=$ 6

Using $2(a^2+b^2)=(a+b)^2+(a-b)^2$

Putting the values

$$2(a^2+b^2)=(10)^2+(6)^2$$
$$2(a^2+b^2)=100+36$$
$$a^2+b^2=\frac{136}{2}=68$$

Q3ii). If $a+b=5$ and $a-b=\sqrt{17}$ then find the value of ab

Sol: Given $a+b=5$ and $a-b=\sqrt{17}$

Using $4ab=(a+b)^2-(a-b)^2$

Putting the values

$$4ab=(5)^2-\left(\sqrt{17}\right)^2$$
$$4ab=25-17$$
$$ab=\frac{8}{4}=2$$

Q4. Find the value of $4xy$ when $x+y=17$ and $x-y=5$

Sol: Given $x+y=17$ and $x-y=5$

Using $4xy=(x+y)^2-(x-y)^2$

Putting the values

$$4xy=(17)^2-(5)^2$$
$$4xy=289-25$$
$$xy=264$$

Q5. If $x+y=11$ and $x-y=3$, find $8xy\left(x^2+y^2\right)$

Sol: Given $x+y=11$ and $x-y=3$

Using $2\left(x^2+y^2\right)=(x+y)^2+(x-y)^2$

Putting the values

$$2\left(x^2+y^2\right)=(11)^2+(3)^2$$
$$2\left(x^2+y^2\right)=121+9$$
$$2\left(x^2+y^2\right)=130.........(1)$$

Now Using $4xy=(x+y)^2-(x-y)^2$

Putting the values

$$4xy=(11)^2-(3)^2$$
$$4xy=121-9$$
$$4xy=112................(2)$$

Multiplying eq (1) and (2) we get

$$4xy\times 2\left(x^2+y^2\right)=112\times 130$$
$$8xy\left(x^2+y^2\right)=14560$$

Q6. If $u+v=7$ and $uv=12$, find $u-v$

Sol: Given $u+v=7$ & $uv=12$

Using $(u-v)^2=(u+v)^2-4uv$

Putting the values

$$(u-v)^2=(7)^2-4(12)$$
$$(u-v)^2=49-48$$
$$(u-v)^2=1$$

Taking square root on both sides

$$u-v=\pm 1$$

FORMULA6:

$$(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$$

PROOF: TAKING LHS

$$(a+b+c)^2=(a+b+c)(a+b+c)$$
$$=a(a+b+c)+b(a+b+c)+c(a+b+c)$$
$$=a^2+ab+ac+ba+b^2+bc+ca+cb+c^2$$
$$=a^2+b^2+c^2+ab+ab+bc+bc+ca+ca$$
$$=a^2+b^2+c^2+2ab+2bc+2ca$$
$$=a^2+b^2+c^2+2(ab+bc+ca)$$

Example: Find the value of $a^2+b^2+c^2$ when $a+b+c=9$ and $ab+bc+ca=20$

Sol: Given $a+b+c=9$ & $ab+bc+ca=20$

using $(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

Putting values $(9)^2=a^2+b^2+c^2+2(20)$

$$81=a^2+b^2+c^2+40$$
$$81-40=a^2+b^2+c^2$$

Or $\quad a^2+b^2+c^2=41$

Example: Find the value of $a+b+c$ when $a^2+b^2+c^2=29$ and $ab+bc+ca=26$

Sol: $a^2+b^2+c^2=29$ & $ab+bc+ca=26$

using $(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

Putting values $(a+b+c)^2=29+2(26)$

$$(a+b+c)^2=29+52$$
$$(a+b+c)^2=81$$

Taking square root on both sides

$$\sqrt{(a+b+c)^2}=\sqrt{81}$$
$$a+b+c=9$$

Example: Find the value of $ab+bc+ca$ when $a+b+c=14$ and $a^2+b^2+c^2=78$

Sol: Given $a+b+c=14$ & $a^2+b^2+c^2=78$

using $(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

Putting values $(14)^2=78+2(ab+bc+ca)$

$$196-78=2(ab+bc+ca)$$
$$\frac{118}{2}=ab+bc+ca$$

Or $\quad ab+bc+ca=59$

Example: Find the value of $ab+bc+ca$ when $a+b+c=12$ and $a^2+b^2+c^2=78$

Sol: $a+b+c=12$ & $a^2+b^2+c^2=78$

using $(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

Putting values $(12)^2=78+2(ab+bc+ca)$

$$144-78=2(ab+bc+ca)$$
$$\frac{66}{2}=ab+bc+ca$$

Or $\quad ab+bc+ca=33$

FORMULA:

$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

Exercise No 4.4

Q1i): Find the value of $a^2+b^2+c^2$ when $a+b+c=5$ and $ab+bc+ca=-4$

Sol: Given $a+b+c=5$ and $ab+bc+ca=-4$

using $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

Putting values $(5)^2 = a^2+b^2+c^2+2(-4)$

$$25 = a^2+b^2+c^2-8$$

$$25+8 = a^2+b^2+c^2$$

Or $a^2+b^2+c^2=33$

Q1ii): Find the value of $a^2+b^2+c^2$ when $a+b+c=5$ and $ab+bc+ca=-2$

Sol: Given $a+b+c=5$ & $ab+bc+ca=-2$

using $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

Putting values $(5)^2 = a^2+b^2+c^2+2(-2)$

$$25 = a^2+b^2+c^2-4$$

$$25+4 = a^2+b^2+c^2$$

Or $a^2+b^2+c^2=29$

Q2i): Find the value of $a+b+c$ when $a^2+b^2+c^2=38$ and $ab+bc+ca=-1$

Sol: Given $a^2+b^2+c^2=38$ & $ab+bc+ca=-1$

using $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

Putting values $(a+b+c)^2 = 38+2(-1)$

$$(a+b+c)^2 = 38-2$$

$$(a+b+c)^2 = 36$$

Taking square root on both sides

$$\sqrt{(a+b+c)^2} = \sqrt{36}$$

$$a+b+c=6$$

Q2ii): Find the value of $a+b+c$ when $a^2+b^2+c^2=10$ and $ab+bc+ca=11$

Sol: Given $a^2+b^2+c^2=10$ & $ab+bc+ca=11$

using $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

Putting values $(a+b+c)^2 = 10+2(11)$

$$(a+b+c)^2 = 10+22$$

$$(a+b+c)^2 = 32$$

Taking square root on both sides

$$\sqrt{(a+b+c)^2} = \sqrt{32} = \sqrt{16\times 2}$$

$$a+b+c = 4\sqrt{2}$$

Q3i): Find the value of $ab+bc+ca$ when $a+b+c=12$ and $a^2+b^2+c^2=56$

Sol: Given $a+b+c=12$ and $a^2+b^2+c^2=56$

using $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

Putting values $(12)^2 = 56+2(ab+bc+ca)$

$$144-56 = 2(ab+bc+ca)$$

$$\tfrac{88}{2} = ab+bc+ca$$

Or $ab+bc+ca=44$

Q3ii): Find the value of $ab+bc+ca$ when $a+b+c=5$ and $a^2+b^2+c^2=12$

Sol: Given $a+b+c=5$ and $a^2+b^2+c^2=12$

using $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

Putting values $(5)^2 = 12+2(ab+bc+ca)$

$$25-12 = 2(ab+bc+ca)$$

$$\frac{13}{2} = ab+bc+ca$$

Or $ab+bc+ca = \frac{13}{2}$

Q4. Prove that $x^2+y^2+z^2-xy-yz-zx$

$$=\left(\frac{x-y}{\sqrt{2}}\right)^2+\left(\frac{y-z}{\sqrt{2}}\right)^2+\left(\frac{z-x}{\sqrt{2}}\right)^2$$

Proof: Taking RHS

$$\left(\frac{x-y}{\sqrt{2}}\right)^2+\left(\frac{y-z}{\sqrt{2}}\right)^2+\left(\frac{z-x}{\sqrt{2}}\right)^2$$

$$=\frac{x^2+y^2-2xy}{2}+\frac{y^2+z^2-2yz}{2}+\frac{z^2+x^2-2zx}{2}$$

$$=\frac{x^2+y^2-2xy+y^2+z^2-2yz+z^2+x^2-2zx}{2}$$

$$=\frac{x^2+x^2+y^2+y^2+z^2+z^2-2xy-2yz-2zx}{2}$$

$$=\frac{2x^2+2y^2+2z^2-2xy-2yz-2zx}{2}$$

$$=\frac{2(x^2+y^2+z^2-xy-yz-zx)}{2}$$

$$=x^2+y^2+z^2-xy-yz-zx=LHS$$

Q5. write $2[x^2+y^2+z^2-xy-yz-zx]$ as the sum of three square.

Sol: Given $2[x^2+y^2+z^2-xy-yz-zx]$

$$=2x^2+2y^2+2z^2-2xy-2yz-2zx$$

$$=x^2+x^2+y^2+y^2+z^2+z^2-2xy-2yz-2zx$$

Rearranging the terms

$$=x^2+y^2-2xy+y^2+z^2-2yz+z^2+x^2-2zx$$

$$=(x-y)^2+(y-z)^2+(z-x)^2$$

Q6. Find the value of $a^2+b^2+c^2-ab-bc-ca$ when $a-b=2$, $b-c=3$, $c-a=4$

Sol: Given $a-b=2$, $b-c=3$, $c-a=4$ using

$$2[a^2+b^2+c^2-ab-bc-ca]=(a-b)^2+(b-c)^2+(c-a)^2$$

putting the values

$$2[a^2+b^2+c^2-ab-bc-ca]=(2)^2+(3)^2+(4)^2$$

$$2[a^2+b^2+c^2-ab-bc-ca]=4+9+16$$

$$a^2+b^2+c^2-ab-bc-ca=\frac{29}{2}$$

FORMULA7: $(a+b)^3 = a^3+3ab(a+b)+b^3$

PROOF TAKING LHS $(a+b)^3$

$=(a+b)(a+b)^2$

$=(a+b)(a^2+2ab+b^2)$

$=a(a^2+2ab+b^2)+b(a^2+2ab+b^2)$

$=a^3+2a^2b+ab^2+a^2b+2ab^2+b^3$

$=a^3+2a^2b+a^2b+ab^2+2ab^2+b^3$

$=a^3+3a^2b+3ab^2+b^3$

$=a^3+3ab(a+b)+b^3=RHS$

FORMULA8: $(a-b)^3=a^3-3ab(a-b)-b^3$

PROOF TAKING LHS $(a-b)^3$

$=(a-b)(a-b)^2$

$=(a-b)(a^2-2ab+b^2)$

$=a(a^2-2ab+b^2)-b(a^2-2ab+b^2)$

$=a^3-2a^2b+ab^2-a^2b+2ab^2-b^3$

$=a^3-2a^2b-a^2b+ab^2+2ab^2-b^3$

$=a^3-3a^2b+3ab^2-b^3$

$=a^3-3ab(a-b)-b^3=RHS$

FORMULAE

$(a+b)^3=a^3+3ab(a+b)+b^3$
$(a-b)^3=a^3-3ab(a-b)-b^3$

Example: Find the value of a^3+b^3, when $a+b=$ 5 and $ab=$ 10

Sol: Given $a+b=$ 5, $ab=$ 10 & a^3+b^3=?

Using $(a+b)^3=a^3+b^3+3ab(a+b)$

Putting the values

$(5)^3=a^3+b^3+3(10)(5)$

$125=a^3+b^3+150$

$125-150=a^3+b^3$

Or $a^3+b^3=-25$

Example: Find the value of a^3-b^3, when $a-b=$ 2 and $ab=$ 15

Sol: Given $a-b=$ 2, $ab=15$ and a^3-b^3=?

Using $(a-b)^3=a^3-b^3-3ab(a-b)$

Putting the values

$(2)^3=a^3-b^3-3(15)(2)$

$8=a^3+b^3-90$

$8+90=a^3-b^3$

Or $a^3-b^3=98$

Example: Find value of $x^3+\frac{1}{x^3}$, when $x+\frac{1}{x}=$ 5

Sol: Given $x+\frac{1}{x}=$ 5 & $x^3+\frac{1}{x^3}$ = ?

Using $\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$

Putting the values

$(5)^3=x^3+\frac{1}{x^3}+3(5)$

$125=x^3+\frac{1}{x^3}+15$

$125-15=x^3+\frac{1}{x^3}$

Or $x^3+\frac{1}{x^3}=110$

Exercise 4.5

Q1: i). Find the value of a^3+b^3, when $a+b=$ 4 and $ab=$ 5

Sol: Given $a+b=$ 4, $ab=$ 5, a^3+b^3=?

Using $(a+b)^3=a^3+b^3+3ab(a+b)$

Putting the values

$(4)^3=a^3+b^3+3(5)(4)$

$64=a^3+b^3+60$

$64-60=a^3+b^3$

Or $a^3+b^3=4$

Q1: ii). Find the value of a^3+b^3, when $a+b=$ 3 and $ab=$ 20

Sol: Given $a+b=$ 3 and $ab=$ 20

Using $(a+b)^3=a^3+b^3+3ab(a+b)$

Putting the values

$(3)^3=a^3+b^3+3(20)(3)$

$27=a^3+b^3+180$

$27-180=a^3+b^3$

Or $a^3+b^3=-153$

Q1: iii). Find the value of a^3+b^3, when $a+b=$ 4 and $ab=$ 2

Sol: Given $a+b=$ 4 and $ab=$ 2

Using $(a+b)^3=a^3+b^3+3ab(a+b)$

Putting the values $(4)^3=a^3+b^3+3(2)(4)$

$64=a^3+b^3+24$

$64-24=a^3+b^3$

Or $a^3+b^3=40$

Q2: i). Find the value of a^3-b^3, when $a-b=$ 5 and $ab=$ 7

Sol: Given $a-b=$ 5 and $ab=$ 7

Using $(a-b)^3=a^3-b^3-3ab(a-b)$

Putting the values $(5)^3=a^3-b^3-3(7)(5)$

$125=a^3+b^3-105$

$125+105=a^3-b^3$

Or $a^3-b^3=230$

Q2: ii). Find the value of a^3-b^3, when $a-b=$ 2 and $ab=$ 15

Sol: Given $a-b=$ 2 and $ab=$ 15

Using $(a-b)^3=a^3-b^3-3ab(a-b)$

Putting the values

$$(2)^3=a^3-b^3-3(15)(2)$$

$$8=a^3+b^3-90$$

$$8+90=a^3-b^3$$

Or $\quad a^3-b^3=98$

Q2: iii). Find the value of a^3-b^3, when $a-b=$ 7 and $ab=$ 6

Sol: $a-b=$ 7 & $ab=$ 6, a^3-b^3=?

Using $(a-b)^3=a^3-b^3-3ab(a-b)$

Putting the values $(7)^3=a^3-b^3-3(6)(7)$

$$343=a^3+b^3-126$$

$$343+126=a^3-b^3$$

Or $\quad a^3-b^3=469$

Q3: i). Find value of $x^3+\frac{1}{x^3}$, when $x+\frac{1}{x}=\frac{5}{2}$

Sol: Given $x+\frac{1}{x}=\frac{5}{2}$ Find $x^3+\frac{1}{x^3}$

Using $\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$

Putting the values

$$\left(\frac{5}{2}\right)^3=x^3+\frac{1}{x^3}+3\left(\frac{5}{2}\right)$$

$$\frac{125}{8}=x^3+\frac{1}{x^3}+\frac{15}{2}$$

$$\frac{125}{8}-\frac{15}{2}\times\frac{4}{4}=x^3+\frac{1}{x^3}$$

Or $\quad x^3+\frac{1}{x^3}=\frac{125-60}{8}$

$$x^3+\frac{1}{x^3}=\frac{65}{8}$$

Q3: ii). Find value of $x^3+\frac{1}{x^3}$, when $x+\frac{1}{x}=$ 2

Sol: Given $x+\frac{1}{x}=$ 2 Find $x^3+\frac{1}{x^3}$

Using $\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$

Putting the values

$$(2)^3=x^3+\frac{1}{x^3}+3(2)$$

$$8=x^3+\frac{1}{x^3}+6$$

$$8-6=x^3+\frac{1}{x^3}$$

Or $\quad x^3+\frac{1}{x^3}=2$

Q4: i). Find value of $x^3-\frac{1}{x^3}$, when $x-\frac{1}{x}=\frac{3}{2}$

Sol: Given $x-\frac{1}{x}=\frac{3}{2}$ Find $x^3-\frac{1}{x^3}$

Using $\left(x-\frac{1}{x}\right)^3=x^3-\frac{1}{x^3}-3\left(x-\frac{1}{x}\right)$

Putting the values

$$\left(\frac{3}{2}\right)^3=x^3-\frac{1}{x^3}-3\left(\frac{3}{2}\right)$$

$$\frac{27}{8}=x^3-\frac{1}{x^3}-\frac{9}{2}$$

$$\frac{27}{8}+\frac{9}{2}\times\frac{4}{4}=x^3-\frac{1}{x^3}$$

Or $\quad x^3-\frac{1}{x^3}=\frac{27+36}{8}$

$$x^3-\frac{1}{x^3}=\frac{63}{8}$$

Q4: ii). Find value of $x^3-\frac{1}{x^3}$, when $x-\frac{1}{x}=\frac{7}{3}$

Sol: Given $x-\frac{1}{x}=\frac{7}{3}$ Find $x^3-\frac{1}{x^3}$

Using $\left(x-\frac{1}{x}\right)^3=x^3-\frac{1}{x^3}-3\left(x-\frac{1}{x}\right)$

Putting the values

$$\left(\frac{7}{3}\right)^3=x^3-\frac{1}{x^3}-3\left(\frac{7}{3}\right)$$

$$\frac{343}{27}=x^3-\frac{1}{x^3}-\frac{21}{3}$$

$$\frac{343}{27}+\frac{21}{3}\times\frac{9}{9}=x^3-\frac{1}{x^3}$$

Or $\quad x^3-\frac{1}{x^3}=\frac{343+189}{27}$

$$x^3-\frac{1}{x^3}=\frac{532}{27}$$

Q4: iii). Find value of $x^3-\frac{1}{x^3}$, when $x-\frac{1}{x}=\frac{15}{4}$

Sol: Given $x-\frac{1}{x}=\frac{15}{4}$ Find $x^3-\frac{1}{x^3}$

Using $\left(x-\frac{1}{x}\right)^3=x^3-\frac{1}{x^3}-3\left(x-\frac{1}{x}\right)$

Putting the values

$$\left(\frac{15}{4}\right)^3=x^3-\frac{1}{x^3}-3\left(\frac{15}{4}\right)$$

$$\frac{3375}{64}=x^3-\frac{1}{x^3}-\frac{45}{4}$$

$$\frac{3375}{64}+\frac{45}{4}\times\frac{16}{16}=x^3-\frac{1}{x^3}$$

Or $x^3-\frac{1}{x^3}=\frac{3375+720}{64}$

$x^3-\frac{1}{x^3}=\frac{4095}{64}$

Q5. If $3a+\frac{1}{a}=4$, find $27a^3+\frac{1}{a^3}$

Sol: Given $3a+\frac{1}{a}=4$, find $27a^3+\frac{1}{a^3}$

Using $(a+b)^3=a^3+b^3+3ab(a+b)$

Accordingly $\left(3a+\frac{1}{a}\right)^3=27a^3+\frac{1}{a^3}+3(3a)\frac{1}{a}\left(3a+\frac{1}{a}\right)$

Putting the values

$(4)^3=27a^3+\frac{1}{a^3}+9(4)$

$64=27a^3+\frac{1}{a^3}+36$

$64-36=27a^3+\frac{1}{a^3}$

Or $27a^3+\frac{1}{a^3}=28$

Q6. If $x-\frac{1}{2x}=6$, find $x^3-\frac{1}{8x^3}$

Sol: Given $x-\frac{1}{2x}=6$, find $x^3-\frac{1}{8x^3}$

Using $(a-b)^3=a^3-b^3-3ab(a-b)$

Accordingly $\left(x-\frac{1}{2x}\right)^3=x^3-\frac{1}{8x^3}-3x\frac{1}{2x}\left(x-\frac{1}{2x}\right)$

Putting the values

$(6)^3=x^3-\frac{1}{8x^3}-\frac{3}{2}(6)$

$216=x^3-\frac{1}{8x^3}-9$

$216+9=x^3-\frac{1}{8x^3}$

Or $x^3-\frac{1}{8x^3}=225$

Q7. If $a+b=6$ show that
$a^3+b^3+18ab=216$

Sol: Given $a+b=6$

Using $a^3+b^3+3ab(a+b)=(a+b)^3$

Putting the value of $a+b=6$

$a^3+b^3+3ab(6)=(6)^3$

$a^3+b^3+18ab=216$

Hence proved

Q8. If $u-v=3$ show that
$u^3-v^3-9uv=27$

Sol: Given $u-v=3$

Using $u^3-v^3-3uv(u-v)=(u-v)^3$

Putting the value of $u-v=3$

$u^3-v^3-3uv(3)=(3)^3$

$u^3-v^3-9uv=27$

Hence proved

Q9. If $a+\frac{1}{a}=2$, find $a^2+\frac{1}{a^2}$, $a^3+\frac{1}{a^3}$ and $a^4+\frac{1}{a^4}$

Sol: Given $a+\frac{1}{a}=2$

using $\left(a+\frac{1}{a}\right)^2=a^2+\frac{1}{a^2}+2a\frac{1}{a}$

Putting $a+\frac{1}{a}=2$ in above identity

$(2)^2=a^2+\frac{1}{a^2}+2$

$4-2=a^2+\frac{1}{a^2}$

Or $a^2+\frac{1}{a^2}=2$

Again squaring both sides

$\left(a^2+\frac{1}{a^2}\right)^2=2^2$

$a^4+\frac{1}{a^4}+2a^2\frac{1}{a^2}=4$

$a^4+\frac{1}{a^4}=4-2$

$a^4+\frac{1}{a^4}=2$

Now using $\left(a+\frac{1}{a}\right)^3=a^3+\frac{1}{a^3}+3\left(a+\frac{1}{a}\right)$

Putting $a+\frac{1}{a}=2$ in above identity

$(2)^3=a^3+\frac{1}{a^3}+3(2)$

$8=a^3+\frac{1}{a^3}+6$

$8-6=a^3+\frac{1}{a^3}$

Or $a^3+\frac{1}{a^3}=2$

FORMULA 9: $a^3+b^3=(a+b)(a^2-ab+b^2)$

Proof: Taking RHS $(a+b)(a^2-ab+b^2)$

$=a(a^2-ab+b^2)+b(a^2-ab+b^2)$

$=a^3-a^2b+ab^2+a^2b-ab^2+b^3$

Rearranging

$=a^3-a^2b+a^2b+ab^2-ab^2+b^3$

$=a^3+b^3=LHS$

Hence proved

FORMULA 10: $a^3-b^3=(a-b)(a^2+ab+b^2)$

Proof: Taking RHS $(a-b)(a^2+ab+b^2)$

$=a(a^2+ab+b^2)-b(a^2+ab+b^2)$

$=a^3+a^2b+ab^2-a^2b-ab^2-b^3$

Rearranging

$=a^3+a^2b-a^2b+ab^2-ab^2-b^3$

$=a^3-b^3=LHS$

Hence proved

Example 19: Find the value of a^3+b^3 when $a+b=$ 4 and $ab=$ 3

Sol: Using $(a+b)^3=a^3+b^3+3ab(a+b)$

Putting the values

$(4)^3=a^3+b^3+3(3)(4)$

$64=a^3+b^3+36$

$64-36=a^3+b^3$

Or $a^3+b^3=28$

Example 20: Find value of a^3-b^3, when $a-b=$ 2 and $ab=$ 10

Sol: Using $(a-b)^3=a^3-b^3-3ab(a-b)$

Putting the values

$(2)^3=a^3-b^3-3(10)(2)$

$8=a^3+b^3-60$

$8+60=a^3-b^3$

Or $a^3-b^3=68$

Example: Find product $\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)$

Sol: Given $\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}-1\right)$

$=x\left(x^2+\frac{1}{x^2}-1\right)+\frac{1}{x}\left(x^2+\frac{1}{x^2}-1\right)$

$=x^3+\frac{x}{x^2}-x+\frac{x^2}{x}+\frac{1}{x^3}-\frac{1}{x}$

$=x^3+\frac{1}{x}-x+x+\frac{1}{x^3}-\frac{1}{x}$

$=x^3+\frac{1}{x^3}$

Example: Find product $\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+1\right)$

Sol: Given $\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+1\right)$

$=x\left(x^2+\frac{1}{x^2}+1\right)-\frac{1}{x}\left(x^2+\frac{1}{x^2}+1\right)$

$=x^3+\frac{x}{x^2}+x-\frac{x^2}{x}-\frac{1}{x^3}-\frac{1}{x}$

$=x^3+\frac{1}{x}+x-x-\frac{1}{x^3}-\frac{1}{x}$

$=x^3-\frac{1}{x^3}$

Example: Find the continued product of $(x+y)(x-y)(x^2+xy+y^2)(x^2-xy+y^2)$

Sol $(x+y)(x-y)(x^2+xy+y^2)(x^2-xy+y^2)$

Rearranging

$=(x-y)(x^2+xy+y^2)(x+y)(x^2-xy+y^2)$

$=(x^3-y^3)(x^3+y^3)$

$=(x^3)^2-(y^3)^2$

$=x^6-y^6$

Formulae:

$a^3+b^3=(a+b)(a^2-ab+b^2)$
$a^3-b^3=(a-b)(a^2+ab+b^2)$

Exercise No 4.6

Q1:i). Find Product $(a-1)(a^2+a+1)$

Sol: Given $(a-1)(a^2+a+1)$

$=a(a^2+a+1)-1(a^2+a+1)$

$=a^3+a^2+a-a^2-a-1$

Rearranging

$=a^3+a^2-a^2+a-a-1$

$=a^3-1$

Hence the required product = a^3-1

Q1:ii). Find Product $(3-b)(9+3b+b^2)$

Sol: Given $(3-b)(9+3b+b^2)$

$=3(9+3b+b^2)-b(9+3b+b^2)$

$=27+9b+3b^2-9b-3b^2-b^3$

Rearranging

$=27+9b-9b+3b^2-3b^2-b^3$

$=27-b^3$

Hence the required product $=27-b^3$

Q1:iii). Find Product $(8+b)(64-8b+b^2)$

Sol: Given $(8+b)(64-8b+b^2)$

$=8(64-8b+b^2)+b(64-8b+b^2)$

$=512-64b+8b^2+64b-8b^2+b^3$

Rearranging

$=512-64b+64b+8b^2-8b^2+b^3$

$=512+b^3$

Hence the required product $=512+b^3$

Q1:iv). Find Product $(a+2)(a^2-2a+4)$

Sol: Given $(a+2)(a^2-2a+4)$

$=a(a^2-2a+4)+2(a^2-2a+4)$

$=a^3-2a^2+4a+2a^2-4a+8$

Rearranging

$= a^3 - 2a^2 + 2a^2 + 4a - 4a + 8$

$= a^3 + 8$

Hence the required product = $a^3 + 8$

Q2:i). Find Product $\left(2p+\frac{1}{2p}\right)\left(4p^2+\frac{1}{4p^2}-1\right)$

Sol: Given $\left(2p+\frac{1}{2p}\right)\left(4p^2+\frac{1}{4p^2}-1\right)$

$= 2p\left(4p^2+\frac{1}{4p^2}-1\right)+\frac{1}{2p}\left(4p^2+\frac{1}{4p^2}-1\right)$

$= 8p^3+\frac{2p}{4p^2}-2p+\frac{4p^2}{2p}+\frac{1}{8p^3}-\frac{1}{2p}$

Rearranging and simplifying

$= 8p^3+\frac{1}{2p}-\frac{1}{2p}-2p+2p+\frac{1}{8p^3}$

$= 8p^3+\frac{1}{8p^3}$

Hence the required product $= 8p^3+\frac{1}{8p^3}$

Q2:ii). Find Product $\left(\frac{3p}{2}-\frac{2}{3p}\right)\left(\frac{9p^2}{4}+\frac{4}{9p^2}+1\right)$

Sol: Given $\left(\frac{3p}{2}-\frac{2}{3p}\right)\left(\frac{9p^2}{4}+\frac{4}{9p^2}+1\right)$

$= \frac{3p}{2}\left(\frac{9p^2}{4}+\frac{4}{9p^2}+1\right)-\frac{2}{3p}\left(\frac{9p^2}{4}+\frac{4}{9p^2}+1\right)$

$= \frac{27p^3}{8}+\frac{12p}{18p^2}+\frac{3p}{2}-\frac{18p^2}{12p}-\frac{8}{27p^3}-\frac{2}{3p}$

Rearranging and simplifying

$= \frac{27p^3}{8}+\frac{2}{3p}-\frac{2}{3p}+\frac{3p}{2}-\frac{3p}{2}-\frac{8}{27p^3}$

$= \frac{27p^3}{8}-\frac{8}{27p^3}$

Hence required product $= \frac{27p^3}{8}-\frac{8}{27p^3}$

Q2:iii). Find Product $\left(3p-\frac{1}{3p}\right)\left(9p^2+\frac{1}{9p^2}+1\right)$

Sol: Given $\left(3p-\frac{1}{3p}\right)\left(9p^2+\frac{1}{9p^2}+1\right)$

$= 3p\left(9p^2+\frac{1}{9p^2}+1\right)-\frac{1}{3p}\left(9p^2+\frac{1}{9p^2}+1\right)$

$= 27p^3+\frac{3p}{9p^2}+3p-\frac{9p^2}{3p}-\frac{1}{27p^3}-\frac{1}{3p}$

Rearranging and simplifying

$= 27p^3+\frac{1}{3p}-\frac{1}{3p}+3p-3p-\frac{1}{27p^3}$

$= 27p^3-\frac{1}{27p^3}$

Hence required product $= 27p^3-\frac{1}{27p^3}$

Q2:iv). Find Product $\left(5p+\frac{1}{5p}\right)\left(25p^2+\frac{1}{25p^2}-1\right)$

Sol: Given $\left(5p+\frac{1}{5p}\right)\left(25p^2+\frac{1}{25p^2}-1\right)$

$= 5p\left(25p^2+\frac{1}{25p^2}-1\right)+\frac{1}{5p}\left(25p^2+\frac{1}{25p^2}-1\right)$

$= 125p^3+\frac{5p}{25p^2}-5p+\frac{25p^2}{5p}+\frac{1}{125p^3}-\frac{1}{5p}$

Rearranging and simplifying

$= 125p^3+\frac{1}{5p}-\frac{1}{5p}-5p+5p+\frac{1}{125p^3}$

$= 125p^3+\frac{1}{125p^3}$

Hence required product $= 125p^3+\frac{1}{125p^3}$

Q3: i) Find the continued product of

$\left(x^2-y^2\right)\left(x^2+xy+y^2\right)\left(x^2-xy+y^2\right)$

Sol: $\left(x^2-y^2\right)\left(x^2+xy+y^2\right)\left(x^2-xy+y^2\right)$

$= (x+y)(x-y)\left(x^2+xy+y^2\right)\left(x^2-xy+y^2\right)$

Rearranging

$= (x-y)\left(x^2+xy+y^2\right)(x+y)\left(x^2-xy+y^2\right)$

$= \left(x^3-y^3\right)\left(x^3+y^3\right)$

$= \left(x^3\right)^2-\left(y^3\right)^2$

$= x^6-y^6$

Q3: ii) Find the continued product of

$(x+y)(x-y)\left(x^2+y^2\right)\left(x^4+y^4\right)$

Sol: Given $(x+y)(x-y)\left(x^2+y^2\right)\left(x^4+y^4\right)$

$= \left(x^2-y^2\right)\left(x^2+y^2\right)\left(x^4+y^4\right)$

using $a^2-b^2=(a+b)(a-b)$

$= \left(\left(x^2\right)^2-\left(y^2\right)^2\right)\left(x^4+y^4\right)$

$= \left(x^4-y^4\right)\left(x^4+y^4\right)$

$= \left(\left(x^4\right)^2-\left(y^4\right)^2\right) \because \ a^2-b^2=(a+b)(a-b)$

$= x^8-y^8$

Q3: iii) Find the continued product of

$(2x-y)(2x+y)\left(4x^2-2xy+y^2\right)\left(4x^2+2xy+y^2\right)$

Sol: Given

$(2x-y)(2x+y)\left(4x^2-2xy+y^2\right)\left(4x^2+2xy+y^2\right)$

Rearranging

$= (2x-y)\left(4x^2+2xy+y^2\right)(2x+y)\left(4x^2-2xy+y^2\right)$

Making formulae $a^3 \pm b^3 = (a\pm b)\left(a^2 \mp ab+b^2\right)$

$= (2x-y)\left((2x)^2+(2x)(y)+(y)^2\right)(2x+y)\left((2x)^2-(2x)(y)+(y)^2\right)$

$= \left\{(2x)^3-(y)^3\right\}\left\{(2x)^3+(y)^3\right\}$

$= \left(8x^3-y^3\right)\left(8x^3+y^3\right)$

$= \left(8x^3\right)^2-\left(y^3\right)^2$

$= 64x^6-y^6$

Q3: iv) Find the continued product of
$(x-2)(x+2)(x^2-2x+4)(x^2+2x+4)$

Solution: Given that
$(x-2)(x+2)(x^2-2x+4)(x^2+2x+4)$

Rearranging
$=(x-2)(x^2+2x+4)(x+2)(x^2-2x+4)$

Making formulae $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$
$=(x-2)(x^2+x(2)+2^2)(x+2)(x^2-x(2)+2^2)$
$=\{x^3-2^3\}\{x^3+2^3\}$
$=(x^3)^2-(2^3)^2$
$=x^6-2^6$
$=x^6-64$

Q4. Find the product with the help of formula $(\sqrt{x}-\sqrt{y})(x+\sqrt{xy}+y)$

Solution: Given $(\sqrt{x}-\sqrt{y})(x+\sqrt{xy}+y)$

Using formula $(a-b)(a^2+ab+b^2)=a^3-b^3$

First to make LHS of above formula
$=(\sqrt{x}-\sqrt{y})\left((\sqrt{x})^2+(\sqrt{x})(\sqrt{y})+(\sqrt{y})^2\right)$

Now Create RHS of formula
$=(\sqrt{x})^3-(\sqrt{y})^3$
$=\sqrt{x^3}-\sqrt{y^3}$

Q4. Find the product with the help of formula $(x^p+y^q)(x^{2p}-x^py^q+y^{2q})$

Solution: Given $(x^p+y^q)(x^{2p}-x^py^q+y^{2q})$

Using formula $(a+b)(a^2-ab+b^2)=a^3+b^3$

First to make LHS of above formula
$=(x^p+y^q)\left((x^p)^2-(x^p)(y^q)+(y^q)^2\right)$

Now Create RHS of formula
$=(x^p)^3+(y^q)^3$
$=x^{3p}+y^{3q}$

Surds:

A Number of the form $\sqrt[n]{a}$ is called a surd.
Where a is a positive rational number iff
i). it is a irrational number
ii). It is a root
iii). A root of a rational number
where n is called order of surd
$\sqrt{a}$ is called quadratic surd
$\sqrt[3]{a}$ is called cubic surd
$\sqrt[4]{a}$ is called biquadratic surd

Note That: $\sqrt{5+\sqrt{3}}$ is not a surd because $5+\sqrt{3}$ is irrational number
$\sqrt[3]{8}$ is not a surd
$\sqrt{3}$ is a surd because 3 is a positive rational

Conjugate of surd

$5+\sqrt{3}$ and $5-\sqrt{3}$ are Conjugate of surd

Example21: if $x=2+\sqrt{3}$ and find the values of $x+\frac{1}{x}$ and $x^2+\frac{1}{x^2}$

Solution: Given $x=2+\sqrt{3}$ …………………(1)
Since every number has denominator 1

So $\frac{x}{1}=\frac{2+\sqrt{3}}{1}$ Taking reciprocal

$\frac{1}{x}=\frac{1}{2+\sqrt{3}}$ To rationalize the denominator

$\times \& \div$ it by conjugate of denominator $2-\sqrt{3}$

$\frac{1}{x}=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$

$\frac{1}{x}=\frac{2-\sqrt{3}}{2^2-(\sqrt{3})^2}=\frac{2-\sqrt{3}}{4-3}$

$\frac{1}{x}=\frac{2-\sqrt{3}}{1}$

$\frac{1}{x}=2-\sqrt{3}$ ……………………….(2)

Now adding eq (1) and eq (2) we get

$x+\frac{1}{x}=2+\sqrt{3}+2-\sqrt{3}$

$x+\frac{1}{x}=4$

Taking square on both sides

$\left(x+\frac{1}{x}\right)^2=(4)^2$

$x^2+\frac{1}{x^2}+2x\frac{1}{x}=16$

$x^2+\frac{1}{x^2}=16-4$

$x^2+\frac{1}{x^2}=14$

Example22: if $x=\sqrt{3}-\sqrt{2}$ and find the values of $x-\frac{1}{x}$ and $x^2+\frac{1}{x^2}$

Solution: Given $x=\sqrt{3}-\sqrt{2}$ …………………(1)
Since every number has denominator 1

So $\frac{x}{1}=\frac{\sqrt{3}-\sqrt{2}}{1}$ Taking reciprocal

$\frac{1}{x}=\frac{1}{\sqrt{3}-\sqrt{2}}$ To rationalize the denominator

$\times \& \div$ it by conjugate of denominator $\sqrt{3}+\sqrt{2}$

$\frac{1}{x}=\frac{1}{\sqrt{3}-\sqrt{2}}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$\frac{1}{x}=\frac{\sqrt{3}+\sqrt{2}}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}=\frac{\sqrt{3}+\sqrt{2}}{3-2}$$

$$\frac{1}{x}=\frac{\sqrt{3}+\sqrt{2}}{1}$$

$$\frac{1}{x}=\sqrt{3}+\sqrt{2}\ \text{..........................}(2)$$

Now Subtracting eq (1) and eq (2) we get

$$x-\frac{1}{x}=\left(\sqrt{3}-\sqrt{2}\right)-\left(\sqrt{3}+\sqrt{2}\right)$$

$$x-\frac{1}{x}=\sqrt{3}-\sqrt{2}-\sqrt{3}-\sqrt{2}$$

$$x-\frac{1}{x}=-2\sqrt{2}$$

Taking square on both sides

$$\left(x-\frac{1}{x}\right)^2=\left(-2\sqrt{2}\right)^2$$

$$x^2+\frac{1}{x^2}-2x\frac{1}{x}=(-2)^2\left(\sqrt{2}\right)^2$$

$$x^2+\frac{1}{x^2}=4\times2+2$$

$$x^2+\frac{1}{x^2}=10$$

Exercise No 4.8

Q1. State which of the following are surds.

i). $\sqrt[3]{81}$

Answer: Given $\sqrt[3]{81}$

81 is rational and can not be expressed as power of 3 So, it is a surd

ii). $\sqrt{1+\sqrt{5}}$

Answer: Given $\sqrt{1+\sqrt{5}}$

$1+\sqrt{5}$ is an irrational So, it is a not a surd

iii). $\sqrt{\sqrt{5}}$

Answer: Given $\sqrt{\sqrt{5}}$

$\sqrt{5}$ is an irrational So, it is a not a surd

iv). $\sqrt[4]{32}$

Answer: Given $\sqrt[4]{32}$

32 is rational and can not be expressed as power of 4 So, it is a surd

v). π

Answer: Given π

π is an irrational So, it is a not a surd

vi). $\sqrt{1+\pi^2}$

Answer: Given $\sqrt{1+\pi^2}$

$1+\pi^2$ is an irrational So, it is a not a surd

Q2. Express the following as the simplest possible surds.

i). $\sqrt{12}$

Solution: $\sqrt{12}=\sqrt{2\times2\times3}$

$$=\sqrt{2^2\times3}$$

$$=\sqrt{2^2}\sqrt{3}$$

$$=2\sqrt{3}$$

2	12
2	6
3	3
	1

ii). $\sqrt{48}$

Solution: $\sqrt{48}$

$$\sqrt{48}=\sqrt{4^2\times3}$$

$$\sqrt{48}=\sqrt{4^2}\sqrt{3}$$

$$\sqrt{48}=4\sqrt{3}$$

4	48
4	12
3	3
	1

iii). $\sqrt{240}$

Solution:

$$\sqrt{240}=\sqrt{4\times4\times5\times3}$$

$$=\sqrt{4^2\times5\times3}$$

$$=\sqrt{4^2}\sqrt{5\times3}$$

$$=4\sqrt{15}$$

4	240
4	60
5	15
3	3
	1

Q3. Simplify the following surds.

i). $\left(2-\sqrt{3}\right)\left(3+\sqrt{5}\right)$

Solution: $\left(2-\sqrt{3}\right)\left(3+\sqrt{5}\right)$

$$=2\left(3+\sqrt{5}\right)-\sqrt{3}\left(3+\sqrt{5}\right)$$

$$=6+2\sqrt{5}-3\sqrt{3}-\sqrt{15}$$

ii). $\left(\sqrt{3}-4\right)\left(\sqrt{2}+1\right)$

Solution: $\left(\sqrt{3}-4\right)\left(\sqrt{2}+1\right)$

$$=\sqrt{3}\left(\sqrt{2}+1\right)-4\left(\sqrt{2}+1\right)$$

$$=\sqrt{6}+\sqrt{3}-4\sqrt{2}-4$$

iii). $\left(\sqrt{2}+\sqrt{3}\right)\left(\sqrt{5}+\sqrt{2}\right)$

Solution: $\left(\sqrt{2}+\sqrt{3}\right)\left(\sqrt{5}+\sqrt{2}\right)$

$$=\sqrt{2}\left(\sqrt{5}+\sqrt{2}\right)+\sqrt{3}\left(\sqrt{5}+\sqrt{2}\right)$$

$$=\sqrt{10}+\sqrt{4}+\sqrt{15}+\sqrt{6}$$

$$=\sqrt{10}+2+\sqrt{15}+\sqrt{6}$$

iv). $\left(3-2\sqrt{3}\right)\left(3+2\sqrt{3}\right)$

Solution: $\left(3-2\sqrt{3}\right)\left(3+2\sqrt{3}\right)$

Using formula $(a+b)(a-b)=a^2-b^2$

$$=(3)^2-\left(2\sqrt{3}\right)^2$$

$$=9-2^2\left(\sqrt{3}\right)^2$$

$$=9-4\times3$$

$$=9-12$$

$$=-3$$

Q4. Rationalize denominator and simplify

i). $\frac{1}{\sqrt{7}}$

Solution: Multiply and divide it by $\sqrt{7}$

$$=\frac{1}{\sqrt{7}}.\frac{\sqrt{7}}{\sqrt{7}}=\frac{\sqrt{7}}{\left(\sqrt{7}\right)^2}=\frac{\sqrt{7}}{7}$$

ii). $\frac{3}{\sqrt{45}}$

Solution: Take

$\frac{3}{\sqrt{45}} = \frac{3}{\sqrt{3\times3\times5}}$

3	45
3	15
5	5
	1

$= \frac{1}{\sqrt{3^2\times5}} = \frac{1}{\sqrt{3^2}\sqrt{5}} = \frac{1}{3\sqrt{5}}$

Multiply and divide it by $\sqrt{5}$

$= \frac{3}{3\sqrt{5}}.\frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{\left(\sqrt{5}\right)^2} = \frac{\sqrt{5}}{5}$

iii). $\frac{1}{\sqrt{2}-1}$

Sol: Multiply and divide it by $\sqrt{2}+1$

i.e., conjugate of $\sqrt{2}-1$ is $\sqrt{2}+1$

$= \frac{1}{\sqrt{2}-1}.\frac{\sqrt{2}+1}{\sqrt{2}+1}$

$= \frac{\sqrt{2}+1}{\left(\sqrt{2}\right)^2-1^2}$

$= \frac{\sqrt{2}+1}{2-1}$

$= \frac{\sqrt{2}+1}{1}$

$= \sqrt{2}+1$

iv). $\frac{5}{2+\sqrt{5}}$

Solution: Given $\frac{5}{2+\sqrt{5}}$

$= \frac{5}{2+\sqrt{5}}\times\frac{2-\sqrt{5}}{2-\sqrt{5}}$

$= \frac{5\left(2-\sqrt{5}\right)}{2^2-\left(\sqrt{5}\right)^2}$

$= \frac{5\left(2-\sqrt{5}\right)}{4-5}$

$= -5\left(2-\sqrt{5}\right)$

v). $\frac{1}{\sqrt{5}-2}+\frac{1}{\sqrt{5}+2}$

Solution: $\frac{1}{\sqrt{5}-2}+\frac{1}{\sqrt{5}+2}$

$= \frac{\sqrt{5}+2}{\sqrt{5}+2}\frac{1}{\sqrt{5}-2}+\frac{1}{\sqrt{5}+2}\frac{\sqrt{5}-2}{\sqrt{5}-2}$

Using formula $(a+b)(a-b) = a^2-b^2$

$= \frac{\sqrt{5}+2+\sqrt{5}-2}{\left(\sqrt{5}\right)^2-2^2}$

$= \frac{2\sqrt{5}}{5-4} = \frac{2\sqrt{5}}{1} = 2\sqrt{5}$

Q5. If $x=\sqrt{5}+2$, find $x+\frac{1}{x}$ and $x^2+\frac{1}{x^2}$

Solution: Given $x=\sqrt{5}+2$

Since every number has denominator 1,

$\frac{x}{1} = \frac{\sqrt{5}+2}{1}$

Taking reciprocal on both sides

$\frac{1}{x} = \frac{1}{\sqrt{5}+2}$

$\times \& \div$ it by $\sqrt{5}-2$ or by conjugate of $\sqrt{5}+2$

$\frac{1}{x} = \frac{1}{\sqrt{5}+2}.\frac{\sqrt{5}-2}{\sqrt{5}-2}$

Using formula $(a+b)(a-b) = a^2-b^2$

$\frac{1}{x} = \frac{\sqrt{5}-2}{\left(\sqrt{5}\right)^2-2^2}$

$\frac{1}{x} = \frac{\sqrt{5}-2}{5-4}$

$\frac{1}{x} = \frac{\sqrt{5}-2}{1}$

$\frac{1}{x} = \sqrt{5}+2$

Now $x+\frac{1}{x} = \left(\sqrt{5}+2\right)+\left(\sqrt{5}-2\right)$

$x+\frac{1}{x} = \sqrt{5}+2+\sqrt{5}-2$

$x+\frac{1}{x} = 2\sqrt{5}$

Taking square on both sides

$\left(x+\frac{1}{x}\right)^2 = \left(2\sqrt{5}\right)^2$

Using formula $(a+b)^2 = a^2+b^2+2ab$

$x^2+\frac{1}{x^2}+2.x.\frac{1}{x} = 2^2\left(\sqrt{5}\right)^2 \quad \because (ab)^n = a^nb^n$

$x^2+\frac{1}{x^2}+2 = 4\times5$

$x^2+\frac{1}{x^2} = 20-2$

$x^2+\frac{1}{x^2} = 18$

Q6. If $x=\sqrt{2}+\sqrt{3}$ find the value of $x+\frac{1}{x}$ and $x^2+\frac{1}{x^2}$

Solution: Given $x=\sqrt{2}+\sqrt{3}$ ....(1)

Since every number has denominator 1

So $\frac{x}{1} = \frac{\sqrt{2}+\sqrt{3}}{1}$ Taking reciprocal

$\frac{1}{x} = \frac{1}{\sqrt{2}+\sqrt{3}}$ To rationalize the denominator

$\times \& \div$ it by conjugate of denominator $\sqrt{2}-\sqrt{3}$

$$\frac{1}{x}=\frac{1}{\sqrt{2}+\sqrt{3}}\times\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$

$$\frac{1}{x}=\frac{\sqrt{2}-\sqrt{3}}{\left(\sqrt{2}\right)^2-\left(\sqrt{3}\right)^2}=\frac{\sqrt{2}-\sqrt{3}}{2-3}$$

$$\frac{1}{x}=\frac{\sqrt{2}-\sqrt{3}}{-1}=-\left(\sqrt{2}-\sqrt{3}\right)$$

$$\frac{1}{x}=-\sqrt{2}+\sqrt{3}\text{ ....(2)}$$

Now subtracting eq (1) and eq (2) we get

$$x-\frac{1}{x}=\left(\sqrt{2}+\sqrt{3}\right)-\left(-\sqrt{2}+\sqrt{3}\right)$$

$$x-\frac{1}{x}=\sqrt{2}+\sqrt{3}+\sqrt{2}-\sqrt{3}$$

$$x-\frac{1}{x}=2\sqrt{2}$$

Taking square on both sides

$$\left(x-\frac{1}{x}\right)^2=\left(2\sqrt{2}\right)^2$$

$$x^2+\frac{1}{x^2}-2x\frac{1}{x}=(2)^2\left(\sqrt{2}\right)^2$$

$$x^2+\frac{1}{x^2}=4\times2+2$$

$$x^2+\frac{1}{x^2}=10$$

Q7. If $x=5-2\sqrt{6}$, find of $x+\frac{1}{x}$ & $x^2+\frac{1}{x^2}$

Solution: Given $x=5-2\sqrt{6}$

Since every number has denominator 1,

$$\frac{x}{1}=\frac{5-2\sqrt{6}}{1}$$

Taking reciprocal on both sides

$$\frac{1}{x}=\frac{1}{5-2\sqrt{6}}$$

Multiply and divide it by $5+2\sqrt{6}$

i.e. $\times$&$\div$ it by conjugate of $5-2\sqrt{6}$

$$\frac{1}{x}=\frac{1}{5-2\sqrt{6}}.\frac{5+2\sqrt{6}}{5+2\sqrt{6}}$$

Using formula $(a+b)(a-b)=a^2-b^2$

$$\frac{1}{x}=\frac{5+2\sqrt{6}}{5^2-\left(2\sqrt{6}\right)^2}$$

$$\frac{1}{x}=\frac{5+2\sqrt{6}}{25-2^2\left(\sqrt{6}\right)^2}\qquad\therefore(ab)^n=a^n.b^n$$

$$\frac{1}{x}=\frac{5+2\sqrt{6}}{25-4\times6}$$

$$\frac{1}{x}=\frac{5+2\sqrt{6}}{25-24}$$

$$\frac{1}{x}=\frac{5+2\sqrt{6}}{1}$$

$$\frac{1}{x}=5+2\sqrt{6}$$

Now $x+\frac{1}{x}=\left(5-2\sqrt{6}\right)+\left(5+2\sqrt{6}\right)$

$$x+\frac{1}{x}=5-2\sqrt{6}+5+2\sqrt{6}$$

Rearranging like terms

$$x+\frac{1}{x}=5+5-2\sqrt{6}+2\sqrt{6}$$

$$x+\frac{1}{x}=10$$

Taking square on both sides

$$\left(x+\frac{1}{x}\right)^2=(10)^2$$

$$x^2+\frac{1}{x^2}+2.x.\frac{1}{x}=100$$

$$x^2+\frac{1}{x^2}+2=100$$

$$x^2+\frac{1}{x^2}=100-2$$

$$x^2+\frac{1}{x^2}=98$$

Q8. If $x=\frac{1}{\sqrt{2}-1}$, find values of $x-\frac{1}{x}$ & $x^2+\frac{1}{x^2}$

Solution: Given $x=\frac{1}{\sqrt{2}-1}$

Since every number has denominator 1

$$\frac{x}{1}=\frac{1}{\sqrt{2}-1}$$

Taking reciprocal on both sides

$$\frac{1}{x}=\sqrt{2}-1$$

Again taking $x=\frac{1}{\sqrt{2}-1}$

Multiply and divide it by $\sqrt{2}+1$

i.e., $\times$ & $\div$ it by conjugate of $\sqrt{2}-1$

$$x=\frac{1}{\sqrt{2}-1}.\frac{\sqrt{2}+1}{\sqrt{2}+1}$$

$$\frac{1}{x}=\frac{\sqrt{2}+1}{\left(\sqrt{2}\right)^2-1^2}$$

$$\frac{1}{x}=\frac{\sqrt{2}+1}{2-1}$$

$$\frac{1}{x}=\frac{\sqrt{2}+1}{1}$$

$$\frac{1}{x}=\sqrt{2}+1$$

Now $x-\frac{1}{x}=\left(\sqrt{2}+1\right)-\left(\sqrt{2}-1\right)$

$$x-\frac{1}{x}=\sqrt{2}+1-\sqrt{2}+1$$

Rearranging like terms

$$x-\frac{1}{x}=\sqrt{2}-\sqrt{2}+1+1$$

$$x-\frac{1}{x}=2$$

Taking square on both sides

$$\left(x-\frac{1}{x}\right)^2=(2)^2$$

Using formula $(a-b)^2=a^2+b^2-2ab$

$$x^2+\frac{1}{x^2}-2x\frac{1}{x}=4$$

$$x^2+\frac{1}{x^2}-2=4$$

$$x^2+\frac{1}{x^2}=4+2$$

$$x^2+\frac{1}{x^2}=6$$

Q9. If $x=\sqrt{10}+3$, find $x-\frac{1}{x}$ & $x^2+\frac{1}{x^2}$

Solution: Given $x=\sqrt{10}+3$

Since every number has denominator 1, then

$$\frac{x}{1}=\frac{\sqrt{10}+3}{1}$$

Taking reciprocal on both sides

$$\frac{1}{x}=\frac{1}{\sqrt{10}+3}$$

Multiply and divide it by $\sqrt{10}+3$

i.e., $\times$ & $\div$ it by conjugate of $\sqrt{10}-3$

$$\frac{1}{x}=\frac{1}{\sqrt{10}+3}\cdot\frac{\sqrt{10}-3}{\sqrt{10}-3}$$

Using formula in denominator

$(a+b)(a-b)=a^2-b^2$

$$\frac{1}{x}=\frac{\sqrt{10}-3}{\left(\sqrt{10}\right)^2-3^2}$$

$$\frac{1}{x}=\frac{\sqrt{10}-3}{10-9}$$

$$\frac{1}{x}=\frac{\sqrt{10}-3}{1}$$

$$\frac{1}{x}=\sqrt{10}-3$$

Now $x-\frac{1}{x}=\left(\sqrt{10}+3\right)-\left(\sqrt{10}-3\right)$

$$x-\frac{1}{x}=\sqrt{10}+3-\sqrt{10}+3$$

Rearranging like terms

$$x-\frac{1}{x}=\sqrt{10}-\sqrt{10}+3+3$$

$$x-\frac{1}{x}=6$$

Taking square on both sides

$$\left(x-\frac{1}{x}\right)^2=(6)^2$$

Using formula $(a-b)^2=a^2+b^2-2ab$

$$x^2+\frac{1}{x^2}-2.x.\frac{1}{x}=36$$

$$x^2+\frac{1}{x^2}-2=36$$

$$x^2+\frac{1}{x^2}=36+2$$

$$x^2+\frac{1}{x^2}=38$$

Q10. If $x=2-\sqrt{3}$ find the value of $x^4+\frac{1}{x^4}$

Solution: Given $x=2-\sqrt{3}$ ....(1)

Since every number has denominator 1

So $\frac{x}{1}=\frac{2-\sqrt{3}}{1}$ Taking reciprocal

$\frac{1}{x}=\frac{1}{2-\sqrt{3}}$ To rationalize denominator $\times$ &

$\div$ it by conjugate of denominator $2+\sqrt{3}$

$$\frac{1}{x}=\frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$\frac{1}{x}=\frac{2+\sqrt{3}}{2^2-\left(\sqrt{3}\right)^2}=\frac{2+\sqrt{3}}{4-3}$$

$$\frac{1}{x}=\frac{2+\sqrt{3}}{1}$$

$\frac{1}{x}=2+\sqrt{3}$ ....(2)

Now adding eq (1) and eq (2) we get

$$x+\frac{1}{x}=2-\sqrt{3}+2+\sqrt{3}$$

$$x+\frac{1}{x}=4$$

Taking square on both sides

$$\left(x+\frac{1}{x}\right)^2=(4)^2$$

$$x^2+\frac{1}{x^2}+2x\frac{1}{x}=16$$

$$x^2+\frac{1}{x^2}=16-4$$

$$x^2+\frac{1}{x^2}=14$$

Again squaring both sides

$$\left(x^2+\frac{1}{x^2}\right)=(14)^2$$

$$x^4+\frac{1}{x^4}+2x^2\frac{1}{x^2}=196$$

$$x^4+\frac{1}{x^4}=196-4$$

$$x^4+\frac{1}{x^4}=194$$

Review Exercise 4

Q1. Select the correct answer.

i). $mr^2+3mr^2-5mr^2=$

a). $-mr^2$ b). $-mr$

c). mr d). mr^2

ii). $(x^3y^2)(x^2y^3)=$

a). x^5y^5 b). x^5y^4

c). x^4y^5 d). x^4y^4

iii). $(4xy^4)^3=$

a). $64x^3y^8$ b). $64x^8y^3$

c). $64x^3y^{12}$ d). $64x^3y^7$

iv). $(7x+4y)-(3x-6y)=$

a). $3x$ b). $2x+10y$

c). $4x+y$ d). $4x+10y$

v). $(a+b)^2-(a-b)^2=$

a). $4ab$ b). $2(a^2+b^2)$

c). $a^2-4ab+2b^2$ c). a^4-b^4

vi). $(a+b+c)^2=$

a). $a^2+b^2+c^2$

b). $a^2+b^2+c^2+2(a+b+c)$

c). $a^2+b^2+c^2+2(ab+bc+ca)$

d). $a+b+c+2(ab+bc+ca)$

vii). $a^3+b^3=$

a). $(a+b)^3-2ab(a+b)$

b). $(a+b)(a^2+ab+b^2)$

c). $(a-b)(a^2-ab+b^2)$

d). $(a+b)(a^2-ab+b^2)$

viii). Conjugate of $3-\sqrt{5}$ is

a). $-3-\sqrt{5}$ b). $-3+\sqrt{5}$

c). $3+\sqrt{5}$ d). $\sqrt{3}+\sqrt{5}$

ix). Equivalent to $(m^2+4)^{\frac{-1}{2}}$ is

a). $\frac{(m^2+4)}{2}$ b). $-\sqrt{m^2+4}$

c). $\frac{1}{\sqrt{m^2+4}}$ d). $\frac{1}{m+2}$

x). For which of the following expressions $a+b$ is not a factor?

a). a^2-b^2 b). a^2+b^2

c). a^3+b^3 c). a^4-b^4

Q2. Simplify $\frac{12x^4y^5}{25a^3b^4}\cdot\frac{15a^5b^4}{16x^7y^2}$

Solution: Given $\frac{12x^4y^5}{25a^3b^4}\cdot\frac{15a^5b^4}{16x^7y^2}$

$$=\frac{12\times15.a^{5-3}b^{4-4}.y^{5-2}}{16\times25.x^{7-4}}$$

$$=\frac{3\times3.a^2b^0.y^3}{4\times5.x^3}$$

$$=\frac{9a^2y^3}{20x^3}$$

Q3: Evaluate $\frac{2x-3}{x^2-x+1}$ for $x=2$

Solution: given $\frac{2x-3}{x^2-x+1}$ put $x=2$ we get

$$=\frac{2(2)-3}{2^2-2+1}$$

$$=\frac{4-3}{4-1}$$

$$=\frac{1}{3}$$

Q4. Find the values of x^2+y^2 and xy, when $x+y=7$, $x-y=3$

Sol: Given $x+y=7$, $x-y=3$

Using $2(x^2+y^2)=(x+y)^2+(x-y)^2$

Putting the values

$$2(x^2+y^2)=(7)^2+(3)^2$$

$$2(x^2+y^2)=49+9$$

$$x^2+y^2=\frac{58}{2}=29$$

Now Using $4xy=(x+y)^2-(x-y)^2$

Putting the values

$$4xy=(7)^2-(3)^2$$

$$4xy=49-9$$

$$xy=\frac{40}{4}=10$$

Q5. Find the value of $a+b+c$ when $a^2+b^2+c^2=43$ and $ab+bc+ca=3$

Sol: Given $a^2+b^2+c^2=43$ & $ab+bc+ca=3$

using $(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

Putting values $(a+b+c)^2=43+2(3)$

$$(a+b+c)^2=43+6$$

$$(a+b+c)^2=49$$

Taking square root on both sides

$$\sqrt{(a+b+c)^2}=\sqrt{49}$$

$$a+b+c=\pm7$$

Q6. If $a+b+c=6$ and $a^2+b^2+c^2=24$ then find the value of $ab+bc+ca$

Sol: Given $a+b+c=6$ and $a^2+b^2+c^2=24$

using $(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

Putting values $(6)^2=24+2(ab+bc+ca)$

$$36-24=2(ab+bc+ca)$$

$$\frac{12}{2}=ab+bc+ca$$

Or $ab+bc+ca=6$

Q7. If $2x-3y=8$ and $xy=2$ then find the value of $8x^3-27y^3$

Sol: Given $2x-3y=8$ & $xy=2$ Find $8x^3-27y^3$

Using $(a-b)^3=a^3-b^3-3ab(a-b)$ Accordingly

$$(2x-3y)^3=(2x)^3-(3y)^3-3(2x)(3y)(2x-3y)$$

Putting the values

$$(8)^3=8x^3-27y^3-18(2)(8)$$

$$512=8x^3-27y^3-288$$

$$512+288=8x^3-27y^3$$

Or $8x^3+27y^3=800$

Q8. If $x+\frac{1}{x}=8$, then find value of $x^3+\frac{1}{x^3}$

Sol: Given $x+\frac{1}{x}=8$ find $x^3+\frac{1}{x^3}$

Using $\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$

Putting the values

$$(8)^3=x^3+\frac{1}{x^3}+3(8)$$

$$512=x^3+\frac{1}{x^3}+24$$

$$512-24=x^3+\frac{1}{x^3}$$

Or $x^3+\frac{1}{x^3}=488$

Q9. Find product $\left(\frac{4x}{5}-\frac{5}{4x}\right)\left(\frac{16x^2}{25}+\frac{25}{16x^2}+1\right)$

Sol: Given $\left(\frac{4x}{5}-\frac{5}{4x}\right)\left(\frac{16x^2}{25}+\frac{25}{16x^2}+1\right)$

$$=\frac{4x}{5}\left(\frac{16x^2}{25}+\frac{25}{16x^2}+1\right)-\frac{5}{4x}\left(\frac{16x^2}{25}+\frac{25}{16x^2}+1\right)$$

$$=\frac{64x^3}{125}+\frac{100x}{80x^2}+\frac{4x}{5}-\frac{80x^2}{100x}-\frac{125}{64x^3}-\frac{5}{4x}$$

Simplifying and rearranging

$$=\frac{64x^3}{125}+\frac{5}{4x}-\frac{5}{4x}+\frac{4x}{5}-\frac{4x}{5}-\frac{125}{64x^3}$$

$$=\frac{64x^3}{125}-\frac{125}{64x^3}$$

Q10. Simplify $\frac{2x^2}{x^4-16}-\frac{x}{x^2-4}+\frac{1}{x+2}$

Sol: Given $\frac{2x^2}{x^4-16}-\frac{x}{x^2-4}+\frac{1}{x+2}$

$$=\frac{2x^2}{x^4-16}-\frac{x}{x^2-4}+\frac{1}{x+2}\times\frac{x-2}{x-2}$$

$$=\frac{2x^2}{x^4-16}-\frac{x}{x^2-4}+\frac{x-2}{x^2-4}$$

$$=\frac{2x^2}{x^4-16}-\frac{x+x-2}{x^2-4}$$

$$=\frac{2x^2}{x^4-16}-\frac{2x-2}{x^2-4}\times\frac{x^2+4}{x^2+4}$$

$$=\frac{2x^2}{x^4-16}-\frac{2x\left(x^2+4\right)-2\left(x^2+4\right)}{\left(x^2\right)^2-(4)^2}$$

$$=\frac{2x^2}{x^4-16}-\frac{2x^3+8x-2x^2-8}{x^4-16}$$

$$=\frac{2x^2-2x^3-8x+2x^2+8}{x^4-16}$$

$$=\frac{4x^2-2x^3-8x+8}{x^4-16}$$