Chapter 13

Circumcircle:
The circle which passes through the three vertices of a triangle is called the Circumcircle of the triangle.

Inscribed circle or in-circle of triangle:
The circle inscribed with in a triangle so as to touch each of its sides is called the inscribed circle of the triangle. Radius of in-circle is called the in-radius denoted by **r** & the centre of inscribed circle is called incentre denoted by ***I***.

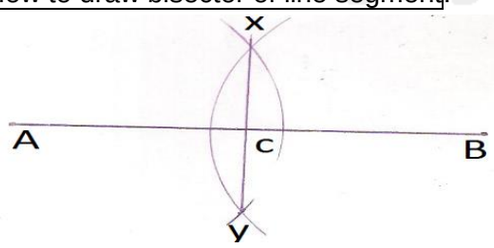
Escribed circle or e-circle of a triangle:
A circle which touches one side externally and the other two produced sides of triangle internally is called an escribed circle of triangle.

A tangent to a point of the circle:
A line touching a circle at a point and perpendicular to its radius through that point is called tangent to that circle at that point.

Direct common tangent:
If the points of contact of a common tangent to the tow circles are on the same side of the line joining their centers, then this common tangent is called direct common tangent.

Transverse Common tangent:
If the points of contact of the common tangent to the two circles lie on opposite side of the line joining the centres of the circles, then these tangent are called transverse common tangent.

How to draw bisector of line segment:

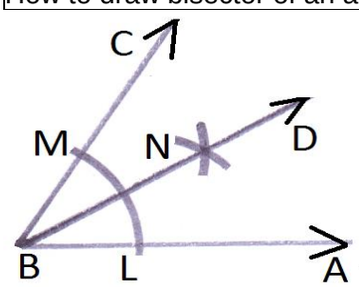


Suppose we have a line segment $\overline{AB} = 5cm$

i). Draw arcs from A and B of radius 3cm (more than half of the given line segment)
ii). By joining the point of intersection we get $\overleftrightarrow{XY}$

i.e., $m\overline{AC} = m\overline{CB}$ **and** $\overleftrightarrow{XY} \perp \overline{AB}$ **or**
$m\angle ACX = m\angle BCX = 90^o$ **or**
$m\angle ACY = m\angle BCY = 90^o$

How to draw bisector of an angle:

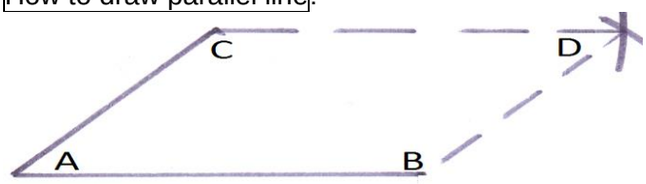


Suppose we have a an angle $\angle ABC$

i). Draw arcs at B of radius 1cm which intersects the rays $\overrightarrow{BA}$ at L and $\overrightarrow{BC}$ at M
ii). draw an arcs from L and M which intersect at N
iii). Join B to N i.e. $\overrightarrow{BN}$

$\overrightarrow{BN}$ is an angle bisector of $\angle ABC$
i.e., $m\angle ABD = m\angle CBD$
or $m\angle ABC = 2m\angle CBD = 2m\angle ABD$

How to draw parallel line:

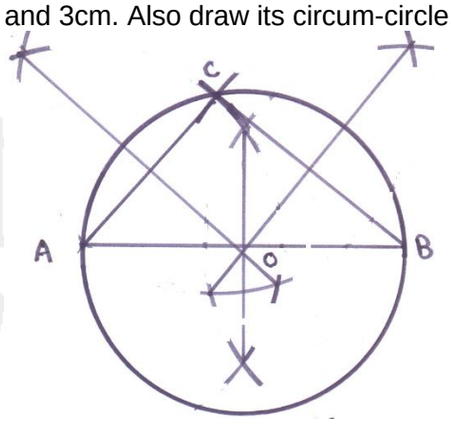


Suppose we have two lines $\overline{AB}$ & $\overline{AC}$

i). Draw arcs at B of radius $\overline{AC}$ & Draw arcs at C of radius $\overline{AB}$ which intersects at D
iii). Join D to B & C we get $\overline{AB} \| \overline{CD}$ & $\overline{AC} \| \overline{BD}$

Exercise 13.1

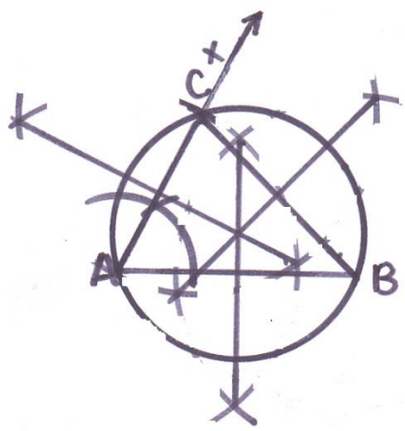
Q1: Construct a triangle with sides 2cm, 2.5cm and 3cm. Also draw its circum-circle.



Steps of construction:

i). Draw a line segment $m\overline{AB} = 3cm$
ii). At point A draw an arc of 2cm
iii). At point B draw an arc of 2.5cm which intersects at point C
iv). Join C to A and B thus ΔABC is formed
v). Draw perpendicular bisector of the sides which intersects at O
vi). At O draw a circle of radius $\overline{OA}$ or $\overline{OB}$ or $\overline{OC}$ which is the required circumcircle.

Q2: Construct triangle ABC such that
$m\overline{AB} = 3", m\overline{AC} = 4"$
and $m\angle A = 60^o$ Draw circum-circle



Steps of construction:

i). Draw a line segment $m\overline{AB} = 3"$

ii). At point A draw an angle of 60^o with the help of compass. i.e. $m\angle BAX = 60^o$

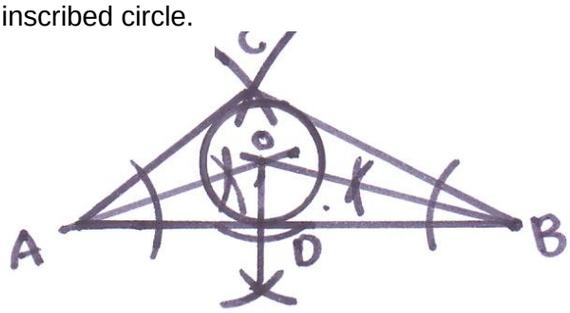
iii). At point A draw an arc of 4" which intersects the ray $\overrightarrow{AX}$ at point C

iv). Join B to C thus ΔABC is formed

v). Draw perpendicular bisector of the sides which intersects at O

vi). At O draw a circle of radius $\overline{OA}$ or $\overline{OB}$ or $\overline{OC}$ which is the required circumcircle.

Q3: Suppose we have a triangle whose sides are 3cm, 4cm and 6cm respectively. Draw its inscribed circle.



Steps of construction:

i). Draw a line segment $m\overline{AB} = 6cm$

ii). At point A draw an arc of 3cm

iii). At B draw an arc of 4cm which intersects at C

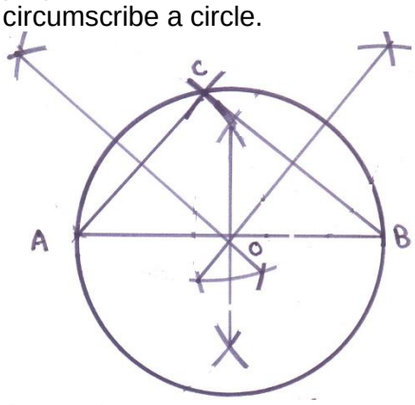
iv). Join C to A and B we get a ΔABC

v). draw angle bisectors of angle A and B which intersects at O

vi). Draw altitude from O to any side of triangle AB which intersects at D

vii). Draw a circle with center O of radius $\overline{OD}$ which is the required inscribed circle

Q4: Construct a triangle ABC with sides $m\overline{AB} = 5cm$, $m\overline{BC} = 6cm$ and $m\overline{CA} = 8cm$ Draw perpendicular bisector of its sides and the then circumscribe a circle.



Steps of construction:

i). Draw a line segment $m\overline{CA} = 8cm$

ii). At point A draw an arc of 5cm

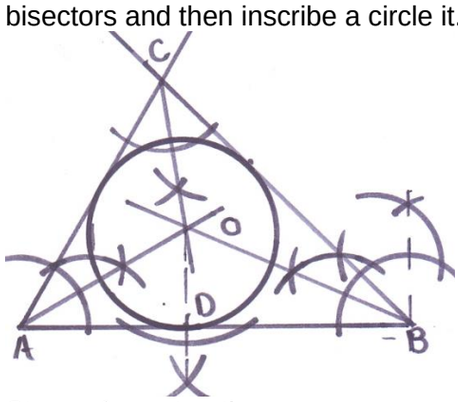
iii). At C draw an arc of 6cm which intersects at B

iv). Join B to A and C thus ΔABC is formed

v). Draw perpendicular bisector of the sides which intersects at O

vi). At O draw a circle of radius $\overline{OA}$ or $\overline{OB}$ or $\overline{OC}$ which is the required circumcircle.

Q5: Draw a triangle ABC with $m\angle A = 60^o$ & $m\angle B = 45^o$ Draw three angle bisectors and then inscribe a circle it.



Steps of construction:

i). Draw a suitable line segment $m\overline{AB} = 6cm$

ii). At point A draw an angle of 60^o with the help of compass. i.e. $m\angle BAX = 60^o$

iii). At B draw an angle of 45^o with the help of compass. i.e. $m\angle ABY = 45^o$

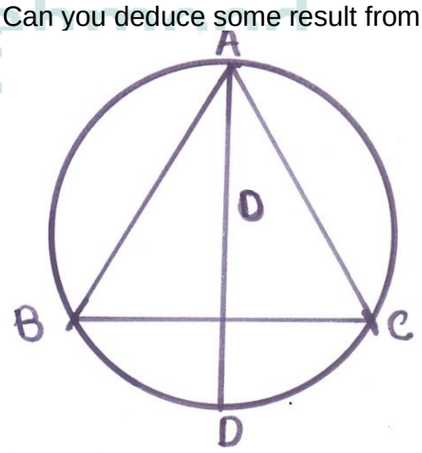
iv). Rays $\overrightarrow{AX}$ & $\overrightarrow{BY}$ intersects at C, thus ΔABC formed

v). To draw angle bisectors of angle A,B and C which intersects at O

vi). Draw altitude from O to any side of triangle AB which intersects at D

vii). Draw a circle with center O of radius $\overline{OD}$ which is the required inscribed circle

Q6: An equilateral triangle in inscribed in a circle. Find the altitude of the triangle if the radius r of the circle varies as under $r = 3$ units, $r = 4$ units, $r = 6$ units, $r = 12$ units. Can you deduce some result from this?



Steps of construction:

i). Draw a circle of radius 3cm

ii). Draw diameter AD

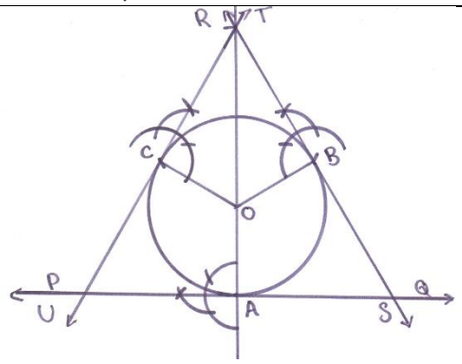
iii). At point D draw an arcs 3cm which intersects the circle at B and C

iv) Join A to B & C & B to C, equilateral triangle is formed

Here Altitude =4.5cm when radius =3cm

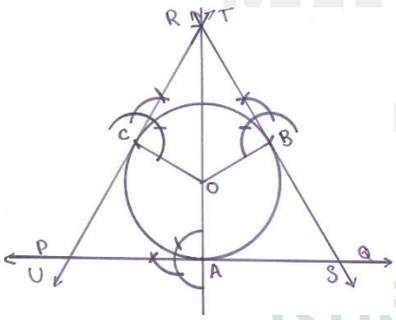
Hence Altitude = $\frac{3}{2}$ radius

Q7: An equilateral triangle is circumscribed about a circle. Find the altitude of the triangle if the radius r of the circle varies as $r = 2$ units, $r = 5$ units, $r = 10$ units. Can you deduce some result from this?



Steps of construction:
i). Draw a circle of radius 2cm
ii). Draw diameter AD
iii). At point O draw an angles of 60^o in a reverse direction or which intersects the circle at B and C i.e. $m\angle AOB = 120^o$ and $m\angle AOC = 120^o$
iv). At point A,B & C draw right angle or tangents
i.e. $\overline{PQ}$ is tangent at A, $\overline{RS}$ is tangent at B and $\overline{TU}$ is tangent of C thus equilateral triangle is formed
Hence Altitude = 3 x radius

Q8: Circumscribe an equilateral triangle about a circle of radius $2'', 3''$ and $1''$



Steps of construction:
i). Draw a circle of radius 2cm
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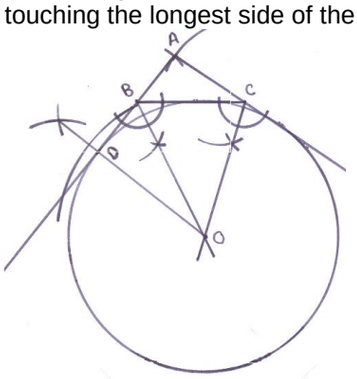
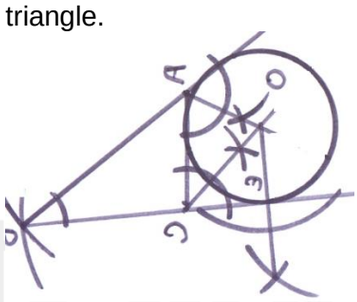
iv). At point A,B & C draw right angle or tangents
i.e. $\overline{PQ}$ is tangent at A, $\overline{RS}$ is tangent at B and $\overline{TU}$ is tangent of C thus equilateral triangle is formed

Q9: Draw a triangle with sides 2.5cm, 3.5cm and 4.5cm long. Draw an escribed circle to the triangle touching the longest side of the triangle.



Steps of construction:
i). Draw a line segment $\overline{AB} = 3.5cm$
ii). At A draw an arc of radius 2.5cm
iii). At B draw an arc of radius 4.5cm which intersects at C, Join C to A and B thus triangle is formed
iv). Produce $\overrightarrow{AB}$ and $\overrightarrow{AC}$ to make exterior angles at B and C
v). Bisect two exterior angles and one interior angle which intersects at O
vi). Draw an altitude from O to the ray $\overrightarrow{AB}$ which intersect at D
vii) draw a circle of radius $\overline{OD}$ which is required circle

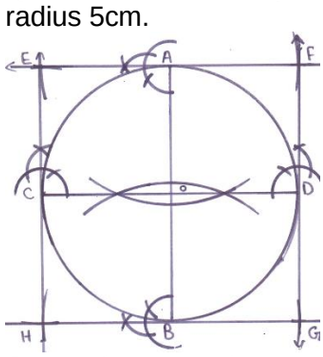
Q10: Draw a triangle with sides 2.5cm, 3.5cm and 4.5cm long. Draw an escribed circle to the triangle touching the smallest side of the triangle.



Steps of construction:
i). Draw a line segment $\overline{AB} = 4.5cm$
ii). At A draw an arc of radius 3.5cm
iii). At B draw an arc of radius 2.5cm which intersects at C, Join C to A and B thus triangle is formed
iv). Produce $\overrightarrow{BA}$ and $\overrightarrow{BC}$ to make exterior angles at A and C
v). Bisect two exterior angles and one interior angle which intersects at O
vi). Draw an altitude from O to the ray $\overrightarrow{BA}$ which intersect at E
vii) draw a circle of radius $\overline{OE}$ which is required circle

Exercise 13.2

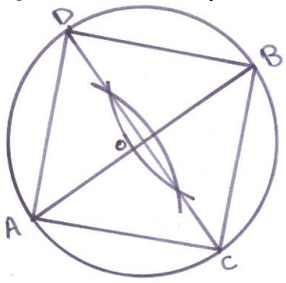
Q1: Circumscribe a square about a circle of radius 5cm.



Steps of constructions
i). Draw a circle of radius 5cm
ii). Draw diameter AB
iii). Draw perpendicular bisector which intersects the circle at C and D
iv). Draw an arc of radius 5cm from C and A which intersects at E continuing this manner we get the points of intersection F,G and H

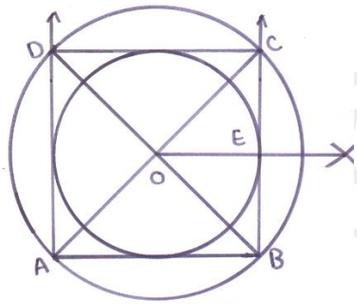
v) Join E to F which passes through A, which is tangent
an so on we get a required square EFGH

Q2: Inscribe a square in a circle of radius 6cm.



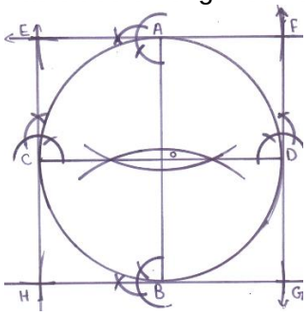
Steps of constructions
i). Draw a circle of radius 5cm
ii). Draw diameter AB
iii). Draw perpendicular bisector which intersects the circle at C and D
iv). Join A & B to C & D. we get required square

Q3: Draw a square of side 6cm. circumscribe a circle that square and then inscribe a circle in the same square. Measure the radii of these two circles.



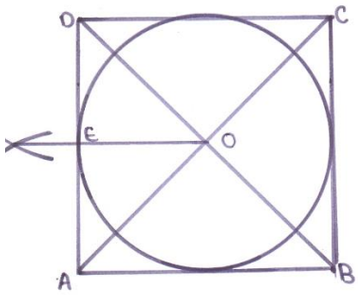
Steps of constructions
i). Draw a line segment $\overline{AB} = 6cm$
ii). Draw an angle of 90^o with the help of protector at point A and B
iii). Draw an arc of radius 6cm at B and A which intersects at C and D
iv). Join C to D. we get a square ABCD
v). Join A to C and B to D (diagonals) which intersects at O
vi). Draw an altitude from O to one of the side $\overline{CD}$ which intersects at E
vii) Draw a circle of radius $\overline{OA}$ which is required circumscribed circle
viii) Draw a circle of radius $\overline{OE}$ which is required inscribed circle
radius of inscribe circle 3cm
radius co circumscribe circle 4.2cm

Q4: First draw a circle of suitable radius, so that the square circumscribed about that circle has sides of length 8units.



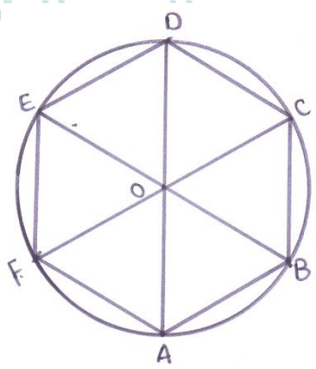
Steps of constructions
i). Draw a circle of radius 4cm
ii). Draw diameter AB
iii). Draw perpendicular bisector which intersects the circle at C and D
iv). Draw an arc of radius 4cm from C and A which intersects at E continuing this manner we get the points of intersection F,G and H
v) Join E to F which passes through A, which is tangent
and so on we get a required square EFGH

Q5: Inscribe a square of side 10cm in a circle what will be the size of the radius?



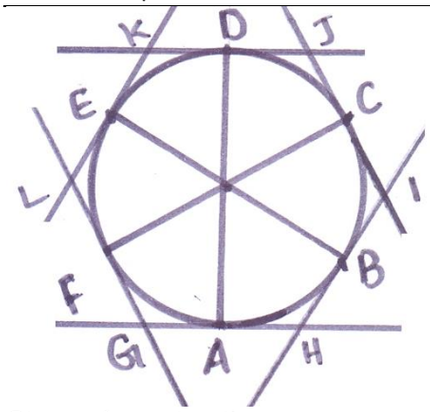
Steps of constructions
i). Draw a line segment $\overline{AB} = 10cm$
ii). Draw an angle of 90^o with the help of protector at point A and B
iii). Draw an arc of radius 6cm at B and A which intersects at C and D
iv). Join C to D. we get a square ABCD
v). Join A to C and B to D (diagonals) which intersects at O
vi). Draw an altitude from O to one of the side $\overline{BC}$ which intersects at E
vii) Draw a circle of radius $\overline{OE}$ which is required inscribed circle

Q6: Inscribe a regular hexagon in a circle of radius 4cm.



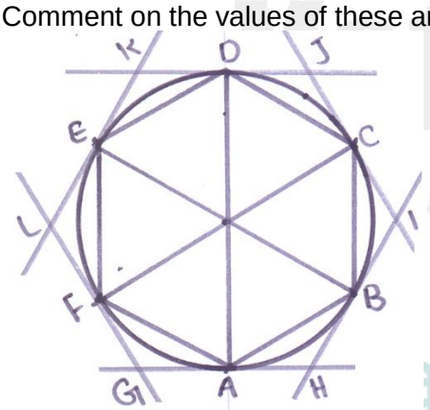
Steps of constructions
i). Draw a circle with centre O of radius 4cm
ii). Draw diameter AD
iii). At A draw an arc of radius 4cm which intersects the circle at B and F
iv). Join B to O up to E and F to O up to C
v) Join A to B & F , C to B and D and E to D and F
we get a required Inscribe regular hexagon ABCDEF

Q7: Construct a circle of radius 4cm and draw a regular hexagon



Steps of constructions
i). Draw a circle with centre O of radius 4cm
ii). Draw diameter AD
iii). At A draw an arc of radius 4cm which intersects the circle at B and F
iv). Join B to O up to E and F to O up to C
v) Draw tangents(right angles) at A,B.C.D,E,F
vi). we get a required circumscribe regular hexagon

Q8: Draw a circle of radius 8cm. circumscribe a regular hexagon about that circle and also inscribe a regular hexagon in the same circle. Find the areas of these geometrical figures. Comment on the values of these areas.



Steps of constructions
i). Draw a circle with centre O of radius 4cm
ii). Draw diameter AD
iii). At A draw an arc of radius 4cm which intersects the circle at B and F
iv). Join B to O up to E and F to O up to C
v) Draw tangents(right angles) at A,B.C.D,E,F
vi). we get a required circumscribe regular hexagon
vii) Join A to B & F , C to B and D and E to D and F,we get a required Inscribe regular hexagon ABCDEF of side 4cm
Now to find Area of inscribe regular hexagon ABCDEF

Area $= \mathcal{S}^2 \times \frac{3\sqrt{3}}{2}$ Side of hexagon AB = 4cm

Area $= 4^2 \times \frac{3\sqrt{3}}{2} = 16 \times \frac{3\sqrt{3}}{2} = 24\sqrt{3}$

Now to find Area of circumscribe regular hexagon GHIJKL

In ΔAOH, $m\overline{OA} = 4cm$ which is radius and $m\angle AOH = 30^o$ which obtained from complete rotation divided by 12 i.e. $\frac{360}{12} = 30^o$ To find AH

$\tan 30^o = \frac{AH}{OA}$

$\frac{1}{\sqrt{3}} = \frac{AH}{4}$

$AH = \frac{4}{\sqrt{3}}$

$\therefore GH = 2AH$

$GH = 2\left(\frac{4}{\sqrt{3}}\right) = \frac{8}{\sqrt{3}}cm$

Area $= \mathcal{S}^2 \times \frac{3\sqrt{3}}{2}$ Side of hexagon GH= $\frac{8}{\sqrt{3}}cm$

Area = $\left(\frac{8}{\sqrt{3}}\right)^2 \frac{3\sqrt{3}}{2} = \frac{64}{3} \times \frac{3\sqrt{3}}{2} = 32\sqrt{3}$

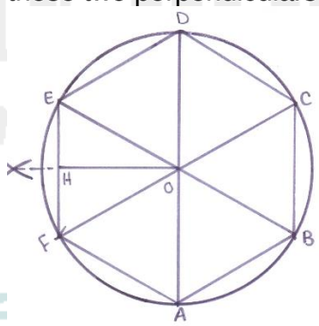
To find the relation
Area of inscribed hexagon = k (area of circumscribed hexagon)

$$24\sqrt{3} = k \times 32\sqrt{3}$$

$$k = \frac{24\sqrt{3}}{32\sqrt{3}} = \frac{3}{4}$$

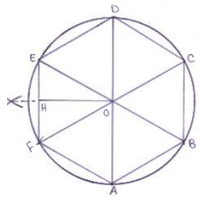
Area of inscribed hexagon $= \frac{3}{4} \times$ area of circumscribed hexagon

Q9: Draw two regular hexagons of perimeters 6cm and 30cm respectively. Determine their centres. From their centers draw perpendicular to any of their sides. What is the relation of these two perpendiculars?



Steps of constructions
i). Draw a circle with centre O of radius 5cm
ii). Draw diameter AD
iii). At A draw an arc of radius 5cm which intersects the circle at B and F
iv). Join B to O up to E and F to O up to C
v) Join A to B & F , C to B and D and E to D and F
we get a required Inscribe regular hexagon ABCDEF
vi). Draw an altitude from O to one of its side $\overline{EF}$ which bisects the sides at H

here $m\overline{OH}$ = 4.3 cm or Altitude $= \frac{\sqrt{3}}{2}$ radius



Steps of constructions
i). Draw a circle with centre O of radius 1cm
ii). Draw diameter AD
iii). At A draw an arc of radius 1cm which intersects the circle at B and F

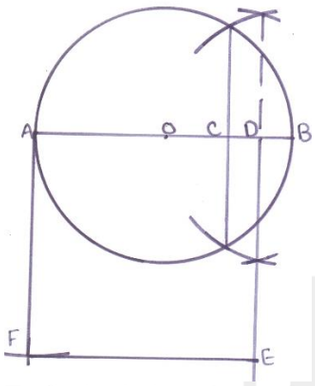
iv). Join B to O up to E and F to O up to C
v) Join A to B & F , C to B and D and E to D and F
we get a required Inscribe regular hexagon ABCDEF

vi). Draw an altitude $\overrightarrow{OG}$ from O to one of side $\overline{EF}$ which intersects at H

vi). Draw an altitude from O to one of its side $\overline{EF}$ which bisects the sides at H

here $m\overline{OH}$ = 0.9 cm or Altitude $= \frac{\sqrt{3}}{2}$ radius

Q10: Can you construct a square whose area equal to the area of given circle? Discuss in detail.



To find relation Area of a circle = k (Area of a square)

$$S^2 = k\pi r^2$$

Here k must be equal to 1 i.e. k = 1, put r = 4

$$S^2 = \frac{22}{7}(1)(4)^2 \qquad \therefore \pi = \frac{22}{7}$$

$$S^2 = (3.142)(16) = 50.272$$

Taking square root

$$S = \sqrt{50.272} = 7.09 \approx 7$$

Steps of constructions

i). Draw a circle with centre O of radius $r = 4cm$

ii). Draw a diameter $\overline{AB}$

iii). Draw perpendicular bisector of $\overline{BO}$ which interests at C

iv). Now draw a perpendicular bisector of $\overline{BC}$ which interests at D i.e. $\overrightarrow{DE}$ is perpendicular bisector of $\overline{BC}$

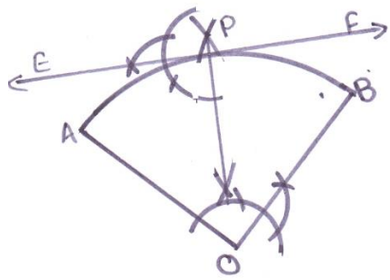
v). Draw an arc of radius $\overline{AD}$ from D which intersects the ray at E and similarly draw arcs of radius $\overline{AD}$ from A and E which interests at F

vi). Join F to A and E we get the required square

Exercise 13.3

Q1: Draw an arc of length 7cm. without using centre draw tangent through a given point A when A is

i). The middle point of arc



Steps of constructions:

i). Since arc length $l = 7cm$ & take $\theta = \frac{\pi}{2} = 90^o$ & using formula $l = r\theta$

$$7 = r\,\tfrac{\pi}{2}$$

$$\frac{7\times 2}{\pi} = r$$

$$r = 4.45 \simeq 4.5cm$$

ii). Draw a line segment $\overline{OC} = 4.5cm$

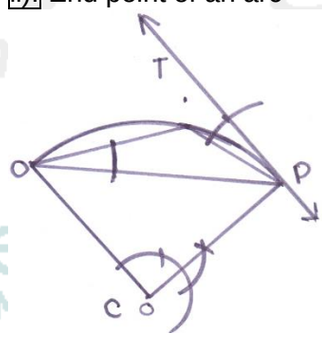
iii). Draw an angle of 90^o at O i.e. $m\angle OCP = 90^o$

iv). Draw an arc AB with centre O which should be the 7cm and join A to B

v). Draw perpendicular bisector of $\overline{AB}$ which intersects the arc at C

vi) Draw an angle of 90^o at P which gives required tangent

ii). End point of an arc



i). Since arc length $l = 7cm$ & take $\theta = \frac{\pi}{2} = 90^o$ & using formula $l = r\theta$

$$7 = r\,\tfrac{\pi}{2}$$

$$\frac{7\times 2}{\pi} = r$$

$$r = 4.45 \simeq 4.5cm$$

ii). Draw a line segment $\overline{OC} = 4.5cm$

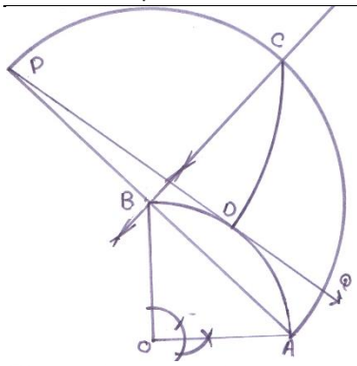
iii). Draw an angle of 90^o at O i.e. $m\angle OCP = 90^o$

iv). Draw an arc OP with centre C which should be the 7cm and join O to P

v). Take any point Q on arc OP and join Q to O and P

vi) Draw an angle of measure $m\angle QOP$ at P i.e. $m\angle QOP \cong m\angle QPT$ which gives required tangent

iii). Outside of an arc



Steps of constructions:

i). Since arc length $l = 7cm$ & take $\theta = \frac{\pi}{2} = 90^o$ & using formula $l = r\theta$

$$7 = r\frac{\pi}{2}$$

$$\frac{7\times 2}{\pi} = r$$

$$r = 4.45 \simeq 4.5cm$$

ii). Draw a line segment $\overline{OC} = 4.5cm$

iii). Draw an angle of 90^o at O i.e. $m\angle OCP = 90^o$

iv). Draw an arc AB with centre O which should be the 7cm & join A to B and produce up to P such that $\overline{AB} = \overline{BP}$

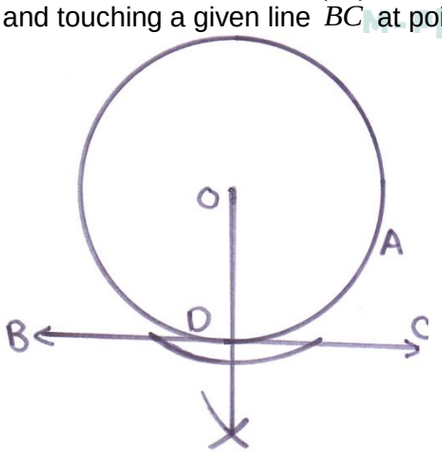
v). Draw perpendicular bisector of $\overline{AP}$ which intersects the arc at B

vi) Draw a semicircle of radius $\overline{AB} = \overline{BP}$ which intersects the perpendicular line at C

vii). Draw an arc of radius $\overline{CP}$ from P which intersects the arc AB at D

viii). Join P to D and Produce which is required tangent

Q2: Draw a circle passing through a point A and touching a given line $\overleftrightarrow{BC}$ at point D



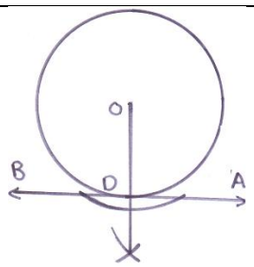
Steps of constructions:

i). Draw a line segment $\overline{BC}$

i). Take a point O, draw altitude from O to the line segment $\overline{BC}$ which intersects at D

ii). Draw circle of radius $\overline{OD}$ with centre O which passing through A

Q3: Describe a circle of radius 4cm, passing through a given point C and touching a give line $\overleftrightarrow{AB}$



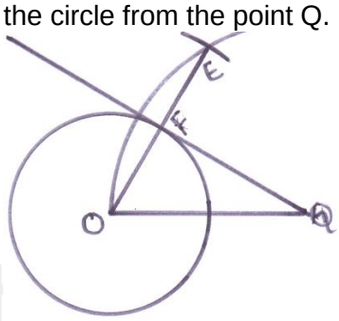
Steps of constructions:

i). Draw a line segment $\overline{AB}$

i). Take a point O, draw altitude from O to the line segment $\overline{AB}$ which intersects at D

ii). Draw circle of radius $\overline{OD}$ with centre O which passing through C

Q4: Radius of a circle is 2.5cm. A point Q is at a distance of 5cm from centre. Draw tangent to the circle from the point Q.



Steps of constructions:

i). Draw a line segment $m\overline{OQ} = 5cm$

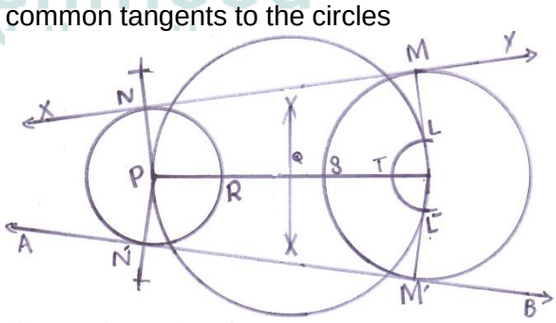
ii). Take a point O, draw circle of radius 2.5cm

iii). Draw an arc of radius 5cm from Q and then from O which intersects at E

iv). Join O to E which intersects the circle at F

v). Join Q to F and produce which is required tangent

Q5: Radii of two circles are 2cm and 3cm and their centers are 8cm apart. Draw direct common tangents to the circles



Steps of constructions:

i). Draw a line segment $m\overline{PO} = 8cm$

ii). Draw a circle of radius 2cm at P and 3cm at O which intersects the line segment $\overline{OP}$ at S and R

iii). Draw perpendicular bisector of $\overline{OP}$ which intersects at Q

iv). Draw a semicircle of radius $\overline{PQ}$ or $\overline{QO}$ at Q

v). Take a point T such that $m\overline{PR} = m\overline{ST}$

vi). Draw an arc of radius $\overline{OT}$ which intersects the semicircle at L

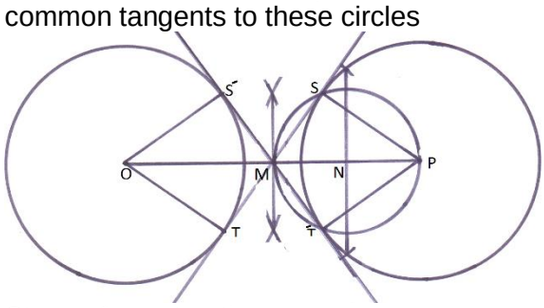
vii). Join O to L and produce up to M

viii). Draw a line from P which is parallel to $\overline{OM}$

i.e. $\overline{PN} \| \overline{OM}$

ix). Join M to N which is the required tangent

Q6: Two congruent circles of radius 4cm each. Their centers are 10cm apart. Draw transverse common tangents to these circles



Steps of constructions:

i). Draw a line segment $m\overline{OP} = 10cm$

ii). Draw congruent circles of radius 4cm at P and O

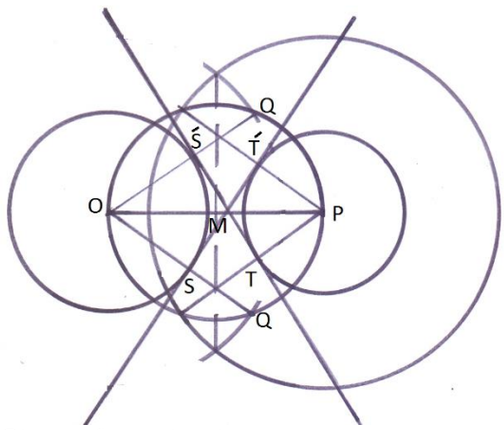
iii). Draw mid point M of OP & Draw mid point N of MP

iv). Draw a circle of radius NP at point N which cuts the circle at S & T'

v).Measure an angle NPS and mark angles MOS' & MOT

iv). Join S to T & S' to T' which is the required tangent

Q7: Radii of two circles are 2cm and 2.5cm and their centers are 5.5cm apart. Draw transverse common tangents to the circles



Steps of constructions:

i). Draw a line segment $m\overline{PO} = 5.5cm$

ii). Draw a circle of radius 2cm at P and 2.5cm at O

iii). Draw perpendicular bisector of $\overline{OP}$

iv). Here is $r_1 + r_2 = 2 + 2.5 = 4.5cm$ Draw arc of radius 4.5cm at O

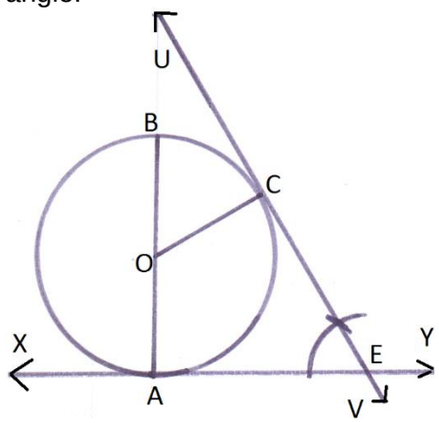
v). at C draw arc of radius half of $\overline{OP}$ which intersects the semi circle at Q, Join O to Q which is intersecting the bigger circle at T

vi). Draw a line from P which is parallel to $\overline{OT}$

i.e. $\overline{PS} \| \overline{TO}$

ix). Join S to T which is the required tangent

Q8: Draw $m\angle ABC = 60^o$ construct a circle having 2.5cm and touching the arms of the angle.



Steps of constructions:

i). Draw a circle of radius 2.5cm

ii). Draw a diameter $\overline{AB}$

iii). At A draw a right angle & at O draw angle of 60^o which intersects the circle at C

iv). Draw a right angle at C which intersects the ray $\overleftrightarrow{XY}$ at E

v). measure the angle at E which is equal to 60^o