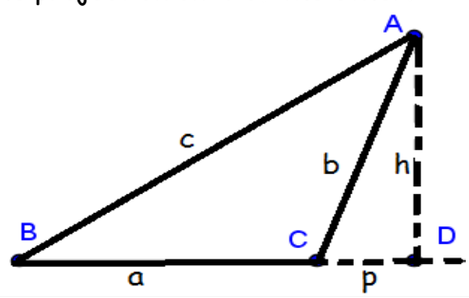
CHPATER 8

Theorem 8.1 Statement: In an obtuse angled triangle, the square on the side opposite to the obtuse angle is equal to the sum of the square on the sides containing the obtuse angle together with twice the rectangle contained be the sides, and the projection on it of the other.

Given: In $\Delta ABC, m\angle C > 90°$, Let us denote $m\overline{AB} = c, m\overline{AC} = b$ and $m\overline{AB} = c$. $m\overline{AD} = h$ is perpendicular from A to $\overline{BC}$ (Produced) so that $m\overline{CD} = p$ is projection of $\overline{AC}$ on $\overline{BC}$

To Prove: $\left(m\overline{AB}\right)^2 = \left(m\overline{AC}\right)^2 + \left(m\overline{BC}\right)^2 + 2\left(m\overline{BC}\right)\left(m\overline{CD}\right)$

Or $c^2 = a^2 + b^2 + 2ap$



Proof	Statements	Reasons
	ΔADC is right angled triangle	
	$\therefore \left(m\overline{AC}\right)^2 = \left(m\overline{AD}\right)^2 + \left(m\overline{CD}\right)^2 \cdots (1)$	Pythagoras theorem
	ΔADB is right angled triangle	
	$\therefore \left(m\overline{AB}\right)^2 = \left(m\overline{AD}\right)^2 + \left(m\overline{BD}\right)^2 \cdots (2)$	
	$\Rightarrow \left(m\overline{AB}\right)^2 = \left(m\overline{AD}\right)^2 + \left(m\overline{BC} + m\overline{CD}\right)^2$	Pythagoras theorem
	$\Rightarrow \left(m\overline{AB}\right)^2 = \left(m\overline{AD}\right)^2 + \left(m\overline{BC}\right)^2 + \left(m\overline{CD}\right)^2 + 2\left(m\overline{BC}\right)\left(m\overline{CD}\right)$	
	$\Rightarrow \left(m\overline{AB}\right)^2 = \left(m\overline{AD}\right)^2 + \left(m\overline{CD}\right)^2 + \left(m\overline{BC}\right)^2 + 2\left(m\overline{BC}\right)\left(m\overline{CD}\right)$	$(x+y)^2 = x^2 + y^2 + 2xy$
	$\Rightarrow \left(m\overline{AB}\right)^2 = \left(m\overline{AC}\right)^2 + \left(m\overline{BC}\right)^2 + 2\left(m\overline{BC}\right)\left(m\overline{CD}\right)$	Using Equation (1)

Theorem 8.2: In any triangle, square on the side opposite to the acute angle is equal to the sum of the squares on the sides containing that acute angle diminished by twice the rectangle contained by one of those sides and the projection on it of the other.

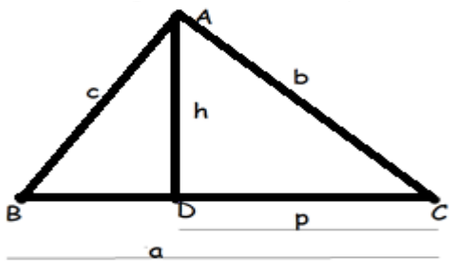


Figure A is acute angled triangle

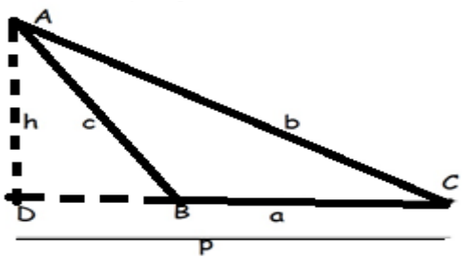


Figure B is obtuse angle triangle

Given: In $\Delta ABC, m\angle C < 90°$, Let us denote $m\overline{AB} = c, m\overline{AC} = b$ and $m\overline{AB} = c$. $m\overline{AD} = h$ is perpendicular from A to $\overline{BC}$ (Produced) so that $m\overline{CD} = p$ is projection of $\overline{AC}$ on $\overline{BC}$

To Prove: $\left(m\overline{AB}\right)^2 = \left(m\overline{BC}\right)^2 + \left(m\overline{CA}\right)^2 - 2\left(m\overline{BC}\right)\left(m\overline{CD}\right)$ Or $c^2 = a^2 + b^2 - 2ap$

Proof	Statements	Reasons
	ΔADC is right angled triangle	
	$\therefore \left(m\overline{AC}\right)^2 = \left(m\overline{AD}\right)^2 + \left(m\overline{CD}\right)^2 \cdots (1)$	Pythagoras theorem
	ΔADB is right angled triangle	
	$\therefore \left(m\overline{AB}\right)^2 = \left(m\overline{AD}\right)^2 + \left(m\overline{BD}\right)^2 \cdots (2)$	Pythagoras theorem
	$\Rightarrow \left(m\overline{AB}\right)^2 = \left(m\overline{AD}\right)^2 + \left(m\overline{BC} - m\overline{CD}\right)^2$	$\therefore m\overline{BC} = m\overline{BD} + m\overline{DC}$
	$\Rightarrow \left(m\overline{AB}\right)^2 = \left(m\overline{AD}\right)^2 + \left(m\overline{BC}\right)^2 + \left(m\overline{CD}\right)^2 - 2\left(m\overline{BC}\right)\left(m\overline{CD}\right)$	$\Rightarrow m\overline{BC} - m\overline{DC} = m\overline{BD}$
	$\Rightarrow \left(m\overline{AB}\right)^2 = \left(m\overline{AD}\right)^2 + \left(m\overline{CD}\right)^2 + \left(m\overline{BC}\right)^2 - 2\left(m\overline{BC}\right)\left(m\overline{CD}\right)$	$\therefore (x-y)^2 = x^2 + y^2 - 2xy$
	$\Rightarrow \left(m\overline{AB}\right)^2 = \left(m\overline{AC}\right)^2 + \left(m\overline{BC}\right)^2 - 2\left(m\overline{BC}\right)\left(m\overline{CD}\right)$	Using Equation (1)

Theorem 8.3 **Statement:** In any triangle, the sum of square of any two sides is equal to twice square of on half the third side together with twice the square on median which bisect the third side.

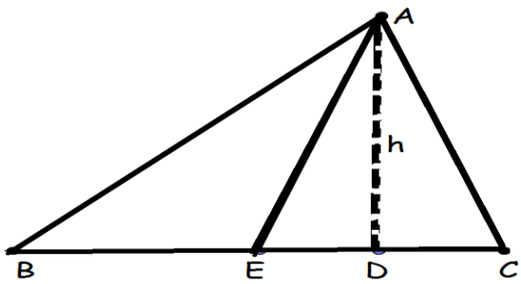


Figure 1 is acute angled triangle

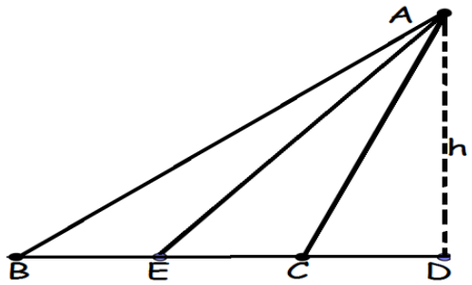


Figure 2 is obtuse angle triangle

Given: In $\Delta ABC, \overline{AE}$, is the median drawn from the Vertex A then $m\overline{BE} = m\overline{EC}$.

To Prove: $\left(m\overline{AB}\right)^2 + \left(m\overline{AC}\right)^2 = 2\left(m\overline{EB}\right)^2 + 2\left(m\overline{AE}\right)^2$

Proof Statements	Reasons
ΔADE is right angled triangle $\therefore \left(m\overline{AE}\right)^2 = \left(m\overline{AD}\right)^2 + \left(m\overline{ED}\right)^2 \cdots(1)$	Pythagoras theorem
ΔADC is right angled triangle $\therefore \left(m\overline{AC}\right)^2 = \left(m\overline{AD}\right)^2 + \left(m\overline{CD}\right)^2 \cdots(2)$	Pythagoras theorem
ΔADB is right angled triangle $\therefore \left(m\overline{AB}\right)^2 = \left(m\overline{AD}\right)^2 + \left(m\overline{BD}\right)^2 \cdots(3)$	Pythagoras theorem
$\Rightarrow \left(m\overline{AB}\right)^2 + \left(m\overline{AC}\right)^2 = \left(m\overline{AD}\right)^2 + \left(m\overline{AD}\right)^2 + \left(m\overline{CD}\right)^2 + \left(m\overline{BD}\right)^2$	Adding equations (2) and (3)
$\therefore m\overline{DE} + m\overline{DC} = m\overline{EC}$	
$\Rightarrow m\overline{DC} = m\overline{EC} - m\overline{DE}$	
$\Rightarrow \left(m\overline{AB}\right)^2 + \left(m\overline{AC}\right)^2 = 2\left(m\overline{AD}\right)^2 + \left(m\overline{EC} - m\overline{ED}\right)^2 + \left(m\overline{BE} + m\overline{ED}\right)^2$	
$\therefore m\overline{BD} = m\overline{BE} + m\overline{ED}$	
$\Rightarrow \left(m\overline{AB}\right)^2 + \left(m\overline{AC}\right)^2 = 2\left(m\overline{AD}\right)^2 + \left(m\overline{BE} - m\overline{ED}\right)^2 + \left(m\overline{BE} + m\overline{ED}\right)^2$	
$\Rightarrow m\overline{DC} = m\overline{EC} - m\overline{DE}$	
$\Rightarrow \left(m\overline{AB}\right)^2 + \left(m\overline{AC}\right)^2 = 2\left(m\overline{AD}\right)^2 + 2\left(m\overline{BE}\right)^2 + 2\left(m\overline{ED}\right)^2$	Since $m\overline{BE} = m\overline{EC}$
$\Rightarrow \left(m\overline{AB}\right)^2 + \left(m\overline{AC}\right)^2 = 2\left(m\overline{AD}\right)^2 + 2\left(m\overline{ED}\right)^2 + 2\left(m\overline{BE}\right)^2$	$\therefore (x+y)^2 + (x-y)^2 = 2x^2 + 2y^2$
$\Rightarrow \left(m\overline{AB}\right)^2 + \left(m\overline{AC}\right)^2 = 2\left\{\left(m\overline{AD}\right)^2 + \left(m\overline{ED}\right)^2\right\} + 2\left(m\overline{BE}\right)^2$	
$\Rightarrow \left(m\overline{AB}\right)^2 + \left(m\overline{AC}\right)^2 = 2\left(m\overline{AE}\right)^2 + 2\left(m\overline{BE}\right)^2$	Using Equation (1)

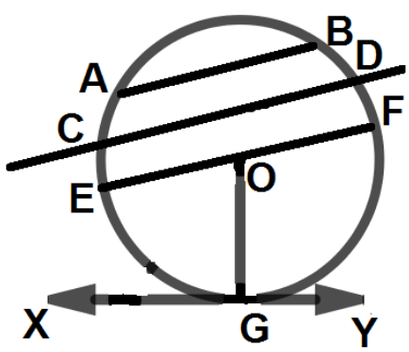
Circle: A circle is a set of points in a plane which are equidistance from a fixed point of the plane. The fixed point is called center of the circle.

Radius of the Circle: The radius of the circle is a straight line drawn from the center to the boundary line or circumference. The plural of the word radius is radii. $\overline{OG}$

Circumference: The circumference of a circle is the boundary line or perimeter of the circle.

Chord: A line segment whose end points are any two points of a circle is called a chord of the circle. $\overline{AB}$

Diameter of the circle: A chord passing through center of circle is called diameter of the circle. $\overline{EF}$



Tangent to a circle: A line which intersects a circle at one point only and none of its points lie in the interior of circle is called a tangent to circle. $\overleftrightarrow{XY}$

Secant to a circle: If there are two distinct points common between a line and a circle, the line is called secant to the circle. If there no point common between a line and a circle, we say that the line and the circle do not intersect. $\overleftrightarrow{CD}$

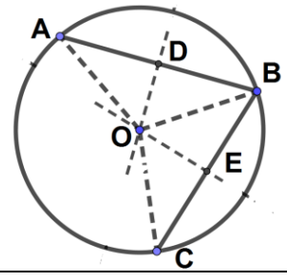
CHPATER 9

Theorem 9.1: One and only one circle can pass through three non-collinear points.

Given: A,B and C are three non-collinear points

To Prove: One and only one circle can pass through the points A,B and C i.e., $\overline{OA} = \overline{OB} = \overline{OC}$

Construction: Join B to A and C, Draw perpendicular bisectors $\overleftrightarrow{XD}$ and $\overleftrightarrow{YE}$ of $\overline{AB}$ and $\overline{BC}$ respectively, which intersect at O.



Proof Statements	Reasons
In $\Delta OAD \leftrightarrow \Delta OBD$	
$\angle ODA \cong \angle ODB$	$\overleftrightarrow{XD} \perp \overline{AB}$ (Construction)
$\overline{AD} \cong \overline{BD}$	$\overleftrightarrow{XD}$ is bisector of $\overline{AB}$
$\overline{OD} \cong \overline{OD}$	Common
$\therefore \Delta OAD \cong \Delta OBD$	SAS Postulate
$\therefore \overline{OA} \cong \overline{OB} \cdots (1)$	Corresponding sides of congruent triangles
Now in $\Delta OBE \leftrightarrow \Delta OCE$	
$\angle OEB \cong \angle OEC$	$\overleftrightarrow{YE} \perp \overline{BC}$
$\overline{BE} \cong \overline{EC}$	$\overleftrightarrow{YE}$ is bisector of $\overline{BC}$
$\overline{OE} \cong \overline{OE}$	Common
$\therefore \Delta OBE \cong \Delta OCE$	SAS Postulate
$\therefore \overline{OB} \cong \overline{OC} \cdots (2)$	Corresponding sides of congruent triangles
$\overline{OA} \cong \overline{OB} \cong \overline{OC}$	From equations (1) & (2)/Transitive property
$\Rightarrow m\overline{OA} \cong m\overline{OB} \cong m\overline{OC}$	

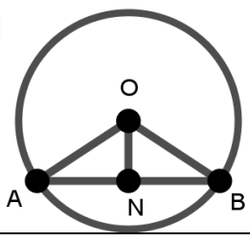
It means that O is the only one point which is equidistance from the three points A,B,C. Therefore, one and only one circle with center O can pass through three non-collinear points i.e., radius $m\overline{OA} \cong m\overline{OB} \cong m\overline{OC}$

Theorem 9.2: A straight line, drawn from a centre of a circle to bisect a chord (which is not diameter) is perpendicular to the chord

Given: A circle with centre O, $\overline{AB}$ is a chord of the circle, N is the midpoint of $\overline{AB}$ which is joined to O

To Prove: $\overline{ON} \perp \overline{AB}$

Construction: Join O to A and B.



Proof Statements	Reasons
In $\Delta OAN \leftrightarrow \Delta OBN$	
$\overline{OA} \cong \overline{OB}$	Radial segments
$\overline{AN} \cong \overline{BN}$	N is mid point of $\overline{AB}$
$\overline{ON} \cong \overline{ON}$	Common
$\therefore \Delta OAN \cong \Delta OBN$	SSS Postulate
$\therefore m\angle 1 \cong m\angle 2$	Corresponding angles of congruent triangles
But $m\angle 1 + m\angle 2 = 180°$	Supplementary angles postulate
$m\angle 1 + m\angle 1 = 180°$	Proved $m\angle 1 \cong m\angle 2$
$2m\angle 1 = 180°$	
$m\angle 1 = \frac{180°}{2}$	
$m\angle 1 = 90°$	Since $m\angle 1 \cong m\angle 2$
$\therefore m\angle 1 = m\angle 2 = 90°$	
Hence $\overline{ON} \perp \overline{AB}$	

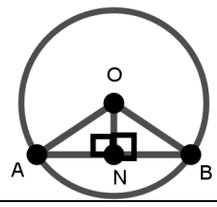
Theorem 9.3: Perpendicular from the centre of a circle on a chord bisect it.

Given: A circle with centre O, $\overline{AB}$ is a chord and $\overline{ON} \perp \overline{AB}$

To Prove: N is the mid point of $\overline{AB}$

i.e., $\overline{AN} \cong \overline{BN}$

Construction: Join O to A and B.



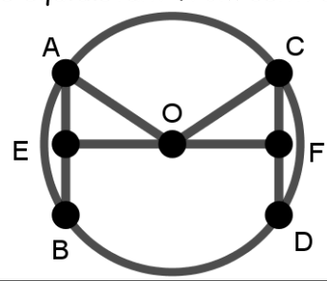
Proof Statements	Reasons
In $\Delta OAN \leftrightarrow \Delta OBN$	
$\overline{OA} \cong \overline{OB}$	Radial segments
$m\angle ONA \cong m\angle ONB$	Given/Right angles
$\overline{ON} \cong \overline{ON}$	Common
$\therefore \Delta OAN \cong \Delta OBN$	HS Postulate
$\therefore \overline{AN} \cong \overline{BN}$	Corresponding sides of congruent triangles

Th 9.4: If two chords of a circle are congruent then they will be equidistant from centre

Given: $\overline{AB}$ and $\overline{CD}$ are two congruent chords of the circle with centre O. i.e., $m\overline{AB} \cong m\overline{CD}$

To Prove: $\overline{AB}$ and $\overline{CD}$ are equidistant from the centre O.

Construction: Join O to A and C. Also draw perpendiculars $\overline{OE}$ and $\overline{OF}$ on the given chords $\overline{AB}$ and $\overline{CD}$ respectively



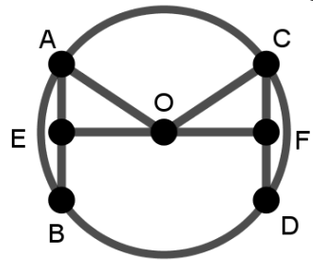
Proof Statements	Reasons
$m\overline{AE} \cong m\overline{EB}$ and $m\angle OEA = 90°$	$\overline{OE}$ is perpendicular bisector of $\overline{AB}$
$m\overline{CF} \cong m\overline{FD}$ and $m\angle OFC = 90°$	$\overline{OF}$ is perpendicular bisector of $\overline{CD}$
$\therefore \quad m\overline{AB} \cong m\overline{CD}$	Given that
$m\overline{AE} + m\overline{EB} \cong m\overline{CF} + m\overline{FD}$	$m\overline{AE} \cong m\overline{EB}$ $\quad m\overline{CF} \cong m\overline{FD}$
$m\overline{AE} + m\overline{AE} \cong m\overline{CF} + m\overline{CF}$	
$2m\overline{AE} \cong 2m\overline{CF}$	
$m\overline{AE} \cong m\overline{CF}$	
In $\Delta OAE \leftrightarrow \Delta OCF$	In the correspondence
$\overline{OA} \cong \overline{OC}$	Radial segments
$m\angle OEA \cong m\angle OFC$	Construction
$\overline{AE} \cong \overline{CF}$	Proved
$\therefore \Delta OAE \cong \Delta OCF$	HS Postulate
$\therefore m\overline{OE} \cong m\overline{OF}$	Corresponding sides of congruent triangles

Theo 9.5: If two chords of a circle which are equidistant from centre are congruent.

Given: $\overline{AB}$ and $\overline{CD}$ are chords of the circle are equidistant from centre O.

To Prove: $m\overline{AB} \cong m\overline{CD}$

Construction: Join O to A and C. Also draw perpendicular bisectors $\overline{OE}$ and $\overline{OF}$ on the given chords $\overline{AB}$ and $\overline{CD}$ respectively.



Proof Statements	Reasons
$m\overline{AE} \cong m\overline{EB}$ and $m\angle OEA = 90°$	$\overline{OE}$ is perpendicular bisector of $\overline{AB}$
$m\overline{CF} \cong m\overline{FD}$ and $m\angle OFC = 90°$	$\overline{OF}$ is perpendicular bisector of $\overline{CD}$
In $\Delta OAE \leftrightarrow \Delta OCF$	In the correspondence
$\overline{OA} \cong \overline{OC}$	Radial segments
$m\overline{OE} \cong m\overline{OF}$	Given
$m\angle OEA \cong m\angle OFC$	Construction

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$\therefore \Delta OAE \cong \Delta OCF$	HS Postulate
$\therefore \overline{AE} \cong \overline{CF}$	Corresponding sides of congruent triangles
$\therefore m\overline{AE} \cong m\overline{CF}$	
$2m\overline{AE} \cong 2m\overline{CF}$	Multiply both sides by 2
$m\overline{AE} + m\overline{AE} \cong m\overline{CF} + m\overline{CF}$	
$m\overline{AE} + m\overline{EB} \cong m\overline{CF} + m\overline{FD}$	$m\overline{AE} \cong m\overline{EB}$ $\quad m\overline{CF} \cong m\overline{FD}$
$m\overline{AB} \cong m\overline{CD}$	

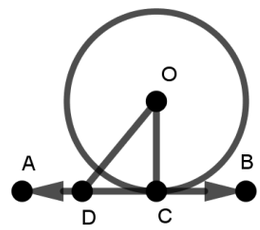
CHPATER 10

Theorem 10.1: If a line is drawn perpendicular to a radial segment of a circle at its outer end point, it is tangent to the circle at that point

Given: $\overline{OC}$ is radial segment of the circle with centre O.
Perpendicular line $\overleftrightarrow{AB}$ at outer end point of the circle C.
i.e., $\overleftrightarrow{AB} \perp \overline{OC}$

To Prove: $\overleftrightarrow{AB}$ is tangent to the circle.

Construction: Take any point D on $\overleftrightarrow{AB}$ other than C Join O to D.



Proof Statements	Reasons
$\because \overline{OC} \perp \overleftrightarrow{AB}$	Given
So the ΔOCD is right angled triangle	$\because m\angle C = 90°$
$\therefore m\angle 1$ and $m\angle 2$ are acute angles	i.e., $m\angle 1 + m\angle 2 = 90°$
Thus $m\angle C > m\angle 2$	$\because m\angle C$ is right and $m\angle 2$ is acute
$\therefore m\overline{OD} > m\overline{OC}$	In a triangle, greater angle has greater opposite side

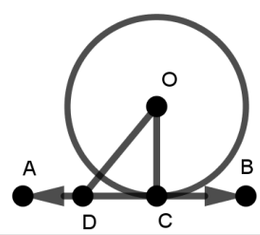
$m\overline{OD}$ is greater then radial segment $m\overline{OC}$ Thus D is exterior of circle
Hence meets the circle at only one point which is C.

Theorem 10.2: The tangent to a circle and the radial segment joining the point of contact and the centre are perpendicular to each other.

Given: $\overleftrightarrow{AB}$ is tangent to the circle at point C, $\overline{OC}$ is radial segment which is obtained by joining the Centre O and the point of contact of the tangent $\overleftrightarrow{AB}$

To Prove: $\overleftrightarrow{AB} \perp \overline{OC}$

Construction: Take any point D on $\overleftrightarrow{AB}$ other than C Join O to D.



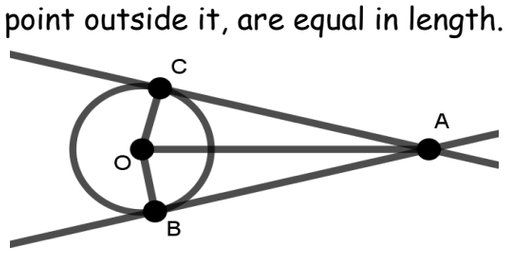
Proof Statements	Reasons
C is the only point common to the circle and the tangent $\overleftrightarrow{AB}$	$\because \overleftrightarrow{AB}$ is tangent to the circle at point C
$\therefore$ D is an exterior point of the circle	Construction (Except C every point of $\overleftrightarrow{AB}$ is outside the circle)
It means, $\therefore m\overline{OD} > m\overline{OC}$	$\because m\angle C$ is right (In a triangle greater angle has greater side opposite to it)
$m\overline{OC}$ is shortest distance between the centre O and the line $\overleftrightarrow{AB}$	
$\therefore \overleftrightarrow{AB} \perp \overline{OC}$	By definition of shortest distance

T 10.3: Two tangents drawn to a circle from a point outside it, are equal in length.

Given: A circle with centre O. A is any point outside the circle $\overline{AB}$ and $\overline{AC}$ are two tangents from a point A.

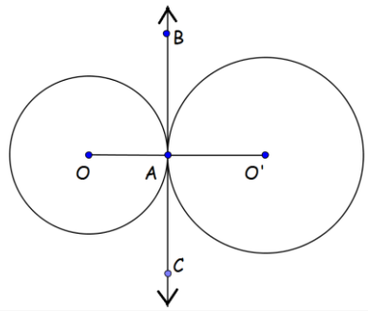
To Prove: $m\overline{AB} \cong m\overline{AC}$

Construction: Join O to A,B and C.



Proof Statements	Reasons
$\Delta AOB \leftrightarrow \Delta AOC$	
$\overline{AO} \cong \overline{AO}$	Common
$\overline{OB} \cong \overline{OC}$	Radial segments
$\angle ABO \cong \angle ACO$	Right angles
$\Delta AOB \cong \Delta AOC$	$HS \cong HS$
$\overline{AB} \cong \overline{AC}$	Corresponding sides of congruent triangles

Theorem 10.4 Statement: The two circles touches externally, the distance between their centres is equal to sum of their radii.



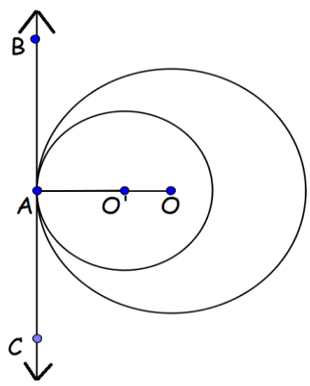
Given: Two circles with centre O and O' which is touching each other externally at point A. $\overline{OA}$ and $\overline{O'A}$ are the radial segments of the circles

To Prove: $m\overline{OO'} \cong m\overline{OA} + m\overline{AO'}$

Construction: Draw a common tangent $\overleftrightarrow{BC}$ at the point A which is common point of the two circles

Proof Statements	Reasons
$m\angle BAO = 90° \cdots (1)$	$\because \overleftrightarrow{BC}$ is tangent to a circle at point A i.e., $\overline{OA} \perp \overleftrightarrow{BC}$
$m\angle BAO' = 90° \cdots (2)$	$\because \overleftrightarrow{BC}$ is tangent to a circle at point A i.e., $\overline{O'A} \perp \overleftrightarrow{BC}$
$m\angle BAO + m\angle BAO' = 90° + 90°$	$m\angle BAO$ and $m\angle BAO'$ are supplementary angles with common vertex A
$m\angle BAO + m\angle BAO' = 180°$	
$\therefore \overline{OA}$ and $\overline{AO'}$ are opposite rays	
Thus O, A and O' are three different collinear points	
$m\overline{OO'} \cong m\overline{OA} + m\overline{AO'}$	Straight line Postulate

Theorem 10.4: The two circles touches internally, the distance between their centre is the difference between their radii.



Given: Two circles with centre O and O' which is touching each other internally at point A. $\overline{OA}$ and $\overline{O'A}$ are the radial segments of the circles

To Prove: $m\overline{OO'} \cong m\overline{OA} - m\overline{O'A}$

Construction: Draw a common tangent $\overleftrightarrow{BC}$ at the point A which is common point of the two circles

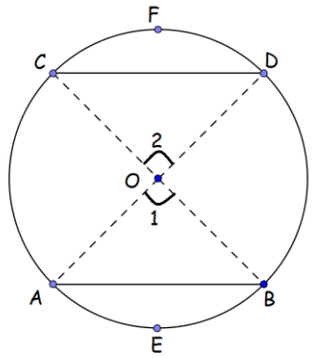
Proof Statements	Reasons
$m\angle BAO = 90° \cdots (1)$	$\because \overleftrightarrow{BC}$ is tangent to a circle at point A i.e., $\overline{OA} \perp \overleftrightarrow{BC}$
$m\angle BAO' = 90° \cdots (2)$	$\because \overleftrightarrow{BC}$ is tangent to a circle at point A i.e., $\overline{O'A} \perp \overleftrightarrow{BC}$
$m\angle BAO = m\angle BAO' = 90°$	$m\angle BAO$ and $m\angle BAO'$ are right angles with common vertex A
Thus O, A and O' are three different collinear points	A,O and O' lies on the same straight line
$m\overline{OA} \cong m\overline{OO'} + m\overline{O'A}$	Straight line Postulate
$m\overline{OA} - m\overline{O'A} \cong m\overline{OO'}$	From the law of equation
$m\overline{OO'} \cong m\overline{OA} - m\overline{O'A}$	

Arc of the Circle: Any portion or part of the circle is called arc of the circle.

Major Arc: An arc which is greater than a semi-circle is called Major arc.

Central Angle: An angle subtended by an arc at the centre of a circle is called central angle of arc.

CHAPTER 11



Theorem 11.1 (a): If two arcs of a circle are congruent then the corresponding chords are equal

Given: A circle with centre O $\overset{\frown}{AEB}$ and $\overset{\frown}{CFD}$ are congruent arcs. i.e., $\overset{\frown}{AEB} \cong \overset{\frown}{CFD}$

To Prove: $m\overline{AB} \cong m\overline{CD}$

Construction: Join O to A,B,C and D respectively and label the central angles $\angle 1$ and $\angle 2$

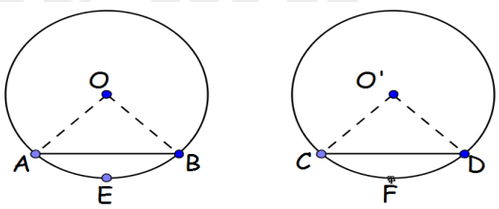
Proof Statements	Reasons
In $\Delta OAB \leftrightarrow \Delta OCD$	In the correspondence
$\overline{OA} \cong \overline{OC}$	Radial segments
$m\angle 1 \cong m\angle 2$	Central angle of two congruent arcs
$\overline{OB} \cong \overline{OD}$	Radial segments
$\therefore \Delta OAB \cong \Delta OCD$	SAS Postulate
$\therefore \overline{AB} \cong \overline{CD}$	Corresponding sides of congruent triangles

Theorem 11.1 (b): If two arcs of the congruent circles are congruent then the corresponding chords are equal

Given: Two congruent circles with centre O and O' respectively. $\overset{\frown}{AEB}$ and $\overset{\frown}{CFD}$ are congruent arcs. i.e., $\overset{\frown}{AEB} \cong \overset{\frown}{CFD}$

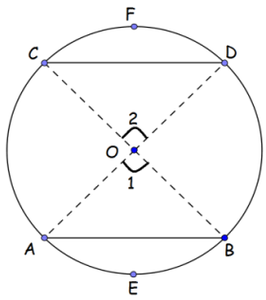
To Prove: $m\overline{AB} \cong m\overline{CD}$

Construction: Join O to A,B and O' to C and D respectively and label the central angles $\angle 1$ and $\angle 2$



Proof Statements	Reasons
In $\Delta OAB \leftrightarrow \Delta O'CD$	
$\overline{OA} \cong \overline{O'C}$	Radii of congruent circles
$m\angle 1 \cong m\angle 2$	Central angle of two congruent arcs
$\overline{OB} \cong \overline{O'D}$	Radii of congruent circles
$\therefore \Delta OAB \cong \Delta O'CD$	SAS Postulate
$\therefore \overline{AB} \cong \overline{CD}$	Corresponding sides of congruent triangles

Theorem 11.2 (a) If two chords of a circle are congruent then the corresponding minor arcs are congruent



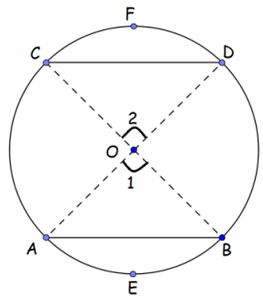
Given: A circle with centre O $\overline{AB}$ and $\overline{CD}$ are congruent chords. i.e., $m\overline{AB} \cong m\overline{CD}$

To Prove: $\overset{\frown}{AEB} \cong \overset{\frown}{CFD}$

Construction: Join O to A,B,C and D respectively and label the central angles $\angle 1$ and $\angle 2$

Proof Statements	Reasons
In $\Delta OAB \leftrightarrow \Delta OCD$	In the correspondence
$\overline{OA} \cong \overline{OC}$	Radial segments
$\overline{AB} \cong \overline{CD}$	Given
$\overline{OB} \cong \overline{OD}$	Radial segments
$\therefore \Delta OAB \cong \Delta OCD$	SSS Postulate
$\therefore m\angle 1 \cong m\angle 2$	Corresponding angles of congruent triangles
$\overset{\frown}{AEB} \cong \overset{\frown}{CFD}$	Congruent arcs has congruent Central angles

Theorem 11.2 (b): If two chords of a circle are congruent then corresponding major arcs are congruent



Given: A circle with centre O $\overline{AB}$ and $\overline{CD}$ are congruent chords. i.e., $m\overline{AB} \cong m\overline{CD}$

To Prove: $\text{AFB} \cong \text{CED}$

Construction: Join O to A,B,C and D respectively and label the central angles $\angle 1$ and $\angle 2$

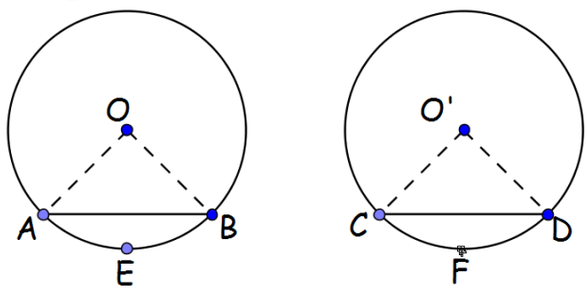
Proof Statements	Reasons
In $\Delta OAB \leftrightarrow \Delta OCD$	In the correspondence
$\overline{OA} \cong \overline{OC}$	Radial segments
$\overline{AB} \cong \overline{CD}$	Given
$\overline{OB} \cong \overline{OD}$	Radial segments
$\therefore \Delta OAB \cong \Delta OCD$	SSS Postulate
$\therefore m\angle 1 \cong m\angle 2$	Corresponding angles of congruent triangles
$\text{AEB} \cong \text{CFD}$	Congruent arcs has congruent Central angles
$m\text{AFB} \cong 360° - m\text{AEB}$ .....(1)	Total angle at a point = 360°
$m\text{CED} \cong 360° - m\text{CFD}$ .....(2)	i.e., m AEB + m AFB=360° & m CFD + m CED=360°
$m\text{AFB} \cong m\text{CED}$	Transitive property

Theorem 11.2 (c) **Statement:** If two chords of the congruent circles are congruent then the corresponding minor arcs are congruent

Given: Two congruent circles with centre O and O′ respectively. $\overline{AB}$ and $\overline{CD}$ are congruent chords. i.e., $m\overline{AB} \cong m\overline{CD}$

To Prove: $\text{AEB} \cong \text{CFD}$

Construction: Join O to A,B and O′ to C and D respectively and label the central angles $\angle 1$ and $\angle 2$



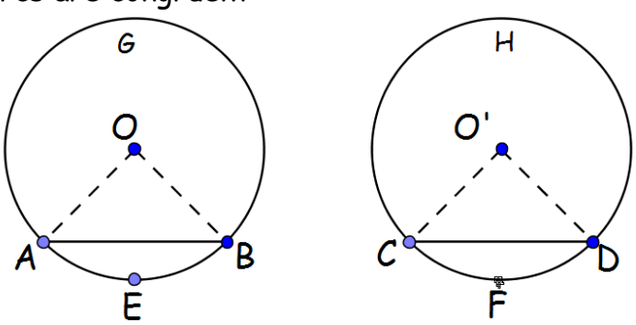
Proof Statements	Reasons
In $\Delta OAB \leftrightarrow \Delta O'CD$	In the correspondence
$\overline{OA} \cong \overline{O'C}$	Radii of congruent circles
$\overline{AB} \cong \overline{CD}$	Given
$\overline{OB} \cong \overline{O'D}$	Radii of congruent circles
$\therefore \Delta OAB \cong \Delta O'CD$	SSS Postulate
$\therefore m\angle 1 \cong m\angle 2$	Corresponding angle of congruent triangles
$m\text{AEB} \cong m\text{CFD}$	Congruent arcs has congruent Central angles

Theorem 11.2 (d) **Statement:** If two chords of the congruent circles are congruent then the corresponding major arcs are congruent

Given: Two congruent circles with centre O and O′ respectively. $\overline{AB}$ and $\overline{CD}$ are congruent chords. i.e., $m\overline{AB} \cong m\overline{CD}$

To Prove: $\text{AGB} \cong \text{CHD}$

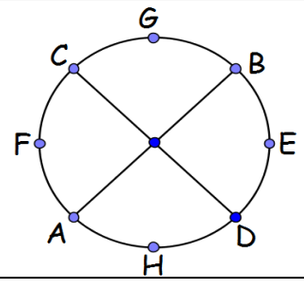
Construction: Join O to A,B and O′ to C and D respectively and label the central angles $\angle 1$ and $\angle 2$



Proof Statements	Reasons

In $\Delta OAB \leftrightarrow \Delta O'CD$	In the correspondence
$\overline{OA} \cong \overline{O'C}$	Radii of congruent circles
$\overline{AB} \cong \overline{CD}$	Given
$\overline{OB} \cong \overline{O'D}$	Radii of congruent circles
$\therefore \Delta OAB \cong \Delta O'CD$	SSS Postulate
$\therefore m\angle 1 \cong m\angle 2$	Corresponding angle of congruent triangles
$mAEB \cong mCFD$	Congruent arcs has congruent Central angles
$mAGB \cong 360^\circ - mAEB$ .....(1)	Total angle at a point = 360°
$mCHD \cong 360^\circ - mCFD$ .....(2)	i.e., m AEB + m AFB=360° & m CFD + m CED=360°
$mAGB \cong mCHD$	Transitive property

Theorem 11.2 (e): If two chords (diameters) of a circle are congruent then the corresponding minor/major arcs are congruent

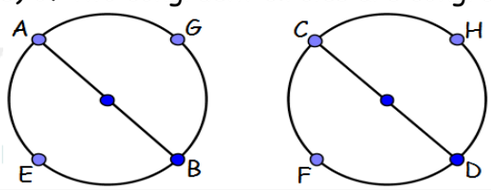


Given: A circle with centre O $\overline{AB}$ and $\overline{CD}$ are diameters.
i.e., $m\overline{AB} \cong m\overline{CD}$

To Prove: $AFB \cong AEB \cong CED \cong CFD$

Proof Statements	Reasons
$m\overline{AB} \cong m\overline{CD}$	Diameters of the same circles
Then length of minor arc = length major arc	
i.e., $mAEB \cong mAFB \cong 180^O$ .....(1)	Congruent arcs has congruent Central angles
similarly $mCFD \cong CED \cong 180^O$ .....(2)	Diameter is a straight line which has supplementary angle
$mAFB \cong mAEB \cong mCED \cong mCFD$	Transitive property

Theorem 11.2 (e): If two chords (diameters) of two congruent circles are congruent then corresponding minor/major arcs are congruent

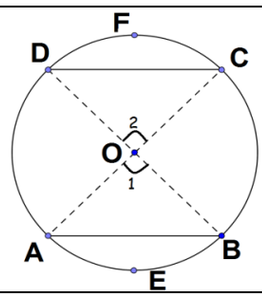


Given: Two circles with centre O and O'. $\overline{AB}$ and $\overline{CD}$ are diameters. i.e., $m\overline{AB} \cong m\overline{CD}$

To Prove: $AGB \cong AEB \cong CHD \cong CFD$

Proof Statements	Reasons
$m\overline{AB} \cong m\overline{CD}$	Diameters of the congruent circles
Then length of minor arc = length major arc	
i.e., $AGB \cong AEB \cong 180^\circ$ .....(1)	Congruent arcs has congruent Central angles
similarly $CFD \cong CHD \cong 180^\circ$ .....(2)	Diameter is a straight line which has supplementary angle
$mAGB \cong mAEB \cong mCHD \cong mCFD$	Transitive property

Theorem 11.3 (a): Equal chords of a circle subtend equal angles at the centre



Given: A circle with centre O and $m\overline{AB} \cong m\overline{CD}$

To Prove: $m\angle 1 \cong m\angle 2$

Construction: Join O to A,B,C and D respectively and label the central angles $\angle 1$ and $\angle 2$

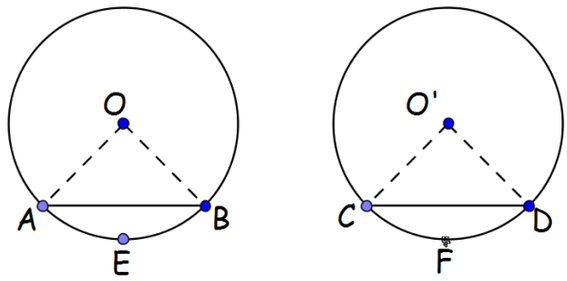
Proof Statements	Reasons
In $\Delta OAB \leftrightarrow \Delta OCD$	
$\overline{OA} \cong \overline{OC}$	Radial segments
$\overline{AB} \cong \overline{CD}$	Given
$\overline{OB} \cong \overline{OD}$	Radial segments
$\therefore \Delta OAB \cong \Delta OCD$	SSS Postulate
$\therefore m\angle 1 \cong m\angle 2$	Corresponding angles of congruent triangles

Theorem 11.3 (b): Equal chords of congruent circles subtend equal angles at their centres

Given: Two congruent circles with centre O and O' respectively. $\overline{AB}$ and $\overline{CD}$ are congruent chords. i.e., $m\overline{AB} \cong m\overline{CD}$

To Prove: $m\angle 1 \cong m\angle 2$

Construction: Join O to A,B and O' to C and D respectively and label the central angles $\angle 1$ and $\angle 2$



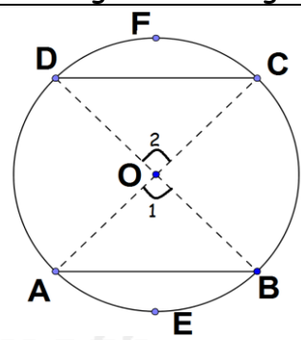
Proof Statements	Reasons
In $\Delta OAB \leftrightarrow \Delta O'CD$	
$\overline{OA} \cong \overline{O'C}$	Radii of congruent circles
$\overline{AB} \cong \overline{CD}$	Given
$\overline{OB} \cong \overline{O'D}$	Radii of congruent circles
$\therefore \Delta OAB \cong \Delta O'CD$	SSS Postulate
$\therefore m\angle 1 \cong m\angle 2$	Corresponding angle of congruent triangles

Theorem 11.4 (a) **Statement:** If the angles subtend by two chords of circle at the centre are equal in measures, then the chords are equal in measure

Given: A circle with centre O $\overline{AB}$ and $\overline{CD}$ are chords of circle and $m\angle 1 \cong m\angle 2$

To Prove: $m\overline{AB} \cong m\overline{CD}$

Construction: Join O to A,B,C and D respectively and label the central Angles $\angle 1$ and $\angle 2$



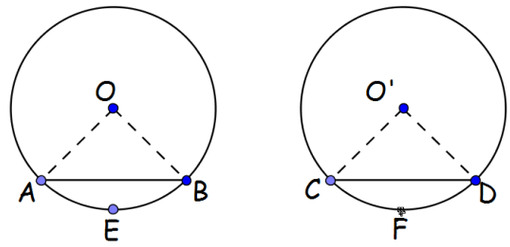
Proof Statements	Reasons
In $\Delta OAB \leftrightarrow \Delta OCD$	
$\overline{OA} \cong \overline{OC}$	Radial segments
$m\angle 1 \cong m\angle 2$	Given
$\overline{OB} \cong \overline{OD}$	Radial segments
$\therefore \Delta OAB \cong \Delta OCD$	SAS Postulate
$\therefore \overline{AB} \cong \overline{CD}$	Corresponding sides of congruent triangles

Theorem 11.4 (b): If the angles subtend by two chords of congruent circle at the centre are equal in measures, then the chords are equal in measure

Given: Two congruent circles with centre O and O' respectively. $\overline{AB}$ and $\overline{CD}$ are chords of circle and $m\angle 1 \cong m\angle 2$

To Prove: $m\overline{AB} \cong m\overline{CD}$

Construction: Join O to A,B and O' to C and D respectively



Proof Statements	Reasons
In $\Delta OAB \leftrightarrow \Delta O'CD$	In the correspondence
$\overline{OA} \cong \overline{O'C}$	Radii of congruent circles
$m\angle 1 \cong m\angle 2$	Central angle of two congruent arcs
$\overline{OB} \cong \overline{O'D}$	Radii of congruent circles
$\therefore \Delta OAB \cong \Delta O'CD$	SAS Postulate
$\therefore \overline{AB} \cong \overline{CD}$	Corresponding sides of congruent triangles

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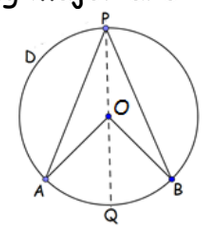
CHAPTER 12

Theorem 12.1 (a): The measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc

Given: A circle with centre O, AQB is minor arc whose central angle is $\angle AOB$. P is any point on the major arc ADB. $\angle APB$ is the angle subtended by the arc at P.

To Prove: $m\angle AOB = 2m\angle APB$

Construction: Draw $\overline{PO}$ to meet circumference at Q



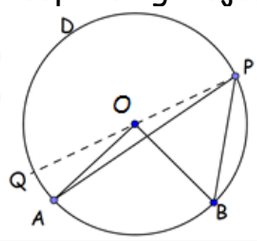
Proof Statements	Reasons
In ΔOAP	
$\overline{OA} \cong \overline{OP}$	Radial segments of circle
$m\angle OAP \cong m\angle OPA$	Congruent sides has congruent angles(Isosceles)
$m\angle AOQ \cong m\angle OAP + m\angle OPA$	Exterior angle = sum of opposite interior angles
$m\angle AOQ \cong m\angle OAP + m\angle OAP$	$\because m\angle OAP \cong m\angle OPA$
$m\angle AOQ \cong 2m\angle OAP \cdots (1)$	
In ΔOPB	
$\overline{OB} \cong \overline{OP}$	Radial segments of circle
$m\angle OAB \cong m\angle OPB$	Congruent sides has congruent angles(Isosceles)
$m\angle BOQ \cong m\angle OBP + m\angle OPB$	Exterior angle = sum of opposite interior angles
$m\angle BOQ \cong m\angle OPB + m\angle OPB$	$\because m\angle OBP \cong m\angle OPB$
$m\angle BOQ \cong m\angle OPB \cdots (2)$	
$m\angle AOQ + m\angle BOQ \cong 2m\angle OPA + 2m\angle OPB$	Adding equation (1) and (2)
$m\angle AOB \cong 2(m\angle OPA + m\angle OPB)$	
$m\angle AOB \cong 2m\angle APB$	

Theorem 12.1 (b): Measure of a central angle of a minor arc of a circle(semi circle), is double that of the angle subtended by the corresponding major arc

Given: A circle with centre O, AQB is minor arc whose central angle is $\angle AOB$. P is any point on the major arc ADB. $\angle APB$ is the angle subtended by the arc at P.

To Prove: $m\angle AOB = 2m\angle APB$

Construction: Draw $\overline{PO}$ to meet circumference at Q



Proof Statements	Reasons
In ΔOAP	
$\overline{OA} \cong \overline{OP}$	Radial segments of circle
$m\angle OAP \cong m\angle OPA$	Congruent sides has congruent angles(Isosceles)
$m\angle AOQ \cong m\angle OAP + m\angle OPA$	Exterior angle = sum of opposite interior angles
$m\angle AOQ \cong m\angle OAP + m\angle OAP$	$\because m\angle OAP \cong m\angle OPA$
$m\angle AOQ \cong 2m\angle OAP \cdots (1)$	
In ΔOPB	
$\overline{OB} \cong \overline{OP}$	Radial segments of circle
$m\angle OAB \cong m\angle OPB$	Congruent sides has congruent angles(Isosceles)
$m\angle BOQ \cong m\angle OBP + m\angle OPB$	Exterior angle = sum of opposite interior angles
$m\angle BOQ \cong m\angle OPB + m\angle OPB$	$\because m\angle OBP \cong m\angle OPB$
$m\angle BOQ \cong m\angle OPB \cdots (2)$	
$m\angle BOQ - m\angle AOQ \cong 2m\angle OPB - 2m\angle OPA$	Subtracting equation (1) from (2)
$m\angle AOB \cong 2(m\angle OPA - m\angle OPB)$	
$m\angle AOB \cong 2m\angle APB$	

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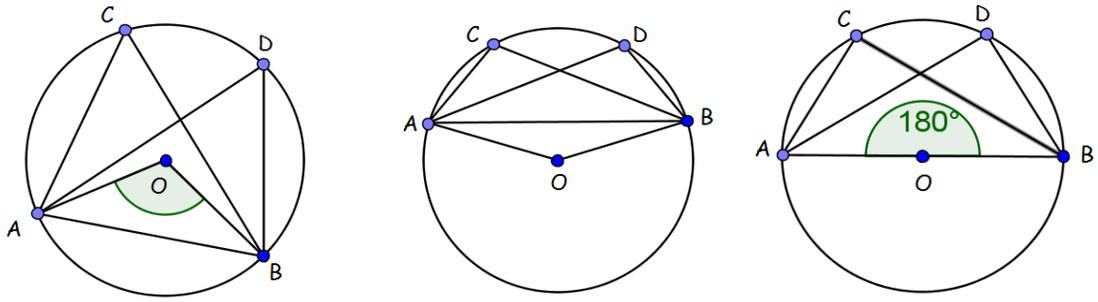
Theorem 12.2 : Two angles in the same segments of a circle are equal



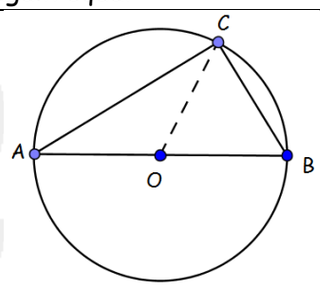
Given: A circle with centre O, $\angle ACB$ and $\angle ADB$ are any two angles in same arc ACDB containing segments $\overline{AB}$

To Prove: $\angle ACB \cong \angle ADB$

Construction: Join O to A and B

Proof Statements	Reasons
$m\angle AOB \cong 2m\angle ACB \cdots (1)$	Central angle = 2 (angle subtended by major arc)
$m\angle AOB \cong 2m\angle ADB \cdots (2)$	Central angle = 2 (angle subtended by major arc)
$\therefore 2m\angle ACB \cong 2m\angle ADB$ $m\angle ACB \cong m\angle ADB$	From equation (1) and (2) Halves of equal quantities is again equal

Theorem 12.3 (a) Statement: The angle in a semi-circle is a right angle.



Given: A circle with centre O, $\overline{AB}$ is a diameter and $\angle ACB$ is any angle in the semi-circle.

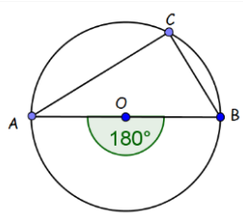
To Prove: $m\angle ACB = 90°$

Construction: Join O to C

Proof Statements	Reasons
In ΔOAC $\overline{OA} \cong \overline{OC}$ $m\angle OAC \cong m\angle OCA \cdots (1)$	 Radial segments of circle Congruent sides has congruent angles(Isosceles)
In ΔOCB $\overline{OB} \cong \overline{OC}$ $m\angle OBC \cong m\angle OCB \cdots (2)$	 Radial segments of circle Congruent sides has congruent angles(Isosceles)
$m\angle OAC + m\angle OBC \cong m\angle OCA + m\angle OCB$ $m\angle OAC + m\angle OBC \cong m\angle ACB$	Adding equation (1) and (2)
But $m\angle OAC + m\angle OBC + m\angle ACB \cong 180°$ $m\angle ACB + m\angle ACB \cong 180°$ $2m\angle ACB \cong 180°$ $m\angle ACB \cong 90°$	Sum of interior angles equals to 180° $m\angle OAC + m\angle OBC \cong m\angle ACB$

Again (Second Method) Theorem 12.3 (a)

Statement: The angle in a semi-circle is a right angle.



Given: A circle with centre O, $\overline{AB}$ is a diameter and $\angle ACB$ is any angle in the semi-circle.

To Prove: $m\angle ACB = 90°$

Proof Statements	Reasons
$m\angle AOB \cong 2m\angle ACB$ $180° \cong 2m\angle ACB$ $\frac{180°}{2} \cong m\angle ACB$ $90° \cong m\angle ACB$	Central angle = 2 (angle subtended by major arc) $\therefore m\angle AOB \cong 180°$

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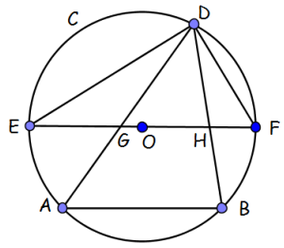
Theorem 12.3 (b) Angle in a segment greater then a semi-circle is less than a right angle

Given: A circle with centre O, $\overline{AB}$ is a chord below the diameter $\overline{EF}$ is a diameter Thus the arc AECDFB if greater than the semi-circular region.



To Prove: $m\angle ADB < 90°$

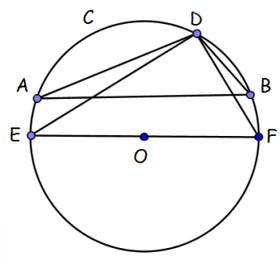
Construction: Join D to E and F such that $\angle EDF$ is in the semi-circle

Proof Statements	Reasons
$m\angle EDF \cong 90°$	Measure of angle in semicircle is right
$\because m\angle EDA + m\angle ADB + m\angle BDF \cong 90°$	$\because m\angle EDF \cong m\angle EDA + m\angle ADB + m\angle BDF$
$\Rightarrow m\angle ADB \cong 90° - (m\angle EDA + m\angle BDF)$	
$\Rightarrow m\angle ADB < 90°$	Whole is greater than its parts

Theorem 12.3 (C)

Statement: The angle in a segment less than a semi-circle is greater than a right angle

Given: A circle with centre O, $\overline{AB}$ is a chord above the diameter $\overline{EF}$ is a diameter Thus the arc ACDB if less than the semi-circular region.



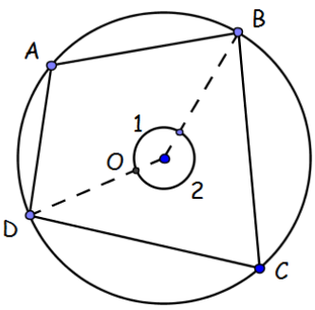
To Prove: $m\angle ADB > 90°$

Construction: Join D to E and F such that $\angle EDF$ is in the semi-circle

Proof Statements	Reasons
$m\angle EDF \cong 90°$	Measure of angle in semi-circle is right
$\because m\angle ADE + m\angle EDF + m\angle FDB \cong m\angle ADB$	$\because m\angle ADB \cong m\angle ADE + m\angle EDF + m\angle FDB$
$\Rightarrow m\angle ADE + 90° + m\angle FDB \cong m\angle ADB$	
$\Rightarrow 90° < m\angle ADB$	Whole is greater than its parts
$\Rightarrow m\angle ADB > 90°$	

Theorem 12.4: The opposite angles of any quadrilateral inscribed in a circle are supplementary.

Given: A circle with centre O, ABCD is quadrilateral inscribed in a circle



To Prove: $m\angle BAD + m\angle BCD \cong 180°$
$m\angle ADC + m\angle ABC \cong 180°$

Construction: Join O to B and D and label the angles such that their sum $m\angle 1 + m\angle 2 \cong 360°$

Proof Statements	Reasons
$2m\angle BCD \cong m\angle 1 \cdots (1)$	Central angle = 2 (angle subtended by major arc)
$2m\angle DAB \cong m\angle 2 \cdots (2)$	Central angle = 2 (angle subtended by major arc)
$2m\angle BCD + 2m\angle DAB \cong m\angle 1 + m\angle 2$	Adding equations (1) and (2)
$2(m\angle BCD + m\angle DAB) \cong 360°$	$m\angle 1 + m\angle 2 \cong 360°$
$m\angle BCD + m\angle DAB \cong \frac{360°}{2}$	Halves of equal quantities is again equal
$m\angle BCD + m\angle DAB \cong 180°$	
Similarly we can prove that	
$m\angle ADC + m\angle ABC \cong 180°$	