Chapter 7

Sexagesimal system: (DMS)
It is a system of measurement of angles, in which angles are measured in degree, minutes and seconds. One complete rotation = 360^o
1 degree is further divided into 60 minutes
1 minute is further divided into 60 seconds
1^o = 60 minutes = 60'
1' = 60 seconds = 60"
Therefore 1^o = 3600 seconds = 3600"

Exp1: convert $15^o\ 30'\ 23''$ to decimal form.

Sol: Since $1' = \left(\frac{1}{60}\right)^o$ and $1'' = \left(\frac{1}{3600}\right)^o$

Therefore $15^o\ 30'\ 23'' = \left(15 + \frac{30}{60} + \frac{23}{3600}\right)^o$

$15^o\ 30'\ 23'' = \left[15 + 0.5 + 0.0694\right]^o$

$15^o\ 30'\ 23'' = 15.50694^o$

Exp2: Convert 38.39^o to $D^o\ M'\ S''$ form.

Sol: Since $1^o = 60'$ and $1' = 60''$

Therefore $38.39^o = 38^o + 0.39^o$

$= 38^o\ \ 0.39 \times 60'$

$= 38^o\ \ 23.4'$

$= 38^o\ \ 23' + 0.4'$

$= 38^o\ \ 23'\ \ 0.4 \times 60''$

$= 38^o\ \ 23'\ \ 24''$

Circular system (Radians)
Angular measurement cannot be conveniently done in degrees. Angular measurement or circular measurement is applied in differentiation and integration of trigonometric ratios.

Radians: Radian is the ratio of the length of an are to radius of the circle.

$\theta = \frac{l}{r}$ If length of are $l = r$

$\theta = \frac{r}{r} = 1$ radian

Relation b/w Radians and Degrees
If r is radius of circle then number of radians in one complete rotation $= \frac{Circumference\ of\ circle}{radius\ of\ circle}$

$\therefore$ Circumference of circle $= 2\pi r$

One complete rotation $= \frac{2\pi r}{r}$

One complete rotation $= 2\pi$

$\therefore$ circumference of circle subtends $= 360^o$

$2\ \pi\ Radian = 360^o$

Or $\pi\ Radian = 180^o$ ………………(1)

$\frac{\pi}{2}\ Radian = 90^o$ divide eq (1) by 2

$\frac{\pi}{3}\ Radian = 60^o$ divide eq (1) by 3

$\frac{\pi}{6}\ Radian = 30^o$ divide eq (1) by 6

Similarly $1\ Radian = \frac{180^o}{\pi} = 57.3^o$

And $1^o = 0.0175\ Radians$

Exp3: Convert $\frac{4\pi}{7}$ radians to degrees.

Sol: Since $\pi\ Radian = 180^o$

Therefore $\frac{4\pi}{7} = \frac{4}{7}\left(180^o\right)$

$\frac{4\pi}{7} = 102.85714^o$

$\frac{4\pi}{7} = 102^o\ 51'\ 26''$

Exp4: Convert $31^o\ 45'$ to radian
Sol: First convert into decimal

Using $1' = \left(\frac{1}{60}\right)^o$ & $1'' = \left(\frac{1}{3600}\right)^o$

$31^o\ 45' = \left(31 + \frac{45}{60}\right)^o$

$31^o\ 45' = 31.75^o$

Now convert into radian $\therefore 1^o = \frac{\pi}{180} Radians$

As $31.75^o = 31.75\left(\frac{\pi}{180}\right)^o$

$31.75^o = 0.5541\ Radians$

Exercise 7.1

Q1: Convert the following angles from $D^oM'S''$ forms to decimal forms.

i). $8^o15'35''$

Sol: Since $1' = \left(\frac{1}{60}\right)^o$ and $1'' = \left(\frac{1}{3600}\right)^o$

$8^o15'35'' = 8^o + \frac{15}{60}^o + \frac{35}{3600}^o$

$8^o15'35'' = 8^o + 0.25^o + 0.0097\bar{2}^o$

$8^o15'35'' = 8^o15'35'' = 8.2597\bar{2}^o$

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ii). $39^o48'55''$

Sol: Since $1' = \left(\frac{1}{60}\right)^o$ and $1'' = \left(\frac{1}{3600}\right)^o$

$39^o48'55'' = 39^o + \frac{48}{60}^o + \frac{55}{3600}^o$

$39^o48'55'' = 39^o + 0.80^o + 0.0152\bar{7}^o$

$39^o48'55'' = 39.8152\bar{7}^o$

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iii). $84^o19'10''$

Sol: Since $1' = \left(\frac{1}{60}\right)^o$ and $1'' = \left(\frac{1}{3600}\right)^o$

$84^\circ19'10'' = 84^\circ + \frac{19}{60}^\circ + \frac{10}{3600}^\circ$

$84^\circ19'10'' = 84^\circ + 0.31\overline{6}^\circ + 0.002\overline{7}^\circ$

$84^\circ19'10'' = 84.319\overline{4}^\circ$

iv). $18^\circ6'21''$

Sol: Since $1' = \left(\frac{1}{60}\right)^\circ$ and $1'' = \left(\frac{1}{3600}\right)^\circ$

$18^\circ6'21'' = 18^\circ + \frac{6}{60}^\circ + \frac{21}{3600}^\circ$

$18^\circ6'21'' = 18^\circ + 0.10^\circ + 0.0058\overline{3}^\circ$

$18^\circ6'21'' = 18.1058\overline{3}^\circ$

Q2: Convert the following angles from decimal form to $D^\circ M' S''$

i). 42.25°

Sol: Since $1^\circ = 60'$ and $1' = 60''$

$42.25^\circ = 42^\circ + 0.25^\circ$

$42.25^\circ = 42^\circ \quad 0.25 \times 60'$

$42.25^\circ = 42^\circ \; 15'$

ii). 57.325°

Sol: Since $1^\circ = 60'$ and $1' = 60''$

$57.325^\circ = 57^\circ + 0.325^\circ$

$57.325^\circ = 57^\circ \quad 0.325 \times 60'$

$57.325^\circ = 57^\circ \; 19.5'$

$57.325^\circ = 57^\circ \; 19' + 0.5'$

$57.325^\circ = 57^\circ \; 19' \; 0.5 \times 60''$

$57.325^\circ = 57^\circ \; 19' \; 30''$

iii). 12.9956°

Sol: Since $1^\circ = 60'$ and $1' = 60''$

$12.9956^\circ = 12^\circ + 0.9956^\circ$

$12.9956^\circ = 12^\circ \quad 0.9956 \times 60'$

$12.9956^\circ = 12^\circ \; 59.736'$

$12.9956^\circ = 12^\circ \; 59' + 0.736'$

$12.9956^\circ = 12^\circ \; 59' \; 0.736 \times 60''$

$12.9956^\circ = 12^\circ \; 59' \; 44.16''$

$12.9956^\circ = 12^\circ \; 59' \; 44''$

iv). 32.625°

Sol: Since $1^\circ = 60'$ and $1' = 60''$

$32.625^\circ = 32^\circ + 0.625^\circ$

$32.625^\circ = 32^\circ \quad 0.625 \times 60'$

$32.625^\circ = 32^\circ \; 37.5'$

$32.625^\circ = 32^\circ \; 37' + 0.5'$

$32.625^\circ = 32^\circ \; 37' \; 0.5 \times 60''$

$32.625^\circ = 32^\circ \; 37' \; 30''$

Q3: Convert the following radian measures of the angles into the measures of degrees.

i). 2 radian

Solution: since $1\text{Radian} = \frac{180^\circ}{\pi}$

$2 \times 1\text{Radian} = 2 \times \frac{180^\circ}{\pi}$

$2\text{Radian} = \frac{360^\circ}{\pi} = 114.5767^\circ$

ii). $\frac{5\pi}{3}$ radians

Solution: since $1\text{Radian} = \frac{180^\circ}{\pi}$

$\frac{5\pi}{3} \times 1\text{Radian} = \frac{5\pi}{3} \times \frac{180^\circ}{\pi}$

$\frac{5\pi}{3}\text{Radian} = 5 \times 60^\circ = 300^\circ$

iii). $\frac{\pi}{6}$ radians

Solution: since $1\text{Radian} = \frac{180^\circ}{\pi}$

$\frac{\pi}{6} \times 1\text{Radian} = \frac{\pi}{6} \times \frac{180^\circ}{\pi}$

$\frac{\pi}{6} \times 1\text{Radian} = 30^\circ$

iv). $\frac{-3\pi}{4}$ radians

Solution: since $1\text{Radian} = \frac{180^\circ}{\pi}$

$\frac{-3\pi}{4} \times 1\text{Radian} = \frac{-3\pi}{4} \times \frac{180^\circ}{\pi}$

$\frac{-3\pi}{4}\text{Radian} = -3 \times 45^\circ$

$\frac{-3\pi}{4}\text{Radian} = -135^\circ$

Q4: Convert following angles in terms of radians

i). 45°

Solution: Since $1^\circ = \frac{\pi}{180}\text{Radian}$

$45 \times 1^\circ = 45 \times \frac{\pi}{180}\text{Radian}$

$45^\circ = \frac{\pi}{4}\text{Radian}$

ii). 120°

Solution: Since $1^\circ = \frac{\pi}{180}\text{Radian}$

$120 \times 1^\circ = 120 \times \frac{\pi}{180}\text{Radian}$

$120^\circ = \frac{2\pi}{3}\text{Radian}$

iii). -210°

Solution: Since $1^\circ = \frac{\pi}{180}\text{Radian}$

$-210 \times 1^\circ = -210 \times \frac{\pi}{180}\text{Radian}$

$-210^\circ = \frac{-7\pi}{6}\text{Radian}$

iv). $60^\circ35'48''$

Sol: First convert into degree form i.e., decimal form

$60^\circ35'48'' = 60^\circ + \frac{35}{60}^\circ + \frac{48}{3600}^\circ$

$60^\circ35'48'' = 60^\circ + 0.58\overline{3}^\circ + 0.01\overline{3}^\circ$

$60^\circ35'48'' = 60.59\overline{6}^\circ$

Now $1^\circ = \frac{\pi}{180}\text{Radian}$

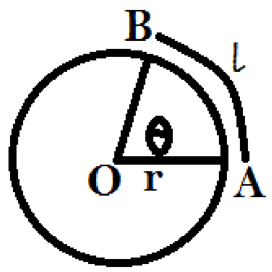
$60.59\overline{6} \times 1^\circ = 60.59\overline{6} \times \frac{\pi}{180}\text{Radian}$

$60.59\overline{6}^\circ = 60.59\overline{6} \times \frac{\pi}{180}\text{Radian}$

$60.59\overline{6}^\circ = 1.058037 \;\text{Radian}$

Length of an arc

Consider a circle with centre O and radius r, which subtends an angle θ radians at the centre O. Let AB is a minor arc of circle whose length l as shown in figure.



By definition $Radian = \dfrac{length\ of\ an\ arc\ AB}{Radius\ of\ circle}$

$$\theta = \frac{l}{r}$$
$$l = r\theta$$

Exp5: Find the length of an arc of a circle of radius 5cm which subtends an angle of $\frac{3\pi}{4}$ radians at the centre.

Sol: Given $r = 5cm$, and $\theta = \frac{3\pi}{4}$ radian

Using $l = r\theta$

$$l = 5\left(\frac{3\pi}{4}\right)$$
$$l = \frac{15\pi}{4} = 11.78cm$$

Exp6: Find the distance travelled by a cyclist moving on a circle of radius 15m, if he makes 3.5 resolutions.

Sol: radius = 15m

And 1 revolution = 2π radians

3.5 revolutions $= 3.5\times 2\pi$

$$\theta = 7\pi$$

Using $l = r\theta$ putting values

$$l = 15\times 7\pi$$
$$l = 105\pi\ m$$

Exp7: An arc of length 2.5cm of a circle subtends an angle θ at the centre O of diameter 6cm. Find the value of θ

Sol: Length of arc l = 2.5 cm

diameter = 6cm

$$d = 2r$$
$$6cm = 2r$$
$$\frac{6cm}{2} = r$$
$$r = 3cm$$

Using $l = r\theta$ putting values

$$2.5 = 3\theta$$
$$\frac{2.5}{3} = \theta$$
$$\Rightarrow \theta = 0.833\ Radians$$

Exp8: If length of an arc of circle is 5cm which subtends an angle of measure 60^o find the radius of circle.

Sol: $l = 5cm,\ \ \theta = 60^o$

First convert the angle into radian measure

Since $\therefore 1^o = \frac{\pi}{180} Radians$

Therefore $\theta = 60^o = 60\left(\frac{\pi}{180}\right) Radians$

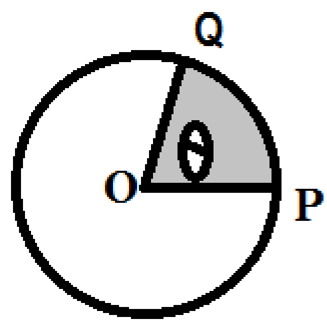
$$\theta = 60^o = \frac{\pi}{3} Radians$$

Using $l = r\theta$ putting values

$$5 = r\left(\frac{\pi}{3}\right)$$
$$\frac{5\times 3}{\pi} = r$$
$$r = \frac{15}{\pi} = 4.78cm$$

Area of Sector

Consider a circle of radius r with centre O. PQ is an arc with subtend an angle θ radians at centre



By proportion

$$\frac{Area\ of\ sector\ POQ}{Area\ of\ circle} = \frac{Central\ angle\ of\ sector}{2\pi}$$
$$\frac{Area\ of\ sector\ POQ}{\pi r^2} = \frac{\theta}{2\pi}$$
$$Area\ of\ sector\ POQ = \frac{\theta}{2\pi}\left(\pi r^2\right)$$
$$Area\ of\ sector\ POQ = \frac{1}{2}r^2\theta$$

Hence Area of sector of a circle with radius r, whose central angle θ radians given by

$$A = \frac{1}{2}r^2\theta$$

Exp9: Find the area of sector with central angle 60^o in a circular region whose radius 5cm.

Sol: Given $r = 5cm, \theta = 60^o$

First convert angle into radian

Since $\therefore 1^o = \frac{\pi}{180} Radians$

Therefore $\theta = 60^o = 60\left(\frac{\pi}{180}\right) Radians$

$$\theta = 60^o = \frac{\pi}{3} Radians$$

Using $A = \frac{1}{2}r^2\theta$ Putting values

$$A = \frac{1}{2}(5)^2\left(\frac{\pi}{3}\right)$$
$$A = \frac{25\pi}{6} = 13.08\ cm^2$$

Exercise 7.2

Q1i): Find l when $\theta=\frac{\pi}{6}$ radian, r = 2cm

Sol: Since $l=r\,\theta$ putting the values

$$l=2\left(\frac{\pi}{6}\right)=\frac{\pi}{3}$$

$$l=1.047\ cm$$

Q1ii). Find l when $\theta=30^\circ$, r = 6 cm

Solution: θ must be in radian so

$$30\times1^\circ=30\times\frac{\pi}{180}\text{Radian}$$

$$30^\circ=\frac{\pi}{6}\text{Radian}$$

$$\Rightarrow\theta=\frac{\pi}{6}\text{Radian}$$

Since $l=r\theta$

$$l=6\left(\frac{\pi}{6}\right)cm$$

$$l=\pi\,cm$$

$$l=3.141592\,cm$$

Qiii). Find l when $\theta=\frac{4\pi}{6}$ radian, r = 6 cm

Sol: Since $l=r\,\theta$ putting the values

$$l=6\left(\frac{4\pi}{6}\right)$$

$$l=4\pi cm$$

$$l=12.57cm$$

Q2i): Find θ when l = 5 cm, r = 2cm

Sol: Since $l=r\,\theta$ putting the values

$$5=2\theta$$

$$\theta=2.5\ Radian$$

Q2ii): Find θ when l = 30 cm, r = 6 cm

Sol: Since $l=r\,\theta$ putting the values

$$30=6\theta$$

$$\theta=5\ Radian$$

Q2iii): Find θ when l = 6 cm, r = 2.87 cm

Sol: Since $l=r\,\theta$ putting the values

$$6=2.87\ \theta$$

$$\theta=\frac{6}{2.87}=2.091\ Radian$$

Q3i). Find r when $\theta=\frac{\pi}{6}$ Radian, $l=$ 2cm

Sol: Since $l=r\theta$ putting the values

$$2=r\left(\frac{\pi}{6}\right)$$

$$r=\frac{12}{\pi}=3.81922\,cm$$

Q3ii). Find r when $\theta=3\frac{1}{2}=\frac{7}{2}$ Radian, $l=\frac{4}{7}$ m

Sol: Since $l=r\,\theta$ putting the values

$$\frac{4}{7}=r\frac{7}{2}$$

$$r=\frac{4\times2}{7\times7}=\frac{8}{49}m$$

$$r=0.163264m$$

$$r=16.3265cm$$

Q3iii). Find r when $\theta=\frac{3\pi}{4}$ Radian, $l=15$ cm

Sol: Since $l=r\,\theta$ putting the values

$$15=r\left(\frac{3\pi}{4}\right)$$

$$r=\frac{15\times4}{3\pi}=\frac{20}{\pi}cm$$

$$r=6.365\ cm$$

Q4: Find area of sector of a circle whose radius is 4 m, with central angle 12 radian.

Solution: Given that r = 4m, $\theta=12\text{Radian}$

$$\text{Area of sector}=\frac{1}{2}r^2\theta$$

$$\text{Area of sector}=\frac{1}{2}(4)^2(12)$$

$$\text{Area of sector}=\frac{1}{2}(16)(12)$$

$$\text{Area of sector}=8\times12=96m^2$$

Q5i): The arc of a circle subtends an angle of 30^o at the centre. The radius of a circle is 5cm Find the length of the arc

Sol: Given that r = 5cm $\theta=30^\circ=\frac{\pi}{6}\text{Radian}$

Since $l=r\,\theta$ putting the values

$$l=5\left(\frac{\pi}{6}\right)=\frac{5\pi}{6}cm$$

$$l=2.6183\ cm$$

Q5i): The arc of a circle subtends an angle of 30^o at the centre. The radius of a circle is 5cm. Area of a sector formed

Sol: Given that r = 5cm $\theta=30^\circ=\frac{\pi}{6}\text{Radian}$

$$\text{Area of sector}=\frac{1}{2}r^2\theta$$

$$\text{Area of sector}=\frac{1}{2}(5)^2\left(\frac{\pi}{6}\right)$$

$$\text{Area of sector}=\frac{1}{2}(25)\left(\frac{\pi}{6}\right)$$

$$\text{Area of sector}=\frac{25\pi}{12}cm^2=6.5458cm^2$$

Q6. An arc of a circle subtends an angle of 2 radian at the centre. If area of sector formed is $64cm^2$. Find the radius of circle

Sol: Given $\theta=2\,Radian,\ A=64cm^2$

Using $A=\frac{1}{2}r^2\theta$ Putting values

$$64=\frac{1}{2}r^2(2)$$

$$\Rightarrow r^2=64$$

$$\Rightarrow r=8$$

Q7. In a circle of radius 10m. Find the distance travel by a point moving on this circle if the point makes 3.5 revolutions.

Sol: radius = 10m

And 1 revolution = 2π radians

3.5 revolutions $= 3.5\times 2\pi$

$\theta = 7\pi$

Using $l = r\theta$ putting values

$l = 10(7\pi)$

$l = 70\pi = 219.94$ m

Q8. What is the circular measure of an angle between the hand of the watch 3 o'clock.

Sol: one complete rotation = 360°.

3 O'clock shows $= \frac{1}{4}$ of complete rotation

3 O'clock shows $= \frac{1}{4}\times 360^o$

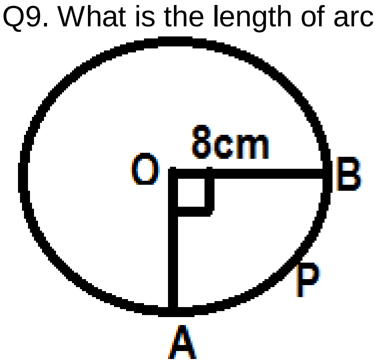
3 O'clock shows $= 90^o$

Since $\therefore 1^o = \frac{\pi}{180} Radians$

Therefore $90^o = 90\times\frac{\pi}{180} Radians = \frac{\pi}{2} Radians$

3 O'clock show $\theta = 90^o = \frac{\pi}{2} Radians$

Q9. What is the length of arc APB?



Sol: from the figure r = 8cm

And $\theta = 90^o = \frac{\pi}{2} Radians$

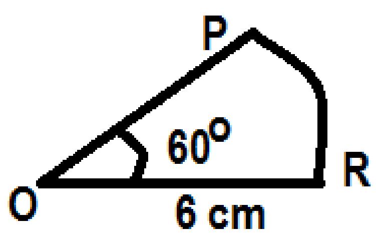
Using $l = r\theta$ putting values

$l = 8\left(\frac{\pi}{2}\right)$

$l = 4\pi\, cm$

$l = 12.59\, cm$

Q10. Find area of sector OPR



Sol: From the figure r = 6cm and $\theta = 60^o$

First convert the angle into radian measure

Since $\therefore 1^o = \frac{\pi}{180} Radians$

Therefore $\theta = 60^o = 60\left(\frac{\pi}{180}\right) Radians$

$\theta = 60^o = \frac{\pi}{3} Radians$

Using $A = \frac{1}{2}r^2\theta$ Putting values

$$A = \frac{1}{2}(6)^2\left(\frac{\pi}{3}\right)$$

$$A = \frac{36\pi}{6} = 6\pi$$

A= 18.85 cm^2

The General Angle (Coterminal Angles)

Angles having same initial and terminal sides are called coterminal angles. e.g. $\theta = 30^o$

Anticlock wise/Positive Clock wise/Negative

$\theta = 30^o + 360^o = 390^o$ $\theta = 30^o - 360^o = -330^o$

When angle in Radians measure e.g. $\theta = \frac{\pi}{3}$

Anticlock wise/Positive Clock wise/Negative

$\theta = \frac{\pi}{3} + 2\pi = \frac{7\pi}{3}$ $\theta = \frac{\pi}{3} - 2\pi = \frac{-5\pi}{3}$

Exp10 Find coterminal angles of 60° And - 60°

Sol: Since $60^o + 360^o = 420^o$

And $60^o - 360^o = -300^o$

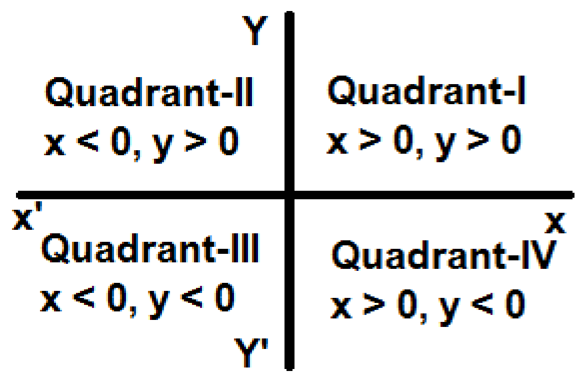
Thus 420^o & -300^o are coterminal angles of 60^o

Now $-60^o + 360^o = 300^o$

And $-60^o - 360^o = -420^o$

Thus 300^o & -420^o are coterminal angles of -60^o

Quadrants: The Cartesian plane is divided into four quadrants and the angel θ is said to be in the quadrant.



Quadrantal Angles

Quadrantal angles are those angles whose terminal sides coinside with co-ordinate axes i.e. x-axis or y-axis. The Quadrantal measure of angles are $0^o, 90^o, 180^o, 270^o, 360^o$ or

$0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ Randians

Exercise 7.3

Q1: Find coterminal angles of following angles.

i). 55 °

Sol: Given angle 55 °

Coterminal Angle Anit-clock wise	Coterminal Angle Clock wise
55 ° + 360 ° = 415 °	55 ° - 360 ° = - 305 °

ii). – 45°

Sol: Given angle -45 °

Coterminal Angle Anit-clock wise	Coterminal Angle Clock wise
-45 ° + 360 ° = 315 °	-45 ° - 360 ° =- 405 °

iii). $\frac{\pi}{6}$ radian

Sol: Given angle $\frac{\pi}{6}$ radian

Coterminal Angle Anit-clock wise	Coterminal Angle Clock wise
$\frac{\pi}{6}+2\pi=\frac{13\pi}{6}$ Radian	$\frac{\pi}{6}-2\pi=\frac{-11\pi}{6}$ Radian

iv). $\frac{-3\pi}{4}$ Radian

Sol: Given angle $\frac{-3\pi}{4}$ radian

Coterminal Angle Anit-clock wise	Coterminal Angle Clock wise
$\frac{-3\pi}{4}+2\pi=\frac{5\pi}{4}$ Radian	$\frac{-3\pi}{4}-2\pi=\frac{-11\pi}{4}$ Radian

Q2: In which quadrant the following angles lie?

i). $\frac{8\pi}{5}$ radian

Solution: Since $\pi\,\text{radian}=180^\circ$

$\frac{8\pi}{5}=\frac{8\times180^\circ}{5}$

$\frac{8\pi}{5}=8\times36^\circ$

$\frac{8\pi}{5}=288^\circ\in 4^{th}$ Quadrant

ii). 75°

Solution: $75^\circ < 90^\circ$

So $75^\circ\in$ 1^{st} Quadrant

iii). -818°

Solution: First we find coterminal angle

$-818^\circ+720^\circ=-98^\circ$

-818° is lies in 3^{rd} quadrant because angle is negative

iv). $\frac{-5\pi}{4}$ Radian

Solution: Since $\pi\,\text{radian}=180^\circ$

$\frac{-5\pi}{4}=\frac{-5\times180^\circ}{4}$

$\frac{-5\pi}{4}=-5\times45^\circ$

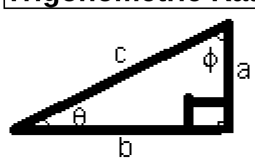
$\frac{-5\pi}{4}=-225^\circ\in 2^{nd}$ Quadrant

because angle is negative

v). 103°

Solution: 103° 2^{nd} Quadrant

Trigonometric Ratios of an acute angle:



ABC is a right angled triangle in which $\angle C$ is right angle. Ratio of any two sides of triangle is called trigonometric ratio. There are following six possible ratios of the sides a, b and c.

$\frac{a}{c},\frac{b}{c},\frac{a}{b},\frac{c}{a},\frac{c}{b},\frac{b}{a}$ Suppose that in ΔABC

$m\angle A=\theta$ (theta).

Ratio = $\frac{\text{length of side opposite to }\theta}{\text{length of hypotenuse}}$ is called sine θ

Ratio = $\frac{\text{length of side adjacent to }\theta}{\text{length of hypotenuse}}$ is called cosine θ

Ratio = $\frac{\text{length of side opposite to }\theta}{\text{length of side adjacent to }\theta}$ is called tangent θ

Ratio = $\frac{\text{length of hypotenuse}}{\text{length of side opposite to }\theta}$ is called cosecant θ

Ratio = $\frac{\text{length of hypotenuse}}{\text{length of side adjacent to }\theta}$ is called secant θ

Ratio = $\frac{\text{length of side adjacent to }\theta}{\text{length of side opposite to }\theta}$ is called cotangent θ

Reciprocal Relations between trigonometric Ratios:

$\sin\theta=\frac{1}{\cos ee\theta}$ $\cos\theta=\frac{1}{see\theta}$

$\tan\theta=\frac{1}{\cot\theta}$ $\cos ee\theta=\frac{1}{\sin\theta}$

$see\theta=\frac{1}{\cos\theta}$ $\cot\theta=\frac{1}{\tan\theta}$

Trigonometric Identities:

$Sin^2x+Cos^2x=1$

$See^2x=1+Tan^2x$

$Cosee^2x=1+Cot^2x$

Hint for trigonometric functions:

S O H	CAH
$\sin\theta=\frac{Opposite}{hypotenuse}$	$\cos\theta=\frac{adjacent}{hypotenuse}$
TOA	
$\tan\theta=\frac{Opposite}{Adjacent}$	

Or Hint for trigonometric functions:

Some **P**eople **H**ave	**C**urly **B**rown **H**air
$\sin\theta=\frac{perpendicular}{hypotenuse}$	$\cos\theta=\frac{Base}{hypotenuse}$
Through **P**roper **B**rushing	
$\tan\theta=\frac{perpendicular}{Base}$	

Exp11i): Find sign of $\sin105^o$

Sol: $\sin105^o$ here angle $105\in$ Quadrant-II

Hence sign of $\sin105^o$ is positive

Exp11ii): Find sign of $\tan\frac{-5\pi}{6}$

Sol: $\tan\frac{-5\pi}{6}$

here $\theta=\tan\frac{-5\pi}{6}=-150^o\in$ Quadrant-III

Hence sign of $\tan\frac{-5\pi}{6}$ is positive

Exp11iii): Find sign of $\sec1030^o$

Sol: Given $\sec1030^o$ coterminal angle of 1030^o

$1030^o \approx 670^o \approx 310^o \in$ Quadrant-IV

Hence sign of $\sec 1030^o$ is positive

Exp11iv): Find sign of $\cot 710^o$

Sol: Given $\cot 710^o$ coterminal angle of 710^o

$710^o \approx 310^o \in$ Quadrant-IV

Hence sign of $\cot 710^o$ is Negative.

Exp12: If $\tan\theta < 0$ and $\cos\theta > 0$ name the quadrant in which θ lies.

Sol: Given $\tan\theta < 0$ and $\cos\theta > 0$

θ lies II, VI and I and VI quadrants

Therefore θ lies in Quadrant VI

Exp13: If $\tan\theta = 1$, find other trigonometric ratios, where θ lies in first quadrant

Sol: Here $\tan\theta = \frac{Perp}{Base} = \frac{1}{1} = 1$

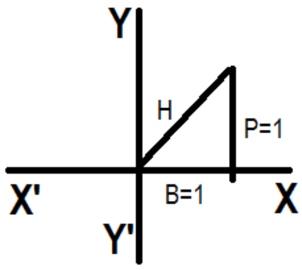
Using Pythagoras theorem

$H^2 = B^2 + P^2$

$H^2 = 1^2 + 1^2$

$H^2 = 2$

$\Rightarrow H = \sqrt{2}$



Now using Hint for trigonometric ratios

Hint	functions	Reciprocal
SPH	$\sin\theta = \frac{1}{\sqrt{2}}$	$\cos ec\theta = \frac{\sqrt{2}}{1}$
CBH	$\cos\theta = \frac{1}{\sqrt{2}}$	$\sec\theta = \frac{\sqrt{2}}{1}$
TPB	$\tan\theta = \frac{1}{1} = 1$	$\cot\theta = \frac{1}{1} = 1$

Exp14: If $\tan\theta = \frac{-2}{3}$ & θ is in 2nd Quadrant

Find the other trigonometric functions.

Sol: Here $\cos\theta = \frac{Base}{Hyp} = \frac{4}{5}$

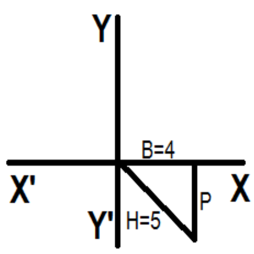
Using Pythagoras theorem

$H^2 = B^2 + P^2$

$H^2 = (-2)^2 + 3^2$

$H^2 = 4 + 9$

$\Rightarrow H = \sqrt{13}$



Now using Hint for trigonometric ratios

Hint	functions	Reciprocal
SPH	$\sin\theta = \frac{3}{\sqrt{13}}$	$\cos ec\theta = \frac{\sqrt{13}}{3}$
CBH	$\cos\theta = \frac{-2}{\sqrt{13}}$	$\sec\theta = \frac{\sqrt{13}}{-2}$
TPB	$\tan\theta = \frac{-2}{3}$	$\cot\theta = \frac{3}{-2}$

Exp15: If $\cos\theta = \frac{4}{5}$ & θ is in 4th Quadrant

Find the other trigonometric functions.

Sol: Here $\tan\theta = \frac{Perp}{Base} = \frac{1}{1} = 1$

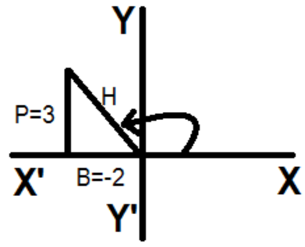
Using Pythagoras theorem

$H^2 = B^2 + P^2$

$5^2 = 4^2 + P^2$

$25 - 16 = P^2$

$\Rightarrow P = \sqrt{9}$



$\Rightarrow P = \pm 3$ but in 4th Quadrant $P = -3$

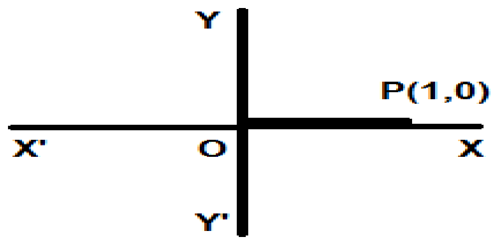
Now using Hint for trigonometric ratios

Hint	functions	Reciprocal
SPH	$\sin\theta = \frac{-3}{5}$	$\cos ec\theta = \frac{5}{-3}$
CBH	$\cos\theta = \frac{4}{5}$	$\sec\theta = \frac{5}{4}$
TPB	$\tan\theta = \frac{-3}{4}$	$\cot\theta = \frac{4}{-3}$

Trigonometric Ratios of 0^o

In XY Plane, terminal side of $\overrightarrow{OP}$ coincide with $\overrightarrow{OX}$, therefore radius $\overrightarrow{OP} = r = 1$

Or $x = 1$, & $y = 0$



$\sin 0^o = \frac{y}{r} = \frac{0}{1} = 0$

$\cos 0^o = \frac{x}{r} = \frac{1}{1} = 1$

$\tan 0^o = \frac{y}{x} = \frac{0}{1} = 0$

$\cos ec 0^o = \frac{r}{y} = \frac{1}{0} = Undefined$

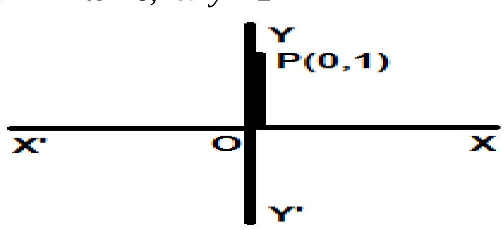
$\sec 0^o = \frac{r}{x} = \frac{1}{1} = 1$

$\cot 0^o = \frac{x}{y} = \frac{1}{0} = undefined$

Trigonometric Ratios of 90^o

In XY Plane, terminal side of $\overrightarrow{OP}$ coincide with $\overrightarrow{OX}$, therefore radius $\overrightarrow{OP} = r = 1$

Or $x = 0$, & $y = 1$



$\sin 90^o = \frac{y}{r} = \frac{1}{1} = 1$

$\cos 90^o = \frac{x}{r} = \frac{0}{1} = 0$

$\tan 90^o = \frac{y}{x} = \frac{1}{0} = Undefined$

$\cos ec 90^o = \frac{r}{y} = \frac{1}{1} = 1$

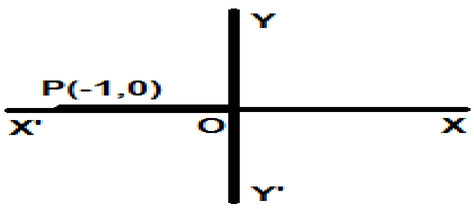
$\sec 90^o = \frac{r}{x} = \frac{1}{0} = Undefined$

$\cot 90^o = \frac{x}{y} = \frac{0}{1} = 0$

Trigonometric Ratios of 180^o

In XY Plane, terminal side of $\overrightarrow{OP}$ coincide with $\overrightarrow{OX}$, therefore radius $\overrightarrow{OP} = r = 1$

Or $x = -1, \ \& \ y = 0$



$\sin 180^o = \frac{y}{r} = \frac{0}{1} = 0$

$\cos 180^o = \frac{x}{r} = \frac{-1}{1} = -1$

$\tan 180^o = \frac{y}{x} = \frac{0}{-1} = 0$

$\cos ec 180^o = \frac{r}{y} = \frac{1}{0} = Undefined$

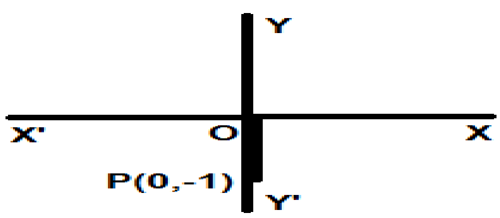
$\sec 180^o = \frac{r}{x} = \frac{1}{-1} = -1$

$\cot 180^o = \frac{x}{y} = \frac{-1}{0} = Undefined$

Trigonometric Ratios of 270^o

In XY Plane, terminal side of $\overrightarrow{OP}$ coincide with $\overrightarrow{OX}$, therefore radius $\overrightarrow{OP} = r = 1$

Or $x = 0, \ \& \ y = -1$



$\sin 270^o = \frac{y}{r} = \frac{-1}{1} = -1$

$\cos 270^o = \frac{x}{r} = \frac{0}{1} = 0$

$\tan 270^o = \frac{y}{x} = \frac{-1}{0} = Undefined$

$\cos ec 270^o = \frac{r}{y} = \frac{1}{-1} = -1$

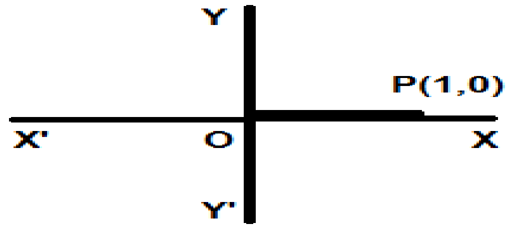
$\sec 270^o = \frac{r}{x} = \frac{1}{0} = Undefined$

$\cot 270^o = \frac{x}{y} = \frac{0}{-1} = 0$

Trigonometric Ratios of 360^o

In XY Plane, terminal side of $\overrightarrow{OP}$ coincide with $\overrightarrow{OX}$, therefore radius $\overrightarrow{OP} = r = 1$

Or $x = 1, \ \& \ y = 0$



$\sin 360^o = \frac{y}{r} = \frac{0}{1} = 0$

$\cos 360^o = \frac{x}{r} = \frac{1}{1} = 1$

$\tan 360^o = \frac{y}{x} = \frac{0}{1} = 0$

$\cos ec 360^o = \frac{r}{y} = \frac{1}{0} = Undefined$

$\sec 360^o = \frac{r}{x} = \frac{1}{1} = 1$

$\cot 360^o = \frac{x}{y} = \frac{1}{0} = undefined$

Exercise 7.4

Q1: Find the sign of the following trigonometric ratios and tell in which quadrant they lie?

i). Sin 98 ^{o}

Solution: since $98^o \in 2^{nd}$ Quadrant

And sin is positive in 2^{nd} Quadrant,

So Sin 98^o is positive

ii). sin 160^o

Solution: since $160^o \in 2^{nd}$ Quadrant

And sine is positive in 2^{nd} Quadrant,

So Sin 160^o is positive

iii). tan 200^o

Solution: since $200^o \in 3^{rd}$ Quadrant

And Tangent is positive in 3^{rd} Quadrant, so,

Tan 200^o is positive

iv). Sec 120^o

Solution: since $120^o \in 2^{nd}$ Quadrant

And Secant is negative in 2^{nd} Quadrant,

so, sec 120^o is negative

v). cosec 198^o

Solution: since $198^o \in 3^{rd}$ Quadrant

And cosecant is negative in 3^{rd} Quadrant,

so, cosec 198^o is negative

vi). sin 460^o

Solution: coterminal angle of 460^o is $100^o \in 2^{nd}$ Q

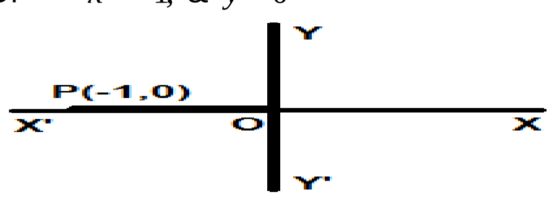
And Sine is positive in 2^{nd} Quadrant,

so, sin 460^o is positive

Q2i): Find trigonometric ratios of -180^o

Sol: In XY Plane, terminal side of $\overrightarrow{OP}$ coincide with $\overrightarrow{OX}$, therefore radius $\overrightarrow{OP} = r = 1$

Or $x = -1, \ \& \ y = 0$



$\sin(-180^o) = \frac{y}{r} = \frac{0}{1} = 0$

$\cos(-180^o) = \frac{x}{r} = \frac{-1}{1} = -1$

$\tan(-180^o) = \frac{y}{x} = \frac{0}{-1} = 0$

$\cos ec(-180^o) = \frac{r}{y} = \frac{1}{0} = Undefined$

$\sec(-180^o) = \frac{r}{x} = \frac{1}{-1} = -1$

$\cot(-180^o) = \frac{x}{y} = \frac{-1}{0} = Undefined$

Q2i): Find trigonometric ratios of -180^o

Solution: Reference angle of -180^0 is 0^0

$Sin(-180^\circ) = \frac{\sqrt{0}}{2} = 0$

$Cos(-180^\circ) = \frac{-\sqrt{4}}{2} = \frac{-2}{2} = -1$

$\tan(-180^\circ) = \frac{\sin(-180^\circ)}{\cos(-180^\circ)} = \frac{0}{-1} = 0$

$\cos ec(-180^\circ) = \frac{1}{\sin(-180^\circ)} = \frac{1}{0} = \infty$

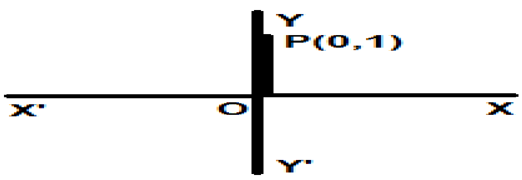
$\sec(-180^\circ) = \frac{1}{\cos(-180^\circ)} = \frac{1}{-1} = -1$

$Cot(-180^\circ) = \frac{\cos(-180^\circ)}{\sin(-180^\circ)} = \frac{-1}{0} = -\infty$

Q2ii): Find trigonometric ratios of -270^o

Sol: In XY Plane, terminal side of $\overrightarrow{OP}$ coincide with $\overrightarrow{OX}$, therefore radius $\overrightarrow{OP} = r = 1$

Or $x = 0$, & $y = 1$



$\sin(-270^o) = \frac{y}{r} = \frac{1}{1} = 1$

$\cos(-270^o) = \frac{x}{r} = \frac{0}{1} = 0$

$\tan(-270^o) = \frac{y}{x} = \frac{1}{0} = Undefined$

$\cos ec(-270^o) = \frac{r}{y} = \frac{1}{1} = 1$

$\sec(-270^o) = \frac{r}{x} = \frac{1}{0} = Undefined$

$\cot(-270^o) = \frac{x}{y} = \frac{0}{1} = 0$

Q2ii): Find trigonometric ratios of -270^o

Solution: Reference angle of -270^0 is 90^0

$Sin(-270^\circ) = \frac{\sqrt{4}}{2} = \frac{2}{2} = 1$

$Cos(-270^\circ) = \frac{\sqrt{0}}{2} = \frac{0}{2} = 0$

$\tan(-270^\circ) = \frac{\sin(-270^\circ)}{\cos(-270^\circ)} = \frac{1}{0} = \infty$

$\cos ec(-270^\circ) = \frac{1}{\sin(-270^\circ)} = \frac{1}{1} = 1$

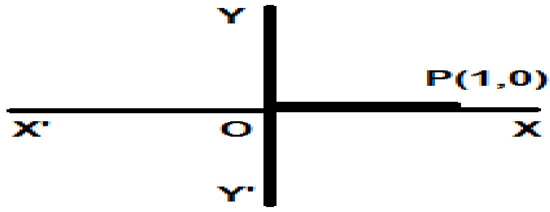
$\sec(-270^\circ) = \frac{1}{\cos(-270^\circ)} = \frac{1}{0} = \infty$

$Cot(-270^\circ) = \frac{\cos(-270^\circ)}{\sin(-270^\circ)} = \frac{0}{1} = 0$

Q2iii): Find trigonometric ratios of 720^o

Sol: In XY Plane, terminal side of $\overrightarrow{OP}$ coincide with $\overrightarrow{OX}$, therefore radius $\overrightarrow{OP} = r = 1$

Or $x = 1$, & $y = 0$



$\sin 720^o = \frac{y}{r} = \frac{0}{1} = 0$

$\cos 720^o = \frac{x}{r} = \frac{1}{1} = 1$

$\tan 720^o = \frac{y}{x} = \frac{0}{1} = 0$

$\cos ec720^o = \frac{r}{y} = \frac{1}{0} = Undefined$

$\sec 720^o = \frac{r}{x} = \frac{1}{1} = 1$

$\cot 720^o = \frac{x}{y} = \frac{1}{0} = undefined$

Q2iii): Find trigonometric ratios of 720^o

Solution: Reference angle of 720^0 is 0^0

$\sin(720^\circ) = \frac{\sqrt{0}}{2} = 0$

$\cos(720^\circ) = \frac{\sqrt{4}}{2} = \frac{2}{2} = 1$

$\tan(720^\circ) = \frac{0}{1} = 0$

$\cos ec(720^\circ) = \frac{1}{\sin(720^\circ)} = \frac{1}{0} = \infty$

$\sec(720^\circ) = \frac{1}{\cos(720^\circ)} = \frac{1}{1} = 1$

$\cot(720^\circ) = \frac{\cos(720^\circ)}{\sin(720^\circ)} = \frac{1}{0} = \infty$

iv). 1470^o

Solution: Reference angle of 1470^0 is 30^0

$Sin(1470^\circ) = \frac{\sqrt{1}}{2} = \frac{1}{2}$

$Cos(1470^\circ) = \frac{\sqrt{3}}{2}$

$\tan(1470^\circ) = \frac{\sin(1470^\circ)}{\cos(1470^\circ)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$

$\cos ec(1470^\circ) = \frac{1}{\sin(1470^\circ)} = \frac{1}{\frac{1}{2}} = \frac{2}{1} = 2$

$\sec(1470^\circ) = \frac{1}{\cos(1470^\circ)} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$

$Cot(1470^\circ) = \frac{\cos(1470^\circ)}{\sin(1470^\circ)} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{1} = \sqrt{3}$

Q3: If $\sec\theta = 2$ where θ lies in 4^{th} quadrant, find the other values of trigonometric ratios.

Sol: 4^{th} Quadrant, sine -ve, cosine +ve

We have $\sec\theta = 2$ or $\frac{1}{\cos\theta} = \frac{2}{1}$

$\Rightarrow \cos\theta = \dfrac{1}{2} = \dfrac{\text{Base}}{\text{Hpy}}$

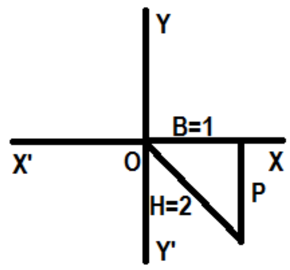
Using Pythagoras theorem

$H^2 = B^2 + P^2$

$2^2 = 1^2 + P^2$

$4 - 1 = P^2$

$\Rightarrow P = \sqrt{3}$



$\Rightarrow P = \pm\sqrt{3}$ but in 4th Quadrant $P = -\sqrt{3}$

Now using Hint for trigonometric ratios

Hint	functions	Reciprocal
SPH	$\sin\theta = \dfrac{-\sqrt{3}}{2}$	$\operatorname{cosec}\theta = \dfrac{2}{-\sqrt{3}}$
CBH	$\cos\theta = \dfrac{1}{2}$	$\sec\theta = \dfrac{2}{1}$
TPB	$\tan\theta = \dfrac{-\sqrt{3}}{1}$	$\cot\theta = \dfrac{1}{-\sqrt{3}}$

Q4: If $\sin\theta = \dfrac{4}{5}$ and $\dfrac{\pi}{2} < \theta < \pi$ then find other trigonometric ratios.

Solution: θ lies in 2nd Quadrant

$\sin\theta = \dfrac{4}{5} = \dfrac{Perp}{Hyp}$

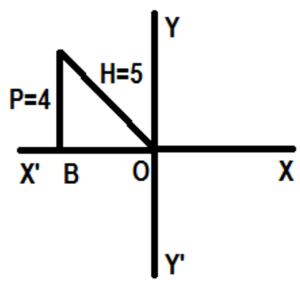
Using Pythagoras theorem

$H^2 = B^2 + P^2$

$5^2 = B^2 + 4^2$

$25 - 16 = P^2$

$\Rightarrow P = \sqrt{9}$



$\Rightarrow P = \pm 3$ but in 4th Quadrant $P = -3$

Now using Hint for trigonometric ratios

Hint	functions	Reciprocal
SPH	$\sin\theta = \dfrac{4}{5}$	$\operatorname{cosec}\theta = \dfrac{5}{4}$
CBH	$\cos\theta = \dfrac{-3}{5}$	$\sec\theta = \dfrac{5}{-3}$
TPB	$\tan\theta = \dfrac{-3}{4}$	$\cot\theta = \dfrac{4}{-3}$

Q5: Find the values of

i) . $2\sin 45^\circ \cos 45^\circ$

Solution: Since $\sin 45^\circ = \frac{\sqrt{2}}{2}$, $\quad Cos 45^\circ = \frac{\sqrt{2}}{2}$

$2\sin 45^\circ \cos 45^\circ = 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$2\sin 45^\circ \cos 45^\circ = \frac{\left(\sqrt{2}\right)^2}{2}$

$2\sin 45^\circ \cos 45^\circ = \frac{2}{2}$

$2\sin 45^\circ \cos 45^\circ = 1$

ii). $\dfrac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ}$

Solution: $\tan 60^\circ = \dfrac{\sqrt{3}}{1}$, $\quad \tan 30^\circ = \dfrac{1}{\sqrt{3}}$

$$\frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ} = \frac{\frac{\sqrt{3}}{1} - \frac{1}{\sqrt{3}}}{1 + \left(\frac{\sqrt{3}}{1}\right)\left(\frac{1}{\sqrt{3}}\right)}$$

$$\frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ} = \frac{\frac{3-1}{\sqrt{3}}}{1+1}$$

$$\frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ} = \frac{1}{2}\left(\frac{2}{\sqrt{3}}\right)$$

$$\frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ} = \frac{1}{\sqrt{3}}$$

iii). $\dfrac{\cos 45^\circ}{\sin 45^\circ + \tan 45^\circ}$

Sol: $\cos 45^\circ = \dfrac{\sqrt{2}}{2}$, $Sin 45^\circ = \dfrac{\sqrt{2}}{2}$, $\tan 45^\circ = 1$

$$\frac{\cos 45^\circ}{\sin 45^\circ + \tan 45^\circ} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2} + 1}$$

$$\frac{\cos 45^\circ}{\sin 45^\circ + \tan 45^\circ} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}+2}{2}} = \frac{\sqrt{2}}{\sqrt{2} + \sqrt{2}\sqrt{2}}$$

$$\frac{\cos 45^\circ}{\sin 45^\circ + \tan 45^\circ} = \frac{\sqrt{2}}{\sqrt{2}\left(1 + \sqrt{2}\right)}$$

$$\frac{\cos 45^\circ}{\sin 45^\circ + \tan 45^\circ} = \frac{1}{1 + \sqrt{2}}$$

iv). $\tan 30^\circ \tan 60^\circ + \tan 45^\circ$

Sol: $\tan 60^\circ = \dfrac{\sqrt{3}}{1}$, $\tan 30^\circ = \dfrac{1}{\sqrt{3}}$, $\tan 45^\circ = 1$

$\tan 30^\circ \tan 60^\circ + \tan 45^\circ = \left(\dfrac{1}{\sqrt{3}}\right)\left(\dfrac{\sqrt{3}}{1}\right) + 1$

$\tan 30^\circ \tan 60^\circ + \tan 45^\circ = 1 + 1$

$\tan 30^\circ \tan 60^\circ + \tan 45^\circ = 2$

v). $\cos\frac{\pi}{3}\cos\frac{\pi}{6} - \sin\frac{\pi}{3}\sin\frac{\pi}{6}$

Sol: we have $\dfrac{\pi}{3} = 60^o$ and $\dfrac{\pi}{6} = 30^o$

$$\cos\frac{\pi}{3}\cos\frac{\pi}{6} - \sin\frac{\pi}{3}\sin\frac{\pi}{6} = \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$$

$$\cos\frac{\pi}{3}\cos\frac{\pi}{6} - \sin\frac{\pi}{3}\sin\frac{\pi}{6} = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

Q6: in which quadrant θ lies?

i). $\sin\theta > 0$, $\quad \tan\theta > 0$

Sol: we have $\sin\theta > 0$, $\quad \tan\theta > 0$

IQ,IIQ $\quad$ IQ, IIIQ

Common Portion = I Quadrant

ii). $\sin\theta < 0$, $\quad \cot\theta > 0$

Sol: we have $\sin\theta < 0$, $\quad \cot\theta > 0$

IIIQ,IVQ, $\quad$ IQ, IIIQ

Common Portion = III Quadrant

iii). $\sin\theta > 0$, $\quad \cos\theta < 0$

Sol: we have $\sin\theta > 0$, $\quad \cos\theta < 0$

IQ,IIQ, $\quad$ IIQ, IIIQ

Common Portion = II Quadrant

iv). $\cos\theta > 0$, $\operatorname{cosec}\theta < 0$

Sol: Since $\cos\theta > 0$, $\operatorname{cosec}\theta < 0$

IQ, IVQ, IIIQ, IVQ

Common Portion = IV Quadrant

v). $\tan\theta < 0$, $\sec\theta > 0$

Sol: we have $\tan\theta < 0$, $\sec\theta > 0$

IIQ, IVQ, IQ, IVQ

Common Portion = IV Quadrant

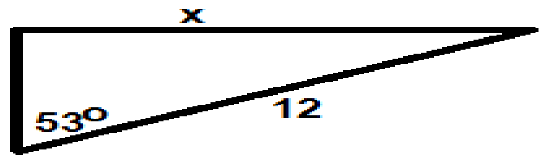
vi). $\cos\theta < 0$, $\tan\theta < 0$

Sol: we have $\cos\theta < 0$, $\tan\theta < 0$

IIQ,IIIQ IIQ, IVQ

Common Portion = II Quadrant

Q7i). Find x measure to two decimal places.



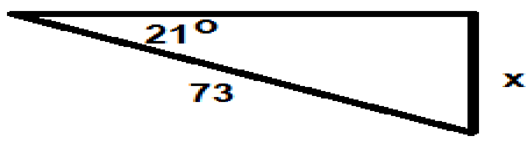
Sol: From figure Perpendicular= x, Hyp=12

Using SPH $\sin 53^o = \dfrac{x}{12}$

$\Rightarrow x = 12\sin 53^o$

$\Rightarrow x = 9.58$

Q7ii). Find x measure to two decimal places.



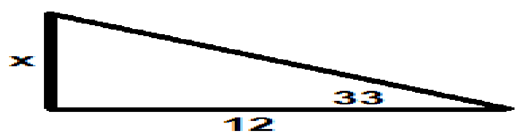
Sol: From figure Perpendicular= x, Hyp=73

Using SPH $\sin 21^o = \dfrac{x}{73}$

$\Rightarrow x = 73\sin 21^o$

$\Rightarrow x = 26.16$

Q7iii). Find x measure to two decimal places.



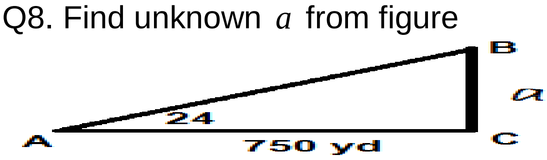
Sol: From figure Perpendicular= x, Base=12

Using TPB $\tan 33^o = \dfrac{x}{12}$

$\Rightarrow x = 12\tan 33^o$

$\Rightarrow x = 7.79$

Q8. Find unknown a from figure



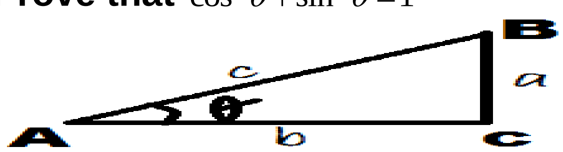
Sol: From Perp = a Base = 750

Using TPB $\tan 24^o = \dfrac{a}{750}$

$\Rightarrow a = 750\tan 24^o$

$\Rightarrow a = 333.92$

Prove that $\cos^2\theta + \sin^2\theta = 1$



In ΔABC , $mAB = c, mBC = a, mCA = b$

And $m\angle ACB = 90^o, m\angle CAB = \theta$

Now using Hint for trigonometric ratios

Hint	functions	Reciprocal
SPH	$\sin\theta = \dfrac{a}{c}$	$\operatorname{cosec}\theta = \dfrac{c}{a}$
CBH	$\cos\theta = \dfrac{b}{c}$	$\sec\theta = \dfrac{c}{b}$
TPB	$\tan\theta = \dfrac{a}{b}$	$\cot\theta = \dfrac{b}{a}$

Using Pythagoras theorem

Base^2 + Perpendicular^2 = Hypotenuse^2

$b^2 + a^2 = c^2$ ....(1)

Dividing eq (1) both sides by c^2

$$\frac{b^2}{c^2} + \frac{a^2}{c^2} = \frac{c^2}{c^2}$$

$$\left(\frac{b}{c}\right)^2 + \left(\frac{a}{c}\right)^2 = 1$$

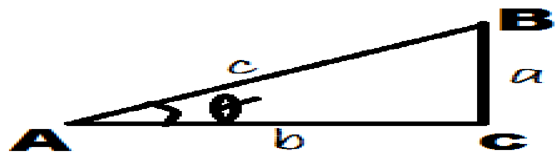
Putting trigonometric functions

$$(\cos\theta)^2 + (\sin\theta)^2 = 1$$

Or $\cos^2\theta + \sin^2\theta = 1$

To calculate $\cos^2 60$ means $(\cos 60)^2 = \left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4}$

Prove that $1 + \tan^2\theta = \sec^2\theta$



In ΔABC , $mAB = c, mBC = a, mCA = b$

And $m\angle ACB = 90^o, m\angle CAB = \theta$

Now using Hint for trigonometric ratios

Hint	functions	Reciprocal
SPH	$\sin\theta = \dfrac{a}{c}$	$\operatorname{cosec}\theta = \dfrac{c}{a}$
CBH	$\cos\theta = \dfrac{b}{c}$	$\sec\theta = \dfrac{c}{b}$
TPB	$\tan\theta = \dfrac{a}{b}$	$\cot\theta = \dfrac{b}{a}$

Using Pythagoras theorem

Base^2 + Perpendicular^2 = Hypotenuse^2

$b^2 + a^2 = c^2$ ....(1)

Dividing eq (1) both sides by b^2

$$\frac{b^2}{b^2} + \frac{a^2}{b^2} = \frac{c^2}{b^2}$$

$$1 + \left(\frac{a}{b}\right)^2 = \left(\frac{c}{b}\right)^2$$

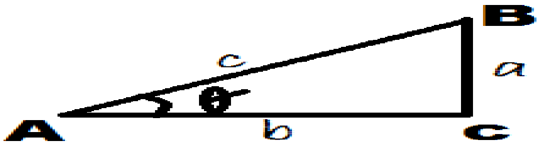
Putting trigonometric functions

$$1 + (\tan\theta)^2 = (\sec\theta)^2$$

Or $1 + \tan^2\theta = \sec^2\theta$

To calculate $\tan^2 60$ means $(\tan 60)^2 = \left(\dfrac{\sqrt{3}}{1}\right)^2 = 3$

Prove that $\cot^2\theta + 1 = \operatorname{cosec}^2\theta$



In ΔABC , $mAB = c, mBC = a, mCA = b$

And $m\angle ACB = 90^o, m\angle CAB = \theta$

Now using Hint for trigonometric ratios

Hint	functions	Reciprocal
SPH	$\sin\theta = \frac{a}{c}$	$\cos ec\theta = \frac{c}{a}$
CBH	$\cos\theta = \frac{b}{c}$	$\sec\theta = \frac{c}{b}$
TPB	$\tan\theta = \frac{a}{b}$	$\cot\theta = \frac{b}{a}$

Using Pythagoras theorem

Base^2 + Perpendicular^2 = Hypotenuse^2

$b^2 + a^2 = c^2$ ....(1)

Dividing eq (1) both sides by a^2

$$\frac{b^2}{a^2} + \frac{a^2}{a^2} = \frac{c^2}{a^2}$$

$$\left(\frac{b}{a}\right)^2 + 1 = \left(\frac{c}{a}\right)^2$$

Putting trigonometric functions

$$(\cot\theta)^2 + 1 = (\cos ec\theta)^2$$

Or $\cot^2\theta + 1 = \cos ec^2\theta$

To calculate $\cot^2 30$ means $(\cot 60)^2 = \left(\frac{\sqrt{3}}{1}\right)^2 = 3$

Exp16: Show that $(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$

Sol: Given $(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$

Taking LHS $= (\sin\theta + \cos\theta)^2$

$= \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$= 1 + 2\sin\theta\cos\theta \qquad \because \sin^2\theta + \cos^2\theta = 1$

= RHS

Hence $(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$

Exp17: Prove that $\sin\theta = \sqrt{1 - \cos^2\theta}$

Sol: Since $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

Taking square root on both sides

$\sin\theta = \sqrt{1 - \cos^2\theta}$ Hence proved

Exp18: Prove that $\frac{\sqrt{1 - \sin^2\theta}}{\sin\theta} = \cot\theta$

Sol: Since $\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

Taking LHS $\frac{\sqrt{1 - \sin^2\theta}}{\sin\theta}$

$= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta$ = RHS

Exp19: Prove that $\sec^2\theta + \tan^2\theta = \frac{1 + \sin^2\theta}{1 - \sin^2\theta}$

Sol: since $\sec\theta = \frac{1}{\cos\theta}$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Taking LHS $\sec^2\theta + \tan^2\theta$

$= \frac{1}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}$

$= \frac{1 + \sin^2\theta}{\cos^2\theta} \qquad \because \sin^2\theta + \cos^2\theta = 1$

$= \frac{1 + \sin^2\theta}{1 - \sin^2\theta} \qquad \because \cos^2\theta = 1 - \sin^2\theta$

Exercise 7.5

Prove that the following trigonometric identities

Q1: $(\sec^2\theta - 1)\cos^2\theta = \sin^2\theta$

Sol: since $\tan^2\theta = \sec^2\theta - 1$

Take LHS $(\sec^2\theta - 1)\cos^2\theta = \tan^2\theta\cos^2\theta$

$(\sec^2\theta - 1)\cos^2\theta = \frac{\sin^2\theta}{\cos^2\theta}\cos^2\theta$

$(\sec^2\theta - 1)\cos^2\theta = \sin^2\theta = RHS$

Hence Proved

Q2: $\tan\theta + \sec\theta = \frac{1 + \sin\theta}{\cos\theta}$

Sol: Take LHS $\tan\theta + \sec\theta = \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}$

$\tan\theta + \sec\theta = \frac{\sin\theta + 1}{\cos\theta}$ = RHS

Hence Proved

Q3: $(\cos\theta - \sin\theta)^2 = 1 - 2\sin\theta\cos\theta$

Sol: LHS $(\cos\theta - \sin\theta)^2 = \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta$

$(\cos\theta - \sin\theta)^2 = 1 - 2\sin\theta\cos\theta$ = RHS

Hence Proved

Q4: $\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

Sol: LHS $\cos^2\theta - \sin^2\theta = \cos^2\theta - (1 - \cos^2\theta)$

$\cos^2\theta - \sin^2\theta = \cos^2\theta - 1 + \cos^2\theta$

$\cos^2\theta - \sin^2\theta = \cos^2\theta + \cos^2\theta - 1$

$\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$ = RHS

Hence Proved

Q5: $\tan\theta + \cot\theta = \sec\theta\cos ec\theta$

Sol: LHS $\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$\tan\theta + \cot\theta = \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$

$\tan\theta + \cot\theta = \frac{1}{\cos\theta\sin\theta}$

$\tan\theta + \cot\theta = \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta}$

$\tan\theta + \cot\theta = \sec\theta\cos ec\theta$ = RHS

Hence Proved

Q6: $\frac{1 - \sin\theta}{\cos\theta} = \frac{\cos\theta}{1 + \sin\theta}$

Sol: LHS $\frac{1 - \sin\theta}{\cos\theta} = \frac{1 - \sin\theta}{\cos\theta} \cdot \frac{1 + \sin\theta}{1 + \sin\theta}$

$\frac{1-\sin\theta}{\cos\theta}=\frac{1^2-\sin^2\theta}{\cos\theta(1+\sin\theta)}$

$\frac{1-\sin\theta}{\cos\theta}=\frac{\cos^2\theta}{\cos\theta(1+\sin\theta)}$

$\frac{1-\sin\theta}{\cos\theta}=\frac{\cos\theta}{1+\sin\theta}=$ RHS

Hence Proved

Q7: $\sin\theta\sqrt{1+\tan^2\theta}=\tan\theta$

Sol: Take LHS $\sin\theta\sqrt{1+\tan^2\theta}=\sin\theta\sqrt{\sec^2\theta}$

$\sin\theta\sqrt{1+\tan^2\theta}=\sin\theta\sec\theta$

$\sin\theta\sqrt{1+\tan^2\theta}=\sin\theta.\frac{1}{\cos\theta}$

$\sin\theta\sqrt{1+\tan^2\theta}=\frac{\sin\theta}{\cos\theta}$

$\sin\theta\sqrt{1+\tan^2\theta}=\tan\theta=$ RHS

Q8: $\cos\theta=\sqrt{1-\sin^2\theta}$

Sol: Since $\sin^2\theta+\cos^2\theta=1$

$\cos^2\theta=1-\sin^2\theta$

Taking square root on both sides

$\cos\theta=\sqrt{1-\sin^2\theta}$ Hence proved

Q9: $(1+\cos\theta)(1-\cos\theta)=\frac{1}{\operatorname{cosec}^2\theta}$

Sol: LHS $(1+\cos\theta)(1-\cos\theta)=1^2-\cos^2\theta$

$(1+\cos\theta)(1-\cos\theta)=\sin^2\theta$

$(1+\cos\theta)(1-\cos\theta)=\frac{1}{\operatorname{cosee}^2\theta}=$ RHS

Hence Proved

Q10. Prove $\cos x-\cos x\sin^2 x=\cos^3 x$

Sol: To prove $\cos x-\cos x\sin^2 x=\cos^3 x$

Taking LHS $\cos x-\cos x\sin^2 x$

$=\cos x-\cos x(1-\cos^2 x)$

$=\cos x-\cos x+\cos^3 x$

$=\cos^3 x$ = RHS

Hence proved

Q11: Prove $\frac{\sin x}{1+\cos x}+\frac{1+\cos x}{\sin x}=2\operatorname{cosec} x$

Sol: Taking LHS $\frac{\sin x}{1+\cos x}+\frac{1+\cos x}{\sin x}$

$=\frac{\sin^2 x+(1+\cos x)^2}{\sin x(1+\cos x)}$

$=\frac{\sin^2 x+1+\cos^2 x+2\cos x}{\sin x(1+\cos x)}$

$=\frac{\sin^2 x+\cos^2 x+1+2\cos x}{\sin x(1+\cos x)}$

$=\frac{1+1+2\cos x}{\sin x(1+\cos x)}$

$=\frac{2+2\cos x}{\sin x(1+\cos x)}$

$=\frac{2(1+\cos x)}{\sin x(1+\cos x)}$

$=\frac{2}{\sin x}$

$=2\operatorname{cosec} x$ = RHS

Hence proved

Q13. Prove $\frac{1}{1+\cos\alpha}+\frac{1}{1-\cos\alpha}=2+2\cot^2\alpha$

Sol: Taking LHS $\frac{1}{1+\cos\alpha}+\frac{1}{1-\cos\alpha}$

$=\frac{1(1-\cos\alpha)+1(1+\cos\alpha)}{(1+\cos\alpha)(1-\cos\alpha)}$

$=\frac{1-\cos\alpha+1+\cos\alpha}{1-\cos^2\alpha}$

$=\frac{2}{\sin^2\alpha}$

$=2\operatorname{cosec}^2\alpha$

$=2(1+\cot^2\alpha)\qquad \because 1+\cot^2\alpha=\operatorname{cosec}^2\alpha$

$=2+2\cot^2\alpha$

Q14. Prove $\cos^4 b-\sin^4 b=1-2\sin^2 b$

Sol: Taking LHS $\cos^4 b-\sin^4 b$

$=(\cos^2 b)^2-(\sin^2 b)^2$

$=(\cos^2 b-\sin^2 b)(\cos^2 b+\sin^2 b)$

$=\cos^2 b-\sin^2 b\qquad \because \cos^2 b+\sin^2 b=1$

$=1-\sin^2 b-\sin^2 b$

$=1-2\sin^2 b$

Q15: $\frac{\sin y+\cos y}{\sin y}-\frac{\cos y-\sin y}{\cos y}=\sec y\operatorname{cosec} y$

Sol: Taking LHS $\frac{\sin y+\cos y}{\sin y}-\frac{\cos y-\sin y}{\cos y}$

$=\frac{\cos y(\sin y+\cos y)-\sin y(\cos y-\sin y)}{\cos y\sin y}$

$=\frac{\cos y\sin y+\cos^2 y-\sin y\cos y+\sin^2 y}{\cos y\sin y}$

$=\frac{\cos^2 y+\sin^2 y}{\cos y\sin y}$

$=\frac{1}{\cos y\sin y}$

$=\sec y\operatorname{cosec} y$ = RHS Hence proved

Q16. Prove $(\sec x-\tan x)^2=\frac{1-\sin x}{1+\sin x}$

Sol: LHS $(\sec x-\tan x)^2$

$=\left(\frac{1}{\cos x}-\frac{\sin x}{\cos x}\right)^2$

$=\frac{(1-\sin x)^2}{(\cos x)^2}$

$=\frac{(1-\sin x)^2}{1-\sin^2 x}$

$$=\frac{(1-\sin x)^2}{(1-\sin x)(1+\sin x)}$$

$$=\frac{(1-\sin x)}{(1+\sin x)}$$ = RHS Hence proved

Q17. Prove that $\sin x\tan x+\cos x=\sec x$

Sol: Taking LHS $\sin x\tan x+\cos x$

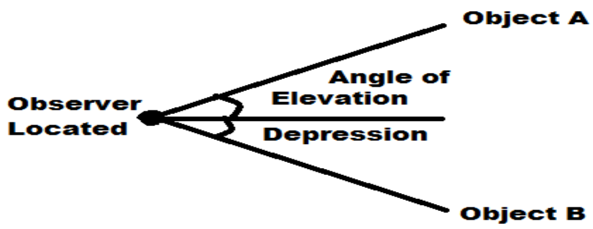
$$=\sin x\frac{\sin x}{\cos x}+\cos x$$

$$=\frac{\sin^2 x+\cos^2 x}{\cos x}$$

$$=\frac{1}{\cos x}$$

$=\sec x$ = RHS Hence proved

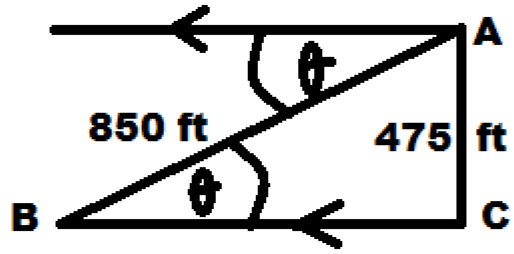
Angle of elevation and Depression



Exp20: An aerial photographs farm house for a company has determine from experience that best photo is taken at a height of approximately 475ft and a distance of 850ft from the farmhouse. What is the angles of depression from the plane to the house?

Sol: when parallel lines are cut by a transversal, alternate interior angles are equal.

According to question



$$\sin\angle B=\frac{475}{850}$$

$$\angle B=\sin^{-1}(0.5588)$$

$$\angle B=33.97^o$$

$$\angle B\approx 34^o$$

Exp21: To measure cloud height at night, a vertical beam of light is directed on a spot on the source, the angle of elevation to the spot is found to be 67.35^o. Find the height of cloud.

Sol: according to Question

Let Base = 135 ft

Height = h ?

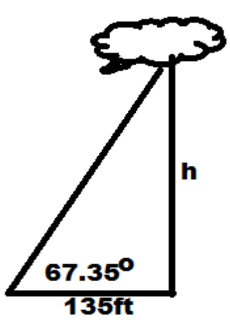
Angle $=67.35^o$

Using TBH

$$\tan 67.35^o=\frac{h}{135}$$

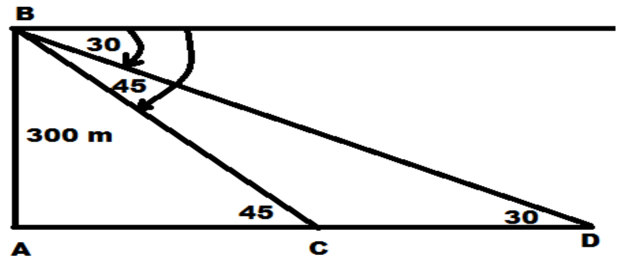
$$\Rightarrow h=135\times\tan 67.35^o$$

$$\Rightarrow h=323.52\ ft$$



Exp22: A light house is 300m above the sea level. Angles of depression of two boats from the top of light house are 30^o and 45^o respectively. If line joining the boats passes through the foot of light house. Find the distance between the boats when they are on the same side of the light house.

Sol: according to Question



In ΔABC

$$\tan 45=\frac{300}{AC}$$

$$1=\frac{300}{AC}$$

$$AC=300$$

In ΔABD

$$\tan 30=\frac{300}{AD}$$

$$\frac{1}{\sqrt{3}}=\frac{300}{AD}$$

$$AD=300\sqrt{3}$$

Now using law for collinear point

$$AD=300\sqrt{3}$$

$$AC+CD=300\sqrt{3}$$

$$300+CD=300\sqrt{3}$$

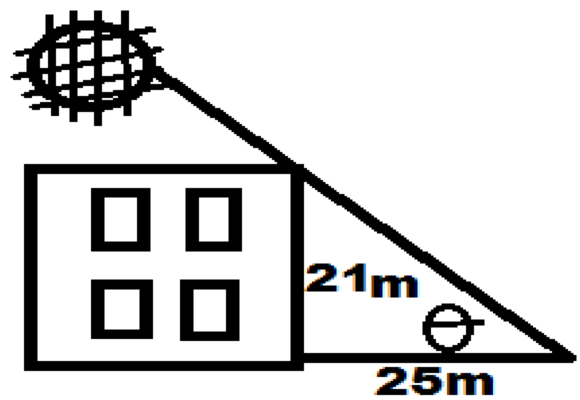
$$CD=300\sqrt{3}-300$$

$$CD=300\left(\sqrt{3}-1\right)m$$

Exercise 7.6

Q1: A building that is 21 meters tall casts a shadow 25 meters long. Find the angle of elevation of the sun to the nearest degree.

Sol: According to Question



Using $\tan\theta=\frac{21}{25}$

$$\theta=\tan^{-1}(0.84)$$

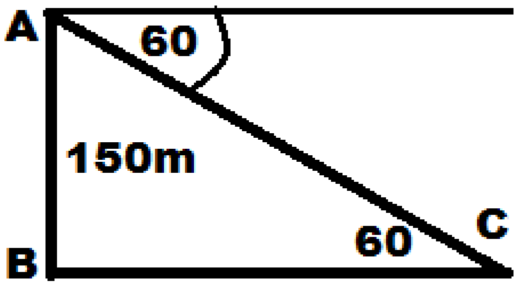
$$\theta=40.03^o$$

Hence Angle of elevation of sun is $\theta\approx 40^o$

Q2: A light house is 150m above the sea level. Angle of depression of a boat form its top is 60^o. Find the distance between the boat and the lighthouse.

Solution: Height of light house = AB

Distance between the boat and light house = BC



In ΔABC

$$\tan 60 = \frac{150}{BC}$$

$$\frac{\sqrt{3}}{1} = \frac{150}{BC}$$

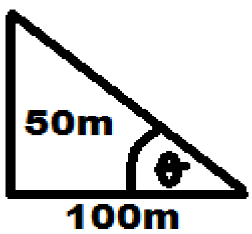
$$BC = \frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

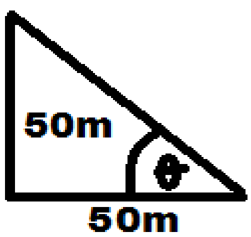
$$BC = \frac{150\sqrt{3}}{3}\text{m}$$

$$BC = 50\sqrt{3}\text{m}$$

Distance b/w the boat and light house $= 50\sqrt{3}\text{m}$

Q3: A tree is 50m high. Find the angle of elevation of its top to a point, on the ground 100m (50m) away from the foot of a tree.
Solution:





Sin θ			Cos θ			Tan θ		
S	P	H	C	B	H	T	P	B
?	Yes	No	?	Yes	No	?	Yes	Yes

$\tan\theta = \frac{\text{Opp}}{\text{adj}}$

$\tan\theta = \frac{50\text{m}}{100\text{m}}$

$\tan\theta = \frac{1}{2}$

$\theta = \tan^{-1}\left(\frac{1}{2}\right)$

$\theta = 26.56505^o$

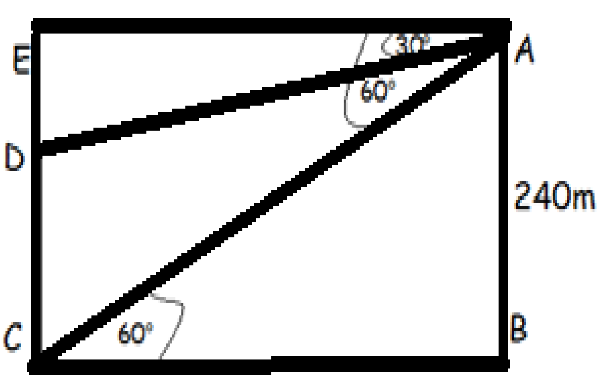
$\tan\theta = \frac{\text{Opp}}{\text{adj}}$

$\tan\theta = \frac{50\text{m}}{50\text{m}}$

$\tan\theta = 1$

$\theta = \tan^{-1}(1) = 45^\circ$

Q4: From top of hill 240m high, measure of angles of depression of top and bottom of minaret are 30^o and 60^o respectively. Find height of minaret.
Solution: ABCE is rectangle then
AE = BC and AB = CE = 240m
Height of minaret = CD



In Triangle ABC

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\frac{\sqrt{3}}{1} = \frac{240\text{m}}{BC}$$

$$\sqrt{3}\ BC = 240\text{m}$$

$$BC = \frac{240\text{m}}{\sqrt{3}} = AE$$

In Triangle AED

$$\tan 30^\circ = \frac{DE}{AE}$$

$$\frac{1}{\sqrt{3}} = \frac{DE}{\frac{240\text{m}}{\sqrt{3}}}$$

$$\frac{240\text{m}}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} = DE$$

$$DE = \frac{240\text{m}}{3} = 80\text{m}$$

$CD + DE = CE$

$CD + 80\text{m} = 240\text{m}$ $\quad \therefore CE = AB$

$CD = 240\text{m} - 80\text{m}$

$CD = 160\text{m}$

Height of minaret = CD = 160m

Q5: A Police Helicopter is flying at 800 feet. A stolen car is sighted at an angle of depression of 72^o. Find the distance of the stolen car, to the nearest foot, from a point directly below the Helicopter.

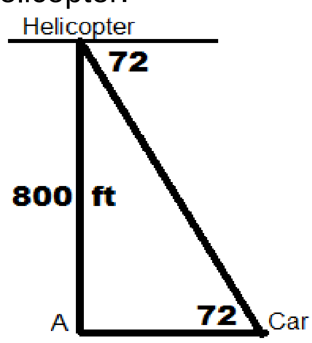
$$\tan 72 = \frac{800}{AC}$$

$$AC = \frac{800}{\tan 72}$$

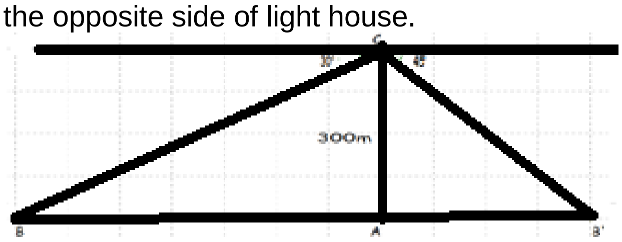
$$AC = \frac{800}{3.0779}$$

$$AC = 259.94\,ft$$

$$AC \approx 260\ ft$$



Q6: A light house is 300m above the sea level. The angle of depression of two boats from the top of light house are 30^o and 45^o respectively. If the line of joining of the boats passes through the foot of light house. Find the distance between two boats when they are on the opposite side of light house.



Solution: In Triangle AB'C

$$\tan 45 = \frac{AC}{AB'}$$

$$1 = \frac{300\text{m}}{AB'}$$

$$AB' = 300\text{m}$$

In Triangle ABC

$$\tan 30 = \frac{AC}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{300\text{m}}{AB}$$

$$AB = 300\sqrt{3}\ \text{m}$$

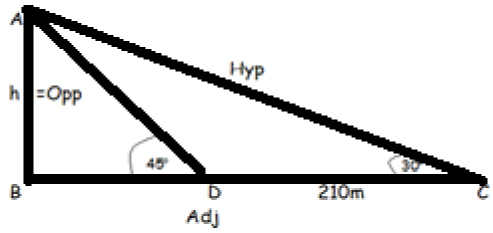
Now distance between boats are

$$BB' = BA + AB'$$

$$BB' = 300\sqrt{3}\ \text{m} + 300\text{m}$$

$$BB' = 300\left(\sqrt{3}+1\right)\text{m}$$

Q7: The angle of elevation of the top of a cliff is 30°. Walking 210 meter from the point towards the cliff, the angle of elevation becomes 45^o.
Find the height of the cliff
Solution: Height of cliff = Opp = AB = h.



In Triangle ABD

$\tan 45^\circ = \frac{AB}{BD}$

$1 = \frac{AB}{BD}$

$BD = AB$

In Triangle ABC

$\tan 30^\circ = \frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{AB}{BD + DC}$

$\frac{1}{\sqrt{3}} = \frac{AB}{AB + DC} \qquad \therefore AB = BD$

$\frac{1}{\sqrt{3}} = \frac{AB}{AB + 210m}$

$AB + 210m = AB\sqrt{3}$

$210m = AB\sqrt{3} - AB$

$210m = AB\left(\sqrt{3} - 1\right)$

$AB = \frac{210m}{\sqrt{3} - 1}$

Thus height of cliff $= \frac{210m}{\sqrt{3} - 1}$

Exercise 7.7

Q1: Fill in the correct circle

i). If an object is above level of observation then angle formed between the horizontal line and observer's line of sight is called

O Angle of depression O obtuse angle

O Angle of elevation O None

ii). $\cot\theta =$ ....

O $\frac{\sin\theta}{\cos\theta}$ O $\frac{1}{\cos\theta}$

O $\frac{\cos\theta}{\sin\theta}$ O $\frac{1}{\sin\theta}$

iii). $1 + \tan^2\theta =$ ....

O $\sin^2\theta$ O $\cos^2\theta$

O $\operatorname{cosec}^2\theta$ O $\sec^2\theta$

iv). If $\tan\theta = 1$ then $\sin\theta =$ .... when θ lies in 3rd quadrant

O $\frac{1}{2}$ O $\frac{-1}{2}$

O $\frac{-1}{\sqrt{2}}$ O $\frac{1}{\sqrt{2}}$

iv). $\sin\left(-350^o\right)$ lies in...............

O 1st quadrant O 2nd quadrant

O 3rd quadrant O 4th quadrant

v). $45^o =$ .... radian

O $\frac{\pi}{3}$ O $\frac{\pi}{4}$

O $\frac{\pi}{4}$ O $\frac{\pi}{2}$

Q2: Convert $45^\circ 35' 30''$ into decimal form.

Sol: Since $1' = \left(\frac{1}{60}\right)^o$ and $1'' = \left(\frac{1}{3600}\right)^o$

$45^\circ 35' 30'' = 45^\circ + \frac{35}{60}^\circ + \frac{30}{3600}^\circ$

$45^\circ 35' 30'' = 45^\circ + 0.58\overline{3}^\circ + 0.008\overline{3}^\circ$

$45^\circ 35' 30'' = 45.591\overline{6}^\circ$

Q3: Convert 216.67° into D°M'S" form.

Sol: Since $1^o = 60'$ and $1' = 60''$

$216.67^\circ = 216^\circ + 0.67^\circ$

$216.67^\circ = 216^\circ \; 0.67 \times 60'$

$216.67^\circ = 216^\circ \; 40.2'$

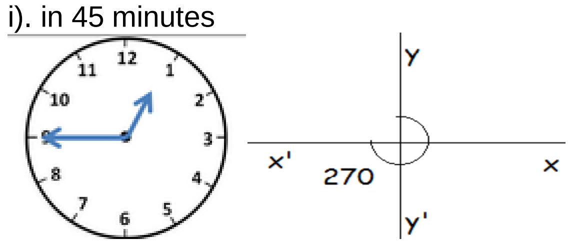
$216.67^\circ = 216^\circ \; 40' + 0.2'$

$216.67^\circ = 216^\circ \; 40' \; 0.2 \times 60''$

$216.67^\circ = 216^\circ \; 40' \; 12''$

Q4: Through how many radians does a minute of a clock turn through

i). in 45 minutes

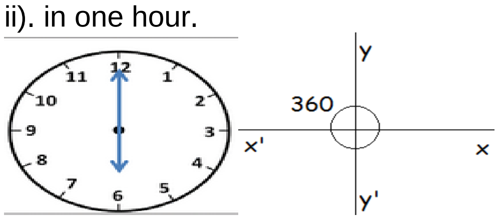


$1^\circ = \frac{\pi}{180}$ Radian

$270 \times 1^\circ = 270 \times \frac{\pi}{180}$ Radian

$270^\circ = \frac{3\pi}{2}$ Radian

ii). in one hour.



$1^\circ = \frac{\pi}{180}$ Radian

$360 \times 1^\circ = 360 \times \frac{\pi}{180}$ Radian

$360^\circ = 2\pi$ Radian

Q5: Find coterminal angle of 190° and -250°.

Solution: For 190°

Coterminal Angle Anit-clock wise	Coterminal Angle Clock wise
190° + 360° = 550°	190° - 360° = - 170°

For - 250°

Coterminal Angle Anit-clock wise	Coterminal Angle Clock wise
-250° + 360° = 110°	-250° - 360° = - 610°

Q6i): Find trigonometric ratio of 390°

Sol: Coterminal angle of 390° is 30° 1st Q

$\sin\left(390^\circ\right) = \frac{1}{2} \qquad \operatorname{cosec}\left(390^\circ\right) = \frac{2}{1} = 2$

$\cos\left(390^\circ\right) = \frac{\sqrt{3}}{2} \qquad \sec\left(390^\circ\right) = \frac{2}{\sqrt{3}}$

$\tan(390^\circ) = \frac{\sin(390^\circ)}{\cos(390^\circ)}$ $\tan(390^\circ) = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$

$Cot(390^\circ) = \frac{\sqrt{3}}{1} = \sqrt{3}$

Q6ii): Find trigonometric ratio of - 240°

Sol: Coterminal angle of -240° is 120° 2nd Q

Reference angle of 120° is 60°

$\sin(-240^o) = \frac{\sqrt{3}}{2}$ $\quad \operatorname{cosec}(-240^o) = \frac{2}{\sqrt{3}}$

$\cos(-240^o) = \frac{-1}{2}$ $\quad \sec(-240^o) = \frac{2}{-1} = -2$

$\tan(-240^o) = \frac{\sqrt{3}}{-1}$ $\quad \cot(-240^o) = \frac{-1}{\sqrt{3}}$

$\tan(-240^o) = -\sqrt{3}$

Q7i) : Prove that $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$

Solution: Take LHS

$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}\sqrt{\frac{1-\sin\theta}{1-\sin\theta}}$

$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sqrt{\frac{(1-\sin\theta)^2}{1^2-\sin^2\theta}}$

$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \frac{1-\sin\theta}{\sqrt{\cos^2\theta}}$

$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \frac{1-\sin\theta}{\cos\theta}$

$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$

$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta = RHS$

Hence proved

Q7i) : Prove that $2\cos\theta\sec\theta - \tan\theta\cot\theta = 1$

Solution: Take LHS

$2\cos\theta\sec\theta - \tan\theta\cot\theta = 2\cos\theta\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\frac{\cos}{\sin}$

$2\cos\theta\sec\theta - \tan\theta\cot\theta = 2\times1 - 1\times1$

$2\cos\theta\sec\theta - \tan\theta\cot\theta = 2-1$

$2\cos\theta\sec\theta - \tan\theta\cot\theta = 1 = RHS$

Hence Proved

Q8: If $\sec\theta = 2$ and θ does not lie in first quadrant. Find remaining trigonometric ratios.

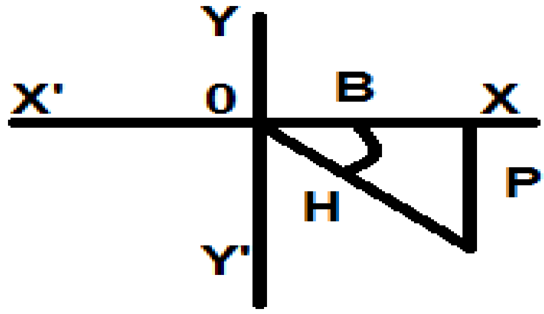
Solution: secant is positive in 1st and 4th Quadrants

So, take 4th Quadrant, sine -ve, cosine +ve

We have $\sec\theta = 2$

$\frac{1}{\cos\theta} = 2$ $\qquad \cos\theta = \frac{adj}{hyp}$

$\Rightarrow \cos\theta = \frac{1}{2}$



$P^2 + B^2 = H^2$

$P^2 + 1^2 = 2^2$

$P^2 = 4-1$

$P^2 = 3$

$P = \pm\sqrt{3}$

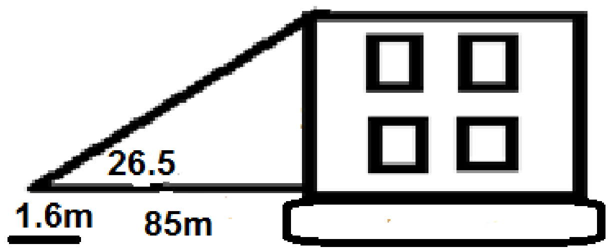
Take $P = -\sqrt{3}$ so

$Sin\theta = \frac{-\sqrt{3}}{2}$ $\quad Cos\theta = \frac{1}{2}$ $\quad Tan\theta = \frac{-\sqrt{3}}{1}$

So there reciprocals

$\operatorname{cosec} = \frac{-2}{\sqrt{3}}$, $\quad \sec\theta = \frac{2}{1}$, $\quad \cot\theta = \frac{-1}{\sqrt{3}}$

Q9. Refer to diagram shown. What is height of building



Using $\tan 26.5 = \frac{height}{85}$

$Height = 85 \times \tan 26.5$

$Height = 42.38$

Total Height = Height of building + Height of observer

Total Height = 1.6m +42.38

Total Height = 43.98m

Total Height $\approx 44m$

Q10. From a point on level ground 125 feet from the base of a tower, the angle of elevation is 57.2o. Approximate the height of tower to nearest foot.

Sol: Form Figure

$\tan 57.2 = \frac{h}{125}$

$h = 125 \times \tan 57.2$

$h = 193.96$

$h \approx 194\,ft$