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Exercise 11.2 (Solutions)
Mathematics 9th (Science)
Punjab Textbook Board

Definitions Alert

Quadrilateral: A figure formed by four non-collinear points in the plane is called a quadrilateral.

Parallelogram: A figure formed by four non-collinear points in the plane is called a parallelogram if its opposite sides are parallel.

Quadrilateral: Angle Sum Property

The sum of all the four interior angles of the quadrilateral is 360 degrees.

Converse of the Same Interior Angles Theorem

If two lines and a transversal form same-side interior angles that are supplementary, then the two lines are parallel.

Sides of Alternate Interior Angles

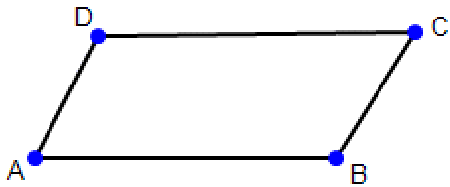
If the pair of alternate interior angles are equal, then the lines are parallel.

SOLUTIONS

Q.1(a) Prove that a quadrilateral is a parallelogram if its opposite angles are congruent.

Solution*: *Given: In a equilateral $ABCD$, $\angle A \cong \angle C$ and $\angle B \cong \angle D$.

To prove: $\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{DC}$.



Proof:

Statement	Reasons
$m\angle A = m\angle C$ ...... (i)	Given
$m\angle B = m\angle D$ ...... (ii)	Given
$m\angle A + m\angle B + m\angle C + m\angle D = 360^\circ$	Angle sum property of quadrilateral
$m\angle C + m\angle D + m\angle C + m\angle D = 360^\circ$	From (i) & (ii)
$2(m\angle C + m\angle D) = 360^\circ$	
$m\angle C + m\angle D = 180^\circ$ .... (iii)	
$\overline{AD} \parallel \overline{BC}$	Converse of the same side interion angles theorem
Similarly	From (i) & (ii) in (iii)
$m\angle A + m\angle B = 180^\circ$	
This gives.	
$\overline{AB} \parallel \overline{DC}$	Converse of the same side interion angles theorem
Hence $ABCD$ is a parallelogram.	

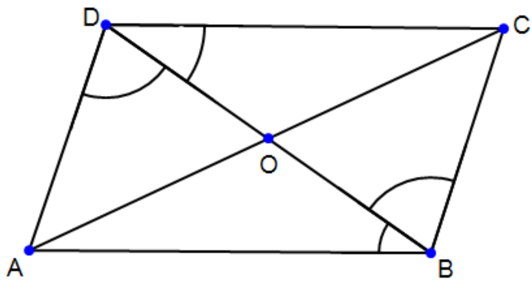
Q.1(b) Prove that a quadrilateral is a parallelogram if its diagonals bisect each other.

Solution: Given: The diagonal AC *and* BD bisect each other at O.

So $\overline{OA} \cong \overline{OC}$, $\overline{OB} \cong \overline{OD}$

To prove:

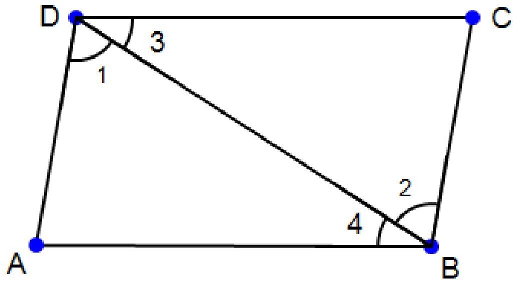
$\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{DC}$.



Proof:

Statement	Reasons
In $\Delta ABO \leftrightarrow \Delta CDO$	
$\overline{OA} \cong \overline{OC}$	Given
$\overline{OB} \cong \overline{OD}$	Given
$\angle AOB \cong \angle COD$	Opposite angles
$\Delta ABO \cong \Delta CDO$	$S.S.A \cong S.S.A$
This gives $\angle ABO \cong \angle CDO$	Corresponding angels of congruent triangles.
Thus $\overline{AB} \parallel \overline{DC}$ …. (i)	Sides of alternate interior angles
In $\Delta ADO \leftrightarrow \Delta BCO$	
$\overline{OA} \cong \overline{OC}$	Given
$\overline{OB} \cong \overline{OD}$	Given
$\angle AOD \cong \angle BOC$	Opposite angles
$\Delta ADO \cong \Delta BCO$	$S.S.A \cong S.S.A$
This gives $\angle ADO \cong \angle CBO$	Corresponding angels of congruent triangles.
Thus $\overline{AD} \parallel \overline{BC}$ …. (ii)	Sides of alternate interior angles
Hence, $ABCD$ is a parallelogram.	From (i) & (ii).

Q.2 Prove that a quadrilateral is a parallelogram if its opposite sides are congruent.



Solution: Given: In a quadrilateral $ABCD$.

$\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$.

To prove: $\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{DC}$.

Construction: Join B *to* D and name the angles $\angle 1, \angle 2, \angle 3, \angle 4,$ as shown in the figure.

Proof:

Statement	Reasons
In $\Delta ABD \leftrightarrow \Delta CDB$	
$\overline{AB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{BC}$	Given
$\overline{BD} \cong \overline{BD}$	Common
$\Delta ABD \cong \Delta CDB$	$S.S.S \cong S.S.S$
Thus $\angle 1 \cong \angle 2$	Corresponding angles of congruent triangles
and $\angle 4 \cong \angle 3$.	Corresponding angles of congruent triangles
$\Rightarrow$ $\overline{AD} \parallel \overline{BC}$	Sides of alternate interior angles.
and $\overline{AB} \parallel \overline{DC}$	Sides of alternate interior angles.
Hence $ABCD$ is a parallelogram.	

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