Chapter# 3: LOGARITHMS

Exercise 3.1:

Q1.Express each of the following numbers in scientific notation.

i. 5700

A. $= 5.7 \times 10^3$

ii. 49,800,000

A. $= 4.98 \times 10^7$

iii. 96,000,000

A. $= 9.6 \times 10^7$

iv. 416.9

A. $= 4.2 \times 10^2$

v. 83,000

A. $= 8.3 \times 10^4$

vi. 0.00643

A. $= 6.43 \times 10^{-3}$

vii. 0.0074

A. $= 7.4 \times 10^{-3}$

viii. 60,000,000

A. $= 6.0 \times 10^7$

ix. 0.00000000395

A. $= 3.95 \times 10^9$

x. $\frac{275,000}{0.0025}$

$$= \frac{2.75 \times 10^5}{2.5 \times 10^{-3}}$$

$$= \frac{2.75}{2.5} \times 10^5 \times 10^3$$

$$= 1.1 \times 10^{5+3}$$

$$= 1.1 \times 10^8$$

Q2. Express the following numbers in ordinary notation.

i. 6×10^{-4}

$= 0.006$

ii. 5.06×10^{10}

$= 50,600,000,000$

iii. 9.018×10^{-6}

$= 0.000009018$

iv. 7.865×10^8

$= 7,86,500,000$

Exercise 3.2:

Q1. Find the common logarithm of the following numbers:

i. 232.92

$\text{Characteristics} = 2$

$\text{Mantissa} = 0.3672$

$\log 232.92 = 2.3672$

ii. 29.326

$\text{Characteristics} = 1$

$\text{Mantissa} = 0.4672$

$\log 29.326 = 1.4672$

iii. 0.00032

$\text{Characteristics} = -4$

$\text{Mantissa} = 0.5051$

$\log 0.00032 = \bar{4}.5051$

iv. 0.3206

$\text{Characteristics} = -1$

$\text{Mantissa} = 0.5059$

$\log 0.3206 = \bar{1}.5059$

Q2. If $\log 31.09 = 1.4926$, find the values of following:

i. log 3.109

$\text{Characteristics} = 0$

$\text{Mantissa} = 0.4926$

$\log 3.109 = 0.4926$

ii. log 310.9

$\text{Characteristics} = 2$

$\text{Mantissa} = 0.4926$

$\log 310.9 = 2.4926$

iii. log 0.003109

$\text{Characteristics} = -3$

$\text{Mantissa} = 0.4926$

$\log 0.003109 = \bar{3}.4926$

iv. log 0.3109

$\text{Characteristics} = -1$

$\text{Mantissa} = 0.4926$

$\log 0.3109 = \bar{1}.4926$

Q3. Find the numbers whose common logarithms are:

i. 3.5621

Since it is log of any number. So,

$\text{Characteristics} = 3$

$\text{Mantissa} = 0.5621$

by looking mantissa in antilog table we get 3.6484

$\Rightarrow$ *Characteristics change the place of decimal.*

So, Antilog $3.5621 = 3648.4$

ii. $\bar{1}.\mathbf{7427}$

Since it is log of any number. So,

Characteristics $= -1$

Mantissa $= 0.7427$

by looking mantissa in antilog table we get 5.5297

$\Rightarrow$ *Characteristics change the place of decimal.*

So, Antilog $5.5297 = 0.55297 \cong 0.5530$

Q4. What replacement for the unknown in each of following will make the statement true?

i. $\log_3 81 = L$

Writing in exponential form:

$3^L = 81$

$3^L = 3^4$

$\Rightarrow L = 4$

ii. $\log_a 6 = 0.5$

Writing in exponential form:

$a^{0.5} = 6$

$a^{1/2} = 6$

Squaring on both sides:

$\Rightarrow a = 36$

iii. $\log_5 n = 2$

Writing in exponential form:

$5^2 = n$

$\Rightarrow n = 25$

iv. $10^p = 40$

Writing in logarithmic form:

$\log_{10} 40 = p$

$\Rightarrow p = 1.6021$

Q5. Evaluate

i. $\log_2 \frac{1}{128}$

$let\ x = \log_2 \frac{1}{128}$

Writing in exponential form:

$2^x = \frac{1}{128}$

$2^x = \frac{1}{2^7}$

$2^x = 2^{-7}$

$\Rightarrow \log_2 \frac{1}{128} = x = -7$

ii. $\log 512\ to\ the\ base\ 2\sqrt{2}$

$let\ x = \log_{2\sqrt{2}} 512$

Writing in exponential form:

$(2\sqrt{2})^x = 512$

$(2^{1+1/2})^x = 2^9$

$2^{\frac{3}{2}x} = 2^9$

$\frac{3}{2}x = 9$

$3x = 2 \times 9$

$x = \frac{2\times 9}{3}$

$\Rightarrow \log_{2\sqrt{2}} 512 = x = 6$

Q6. Evaluate the value of 'x' from the following statements.

i. $\log_2 x = 5$

Writing in exponential form:

$2^5 = x$

$\Rightarrow x = 32$

ii. $\log_{81} 9 = x$

Writing in exponential form:

$81^x = 9$

$(9^2)^x = 9$

$9^{2x} = 9$

$2x = 1$

$\Rightarrow x = 1/2$

iii. $\log_{64} 8 = \frac{x}{2}$

Writing in exponential form:

$64^{x/2} = 8$

$(2^6)^x = 2^3$

$2^{6x} = 2^3$

$6x = 3$

$x = \frac{3}{6}$

$\Rightarrow x = 1/2$

iv. $\log_x 64 = 2$

Writing in exponential form:

$x^2 = 64$

Taking square root on both sides:

$\Rightarrow x = 8$

v. $\log_3 x = 4$

Writing in exponential form:

$3^4 = x$

$\Rightarrow x = 81$

Laws of Logarithm:

In this section we shall prove the laws of logarithm:

i. $\log_a(mn) = \log_a m + \log_a n$
ii. $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$
iii. $\log_a m^n = n\log_a m$
iv. $\log_a n = \log_b n \times \log_a b$

(i) $\log_a(mn) = \log_a m + \log_a n$

Proof:

$let\ \log_a m = x\ and\ \log_a n = y$

Writing in exponential form:

$a^x = m \ldots\ldots(1); \quad a^y = n \ldots\ldots(2)$

Multiplying (1) and (2)

$a^x.a^y = mn$

$a^{x+y} = mn$

Writing in logarithmic form:

$\log_a mn = x + y$

Putting the values of x and y:

$\log_a(mn) = \log_a m + \log_a n$

Note:

(i) $\log_a(mn) \neq \log_a m \times \log_a n$

(ii) $\log_a m + \log_a n \neq \log_a(m+n)$

(iii) $\log_a(mnp..) = \log_a m + \log_a n + \log_a p + ..$

The rule given above is useful in finding the product of two or more numbers using logarithms.

(ii) $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

$let\ \log_a m = x\ and\ \log_a n = y$

Writing in exponential form:

$a^x = m \ldots\ldots(1); \quad a^y = n \ldots\ldots(2)$

Dividing (1) by (2)

$\frac{a^x}{a^y} = \frac{m}{n}$

$a^{x-y} = \frac{m}{n}$

Writing in logarithmic form:

$\log_a\left(\frac{m}{n}\right) = x - y$

Putting the values of x and y:

$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

Note:

(i) $\log_a\left(\frac{m}{n}\right) \neq \frac{\log_a m}{\log_a n}$

(ii) $\log_a m - \log_a n \neq \log_a(m-n)$

(iii) $\log_a\left(\frac{1}{n}\right) = \log_a 1 - \log_a n = -\log_a n \ldots..$

$(\because \log_a 1 = 0)$

(iii) $\log_a m^n = n\log_a m$

$let\ \log_a m = x$

Writing in exponential form:

$a^x = m$

Taking n^th power on both sides:

$(a)^{nx} = m^n$

Writing in logarithmic form:

$\log_a m^n = nx$

Putting value of 'x'

$\log_a m^n = n\log_a m$

(iv) Change of Base Formula:

$\log_a n = \log_b n \times \log_a b$

$let\ \log_b n = x$

Writing in exponential form:

$b^x = n$

Taking log with base 'a' on both sides:

$\log_a b^x = \log_a n$

By 3rd law:

$x.\log_a b = \log_a n$

Putting value of 'x'

$\log_a n = \log_b n \times \log_a b$

Exercise 3.3:

Q1. Write the following into sum or difference.

(i) $\log(A \times B)$

By 1st law of logarithm:

$\log(A \times B) = \log A + \log B =$

(ii) $\log\frac{15.2}{30.5}$

By 2nd law of logarithm:

$\log\frac{15.2}{30.5} = \log 15.2 - \log 30.5$

(iii) $\log\frac{21\times5}{8}$

By 2nd law of logarithm:

$\log\frac{21\times5}{8} = \log(21\times5) - \log 8$

By 1st law of logarithm:

$\log\frac{21\times5}{8} = \log 21 + \log 5 - \log 8$

(iv) $\log\sqrt[3]{\frac{7}{15}}$

$= \log\left(\frac{7}{15}\right)^{1/3}$

By 3rd law of logarithm:

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$= \frac{1}{3}\log\frac{\mathbf{7}}{\mathbf{15}}$

By 2nd law of logarithm:

$= \frac{1}{3}\log 7 - \frac{1}{3}\log 15$

(v) $\log\frac{22^{\frac{1}{3}}}{5^3}$

By 2nd law of logarithm:

$= \log 22^{\frac{1}{3}} - \log 5^3$

By 3rd law of logarithm:

$= \frac{1}{3}\log 22 - 3\log 5$

(vi) $\log\frac{25\times 47}{29}$

By 2nd law of logarithm:

$= \log(25\times 47) - \log 29$

By 1st law of logarithm:

$= \log 25 + \log 47 - \log 29$

Q2. Express $\log x - 2\log x + 3\log(x+1) - \log(x^2-1)$ as a single logarithm.

By re arranging

$= \log x + 3\log(x+1) - \log(x^2-1) - 2\log x$

$= \log x + 3\log(x+1) - \{\log(x^2-1) + 2\log x\}$

By 3rd law:

$= \log x + \log(x+1)^3 - \{\log(x^2-1) + \log x^2\}$

By 1st law:

$= \log x(x+1)^3 - \{\log x^2(x^2-1)\}$

By 2nd law:

$= \log\frac{x(x+1)^3}{x^2(x^2-1)}$

Simplifying

$= \log\frac{(x+1)^3}{x(x-1)(x+1)}$

$= \log\frac{(x+1)^2}{x(x-1)}$

Q3. Write the following in the form of a single logarithm.

(i) $\log 21 + \log 5$

$= \log(21\times 5)$

$= \log 105$

(ii) $\log 25 + 2\log 3$

$= \log 25 + \log 3^2$

$= \log 25 + \log 9$

$= \log(25\times 9)$

$= \log 225$

(iii) $2\log x - 3\log y$

$= \log x^2 - \log y^3$

$= \log\frac{x^2}{y^3}$

(iv) $\log 5 + \log 6 - \log 2$

$= \log\frac{5\times 6}{2}$

$= \log 15$

Q4. Calculate the following:

(i) $\log_3 2 \times \log_2 81$

by 4th law:

$= \frac{\log 2}{\log 3}\times\frac{\log 81}{\log 2}$

simplifing

$= \frac{\log 3^4}{\log 3}$

by 1st law:

$= \frac{4\log 3}{\log 3}$

$= 4$

(ii) $\log_5 3 \times \log_3 25$

by 4th law:

$= \frac{\log 3}{\log 5}\times\frac{\log 25}{\log 3}$

simplifing

$= \frac{\log 5^2}{\log 5}$

by 1st law:

$= \frac{2\log 5}{\log 5}$

$= 2$

Q5. If $\log 2 = 0.3010$, $\log 3 = 0.4771$, $\log 5 = 0.6990$, **then find the values of the following.**

(i) $\log 32$

$= \log 2^5$

$= 5\log 2$

$= 5(0.3010)$

$= 1.5050$

(ii) $\log 24$

$= \log(2^3\times 5)$

$= \log 2^3 + \log 5$

$= 3\log 2 + \log 5$

$= 3(0.3010) + (0.6990)$

$= 1.6020$

(iii) $\log\sqrt[3]{3\frac{1}{3}}$

$= \log(\frac{10}{3})^{\frac{1}{3}}$

$= \frac{1}{3}\log(\frac{10}{3})$

$= \frac{1}{3}\log(\frac{2\times 5}{3})$

$= \frac{1}{3}(\log 2 + \log 5 - \log 3)$

$= \frac{1}{3}(0.3010 + 0.6990 - 0.4771)$

$= \frac{1}{3}(0.5229)$

$= 0.1743$

(iv) $\log\frac{8}{3}$

$= \log 8 - \log 3$

$= \log 2^3 - \log 3$

$= 3\log 2 - \log 3$

$= 3(0.3010) - 0.4771$

$= 0.1249$

(v) $\log 30$

$= \log(2 \times 3 \times 5)$

$= \log 2 + \log 3 + \log 5$

$= 0.3010 + 0.4771 + 0.6990$

$= 1.4771$

Exercise 3.4

Q1. Use log tables to find the values of

i. $\mathbf{0.8176 \times 13.64}$

Let $x = 0.8176 \times 13.64$

Taking log on both sides:

$$\log x = \log(0.8176 \times 13.64)$$

$$\log x = \log(0.8176) + \log(13.64)$$

$$\log x = \bar{1}.9125 + 1.1348$$

$$\log x = -1 + 0.9125 + 1 + 0.1348$$

$$\log x = 1.0473$$

To find 'x' take antilog on both sides:

$$Antilog(\log x) = Antilog(1.0473)$$

$$\mathbf{0.8176 \times 13.64} = x = 11.15$$

ii. $(\mathbf{789.5})^{\frac{1}{8}}$

$$\text{Let } x = (789.5)^{\frac{1}{8}}$$

Taking log on both sides:

$$\log x = \log(789.5)^{\frac{1}{8}}$$

$$\log x = \frac{1}{8}\log(789.5)$$

$$\log x = \frac{1}{8}(2.8974)$$

$$\log x = 0.3622$$

To find 'x' take antilog on both sides:

$$Antilog(\log x) = Antilog(0.3622)$$

$$(789.5)^{\frac{1}{8}} = x = 2.3025$$

iii. $\mathbf{\frac{0.678 \times 9.01}{0.0234}}$

$$\text{Let; } x = \frac{0.678 \times 9.01}{0.0234}$$

Taking log on both sides:

$$\log x = \log(\frac{0.678 \times 9.01}{0.0234})$$

$$\log x = \log(0.678) + \log(9.01) - \log 0.0234$$

$$\log x = \bar{1}.8312 + 0.9547 - \bar{2}.3692$$

$$\log x = -1 + 0.8312 + 0.9547 - (-2 + 0.3692)$$

$$\log x = -1 + 0.8312 + 0.9547 + 2 - 0.3692$$

$$\log x = 1 + 1.4167$$

$$\log x = 2.4167$$

To find 'x' take antilog on both sides:

$$Antilog(\log x) = Antilog(2.4167)$$

$$\frac{0.678 \times 9.01}{0.0234} = x = 261.03$$

iv. $\sqrt[5]{\mathbf{2.709}} \times \sqrt[7]{\mathbf{1.239}}$

$$\text{Let } x = \sqrt[5]{2.709} \times \sqrt[7]{1.239}$$

Taking log on both sides:

$$\log x = \log(\sqrt[5]{2.709} \times \sqrt[7]{1.239})$$

$$\log x = \log \sqrt[5]{2.709} + \log \sqrt[7]{1.239}$$

$$\log x = \frac{1}{5}\log 2.709 + \frac{1}{7}\log 1.239$$

$$\log x = \frac{1}{5}(0.4328) + \frac{1}{7}(0.0931)$$

$$\log x = 0.0866 + 0.0133$$

$$\log x = 0.0999$$

To find 'x' take antilog on both sides:

$$Antilog(\log x) = Antilog(0.0999)$$

$$\sqrt[5]{2.709} \times \sqrt[7]{1.239} = (789.5)^{\frac{1}{8}} = x = 1.2586$$

v. $\frac{(1.23)(0.6975)}{(0.0075)(1278)}$

$$\text{Let; } x = \frac{(1.23)(0.6975)}{(0.0075)(1278)}$$

Taking log on both sides:

$$\log x = \log\frac{(1.23)(0.6975)}{(0.0075)(1278)}$$

$$\log x = \log(1.23) + \log(0.6975) - \log(0.0075) - \log(1278)$$

$$\log x = 0.0899 + \bar{1}.8435 - \bar{3}.8751 - 3.1065$$

$$\log x = 0.0899 - 1 + 0.8435 - (-3) - 0.8751 - 3 - 0.1065$$

$$\log x = 0.0899 - 1 + 0.8435 + 3 - 0.8751 - 3 - 0.1065$$

$$\log x = -1 - 0.0482$$

$$\log x = -1.0482$$

To find 'x' take antilog on both sides:

$$Antilog(\log x) = Antilog(-1.0482)$$

$$\frac{(1.23)(0.6975)}{(0.0075)(1278)} = x = 0.08949$$

vi. $\sqrt[3]{\frac{0.7214 \times 20.37}{60.8}}$

$$\text{Let } x = \sqrt[3]{\frac{0.7214 \times 20.37}{60.8}}$$

Taking log on both sides:

$$\log x = \log\sqrt[3]{\frac{0.7214 \times 20.37}{60.8}}$$

$$\log x = \frac{1}{3}\log\left(\frac{0.7214 \times 20.37}{60.8}\right)$$

$$\log x = \frac{1}{3}(\log 0.7214 + \log 20.37 - \log 60.8)$$

$$\log x = \frac{1}{3}(\bar{1}.8582 + 1.3090 - 1.7839)$$

$$\log x = \frac{1}{3}(\bar{1} + 0.8582 + 1 + 0.3090 - 1 - 0.7839)$$

$$\log x = \frac{1}{3}(-1 + 0.3833)$$

$$\log x = \frac{1}{3}(-0.6167)$$

$$\log x = -0.2056$$

To find 'x' take antilog on both sides:

$$Antilog(\log x) = Antilog(-0.2056)$$

$$\sqrt[3]{\frac{0.7214 \times 20.37}{60.8}} = x = 0.6229$$

vii. $\frac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$

$$\text{Let; } x = \frac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}$$

Taking log on both sides:

$$\log x = \log\left(\frac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}\right)$$

$$\log x = \log 83 + \log\sqrt[3]{92} - \log 127 - \log\sqrt[5]{246}$$

$$\log x = \log 83 + \frac{1}{3}\log 92 - \log 127 - \frac{1}{5}\log 246$$

$$\log x = 1.9191 + \frac{1}{3}(1.9638) - 2.1038 - \frac{1}{5}(2.3909)$$

$$\log x = 1.9191 + 0.6546 - 2.1038 - 0.4782$$

$$\log x = -0.0083$$

To find 'x' take antilog on both sides:

$$Antilog(\log x) = Antilog(-0.0083)$$

$$\frac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}} = x = 0.9811$$

viii. $\frac{(438)^3 \times \sqrt{0.056}}{388^4}$

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$$\text{Let; } x = \frac{(438)^3 \times \sqrt{0.056}}{(388)^4}$$

Taking log on both sides:

$$\log x = \log\left(\frac{(438)^3 \times \sqrt{0.056}}{(388)^4}\right)$$

$$\log x = \log(438)^3 + \log\sqrt{0.056} - \log(388)^4$$

$$\log x = 3\log 438 + \frac{1}{2}\log 0.056 - 4\log 388$$

$$\log x = 3(2.6415) + \frac{1}{2}(\bar{2}.7482) - 4(2.5888)$$

$$\log x = 7.9245 + \frac{1}{2}(-1.2518) - 10.3552$$

$$\log x = 7.9245 - 0.6259 - 10.3552$$

$$\log x = -3.0566$$

To find 'x' take antilog on both sides:

$$Antilog(\log x) = Antilog(-3.0566)$$

$$\frac{(438)^3 \times \sqrt{0.056}}{(388)^4} = x = 0.0008778$$

Q2. A gas is expanding according to the law $PV^n = C$. Find C when P=80, V=3.1 and $n = \frac{5}{4}$.

$$PV^n = C$$

Taking log on both sides:

$$\log PV^n = \log C$$

$$\log P + \log V^n = \log C$$

$$\Rightarrow \log C = \log P + \log V^n$$

$$\log C = \log P + \text{n}\log V$$

$$\log C = \log 80 + \frac{5}{4}\log 3.1$$

$$\log C = 1.9031 + 1.25(0.4914)$$

$$\log C = 1.9031 + 1.25(0.4914)$$

$$\log C = 2.5173$$

Taking antilog on both sides:

$$Antilog(\log C) = Antilog(2.5173)$$

$$C = 329.07$$

Q3.The formula $p = 90(5)^{-\frac{q}{10}}$ applies to the demand of a product, where 'q' is the number of units and p is the price of one unit. How many units will be demanded if the price is Rs. 18.00?

P=18 ; $p = 90(5)^{-\frac{q}{10}}$

Taking log on both sides:

$$\log p = \log 90(5)^{-\frac{q}{10}}$$

$$\log p = \log 90 + \log(5)^{-\frac{q}{10}}$$

$$\log 18 = \log 90 + \left(-\frac{q}{10}\log 5\right)$$

$$1.2553 = 1.9542 - \frac{q}{10}(0.6990)$$

$$\frac{q}{10}(0.6990) = 1.9542 - 1.2553$$

$$\frac{q}{10}(0.6990) = 0.6989$$

$$q = \frac{10 \times 0.6989}{0.6990}$$

$$q = 10\ units$$

Q4 If $A = \pi r^2$ $find\ A$, $when\ \pi = \frac{22}{7}$ $and\ r = 15$.

$$A = \pi r^2$$

Taking log on both sides:

$$\log A = \log \pi r^2$$

$$\log A = \log \pi + \log r^2$$

$$\log A = \log \pi + 2\log r$$

$$\log A = \log\frac{22}{7} + 2\log 15$$

$$\log A = \log 22 - \log 7 + 2\log 15$$

$$\log A = 1.3424 - 0.8450 + 2(1.1761)$$

$$\log A = 2.85$$

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Taking antilog on both sides:

$$Antilog(\log A) = Antilog(2.85) = 707.9$$

$$A = 707.9$$

Q5. If $v = \frac{1}{3}\pi r^2 h$**, find** v **when** $\pi = \frac{22}{7}, r = 2.5$ **and** $h = 4.2$**.**

$$V = \frac{1}{3}\pi r^2 h$$

Taking log on both sides:

$$\log V = \log \frac{\pi r^2 h}{3}$$

$$\log V = \log \pi + \log r^2 + \log h - \log 3$$

$$\log V = \log \frac{22}{7} + 2\log 2.5 + \log 4.2 - \log 3$$

$$\log V = \log 22 - \log 7 + 2\log 2.5 + \log 4.2 - \log 3$$

$$\log V = 1.3424 - 0.8450 + 2(0.3979) + 0.6232 - 0.4771$$

$$\log V = 2.7614 - 1.3221$$

$$\log V = 1.4393$$

Taking antilog on both sides:

$$Antilog(\log V) = Antilog(1.4393)$$

$$V = 27.50$$

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