UNIT 15

PYTHAGORAS THEOREM
Exercise 15

Q.NO.1. Verify that the Δs having the following measure of sides are right angled.

(i). $a = 5cm, b = 12cm, c = 13cm$

Answer:

$$(Hyp)^2 = (Prep)^2 + (Base)^2$$
$$(13)^2 = (12)^2 + (5)^2$$
$$169 = 144 + 25$$
$$169 = 169$$
$$\because \ the\ triangle\ is\ right\ angled.$$

(ii). $a = 1.5cm, b = 2cm,\ c = 2.5\ cm$

Answer.

$$(Hyp)^2 = (Prep)^2 + (Base)^2$$
$$(2.5)^2 = (1.5)^2 + (2.5)^2$$
$$625 = 2.25 + 4$$
$$6.25 = 6.25$$
$$\because \ the\ triangle\ is\ right\ angled.$$

(iii). $a = 9cm, b = 12cm, c = 15cm$

Answer:

$$(Hyp)^2 = (Prep)^2 + (Base)^2$$
$$(15)^2 = (12)^2 + (9)^2$$
$$225 = 144 + 81$$
$$225 = 225$$
$$\because \ the\ triangle\ is\ right\ angled.$$

(iv). $a = 16cm,\ \ b = 30cm,\ \ c = 34cm$

Answer:

$$(Hyp)^2 = (Prep)^2 + (Base)^2$$
$$(34)^2 = (30)^2 + (16)^2$$
$$1156 = 900 + 256$$
$$1156 = 1156$$
$$\because \ the\ triangle\ is\ right\ angled.$$

Q.NO.2. Verify that $a^2 + b^2, a^2 - b^2$ and 2ab are the measures of the sides of a right angled triangle where a and b are any two real numbers $(a > b)$.

Answer:

In the angled triangle

$$H^2 = P^2 + B^2$$
$$(a^2 + b^2)^2 = a^4 + b^4 + 2a^2b^2 \rightarrow (i)$$
$$(a^2 - b^2)^2 = a^4 + b^4 - 2a^2b^2 \rightarrow (ii)$$
$$(2ab)^2 = 4a^2b^2 \rightarrow (iii)$$

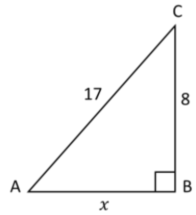
Adding $(ii) and\ (iii) we\ get$

$$(a^2 - b^2)^2 + (2ab)^2$$
$$= a^4 + b^4 - 2a^2b^2 + 4a^2b^2$$
$$= a^4 + b^4 + 2a^2b^2 \rightarrow (iv)$$
$$comparing\ (i) and\ (iv), we\ get$$
$$(a^2 - b^2)^2 + (2ab)^2 = (a^2 + b^2)^2$$

$hence\ a^2 + b^2, a^2 - b^2$ $and\ 2b$ are measures of the sides of a right angled triangle where $a^2 + b^2$ $is\ hypertenous.$

Q.NO.3. The three sides of a triangle are of measure $8, x$ and $17\ respectively$. For what value of x will it becomes base of a right angle triangle?

Answer:



Consider a right angled triangle

$$with\ \overline{AB} = x$$
$$\overline{BC} = 8$$

and $\overline{AC} = 17$

If x is the base of right angled ΔABC then we know by Pythagoras theorem that

$$(Hyp)^2 = (Prep)^2 + (Base)^2$$
$$(17)^2 = (8)^2 + (x)^2$$
$$289 = 64 + x^2$$
$$x^2 = 289 - 64$$
$$x^2 = 225$$
$$x = \sqrt{225}$$
$$As\ \ x\ \ is\ measure\ \ of\ sides$$

So $x = 15$ $units.$

Q.NO.4. In an isosceles Δ the base $\overline{Bc} = 28cm$ and $\overline{AB} = \overline{AC} = 50cm.$ $if\ \overline{AD} \perp \overline{BC},\ then\ find$

(i). Length of $\overline{AD}$

(ii). Area of ΔABC

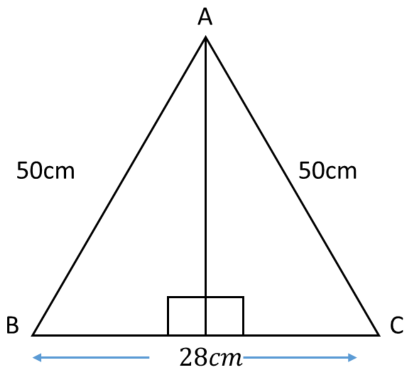
Given:

$$m\overline{AC} = m\overline{AB} = 50cm$$
$$m\overline{BC} = 28cm$$
$$\overline{AD} \perp \overline{BC}$$

To prove:

$$m\overline{AD} = ?$$

Area of ΔABC =?

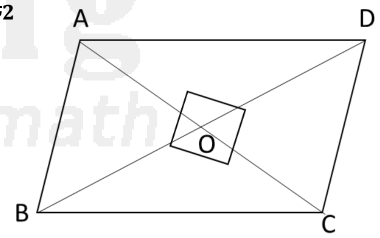


Proof

Statements	**Reasons**
In right angled triangle $m\overline{CD} = 14cm$ $m\overline{AC} = 50cm$ $(m\overline{AD})^2 = (m\overline{AC})^2 - (m\overline{CD})^2$ $= (mAD)^2 = (50)^2 - (14)^2$ $= 2500 - 196$ $= 2304$	$\overline{CD} = \frac{1}{2} m\overline{BC}$ Given $(mAC)^2 = (mAD)^2 - (mCD)^2$ by Pythagoras theorem
$\sqrt{(m\overline{AD})^2} = \sqrt{2304}$ $m\overline{AD} = 18cm$	Taking square root of both sides
(ii). $Area\ of\ \Delta ABC = \frac{Base \times Altitude}{2}$ $= \frac{28 \times 48}{2}$ $= 14 \times 28$ $= 672 sq.cm$	

Q.NO.5. **In a quadrilateral ABCD, the diagonals** $\overline{AC}$ ***and*** $\overline{BD}$ are perpendicular to each other.
Prove that

$$m\overline{AB}^2 + m\overline{CD}^2 = m\overline{AD}^2 + m\overline{BC}^2$$



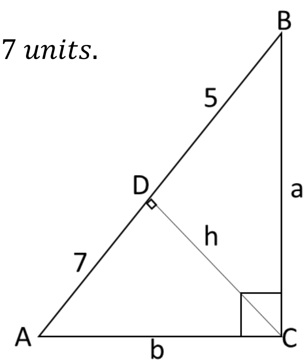
Given:
Quadrilateral ABCD diagonal $\overline{AC}$ ***and*** $\overline{BD}$ ***are*** **perpendicular to each other.**
To prove:

$$m\overline{AB}^2 + m\overline{CD}^2 = m\overline{AD}^2 + m\overline{BC}^2$$

Proof:

Statements	Reasons
In right triangle AOB $m(\overline{AB})^2 = m(\overline{AO})^2 + m(\overline{OB})^2 \rightarrow (i)$	By Pythagoras theorem
In right triangle COD $m(\overline{CD})^2 = m(\overline{OC})^2 + m(\overline{OD})^2 \rightarrow (ii)$	By Pythagoras theorem
In right triangle AOD $m(\overline{AD})^2 = m(\overline{AO})^2 + m(\overline{OD})^2 \rightarrow (iii)$	By Pythagoras theorem
In right triangle BOC $m(\overline{BC})^2 = m(\overline{OB})^2 + m(\overline{OC})^2 \rightarrow (iv)$	By Pythagoras theorem
$m(\overline{AB})^2 + m(\overline{CD})^2 = m(\overline{AO})^2 + m(\overline{OB})^2 + m(\overline{OC})^2 + m(\overline{OD})^2 \rightarrow (v)$	$By\ adding\ (i)\ and\ (ii)$
$m(\overline{AD})^2 + m(\overline{BC})^2 = m(\overline{AO})^2 + m(\overline{OD})^2 + m(\overline{OB})^2 + m(\overline{OC})^2 \rightarrow (vi)$	$By\ adding\ (iii)\ and\ (iv)$
$m(\overline{AB})^2 + m(\overline{CD})^2 = m(\overline{AD})^2 + m(\overline{BC})^2$	$By\ adding\ (v)\ and\ (vi)$

Q.NO.6. In the ΔABC as shown in the figure. $m\angle ACB = 90^0$
$and\ \overline{CD} \perp \overline{AB}$. find the length $a, h, and\ b$ if $m\overline{BD} = 5$ units and $m\overline{AD} = 7\ units.$



Given:
A $\Delta ABC\ as\ shown.$
$m\angle\ ACB = 90^0$
$and\ \ \overline{CD} \perp \overline{AB}$
To prove:
$a, h, and\ b\ in\ right\ angled\ \Delta BDC$
$a^2 = 25 + h^2 \rightarrow (i)$
$b^2 = 49 + h^2 \rightarrow (ii)$

In right angled $\Delta\ ABC$
$a^2 + b^2 = 144 \rightarrow (iii)$

Adding $(i) and\ (ii)$
$a^2 + b^2 = 74 + 2h^2 \rightarrow (iv)$

From (iii) and (iv)

$$74 + 2h^2 = 144$$

$$2h^2 = 144 - 74$$

$$2h^2 = 70$$

$$h^2 = 35$$

$$h = \sqrt{35}$$

Now we will find a and b

Put $h^2 = 35 \ (in\ eq\ (i))$

$$a^2 == 25 + 35$$

$$a^2 = 60$$

$$a = \sqrt{60}$$

$$a = \sqrt{4 \times 15}$$

$$a = 2\sqrt{15}$$

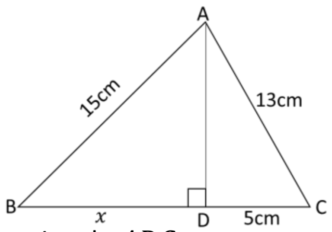
Now put

$$h^2 = 35(in\ eq(ii))$$
$$b^2 = 49 + 35$$
$$b^2 = 48$$
$$b^2 = \sqrt{84}$$
$$b^2 = \sqrt{4 \times 21}$$
$$b = 2\sqrt{21}$$

So $a = 2\sqrt{15}$

$$h = \sqrt{35}$$
$$b = 2\sqrt{21}$$

(ii). Find the value of x in the shown in the figure.



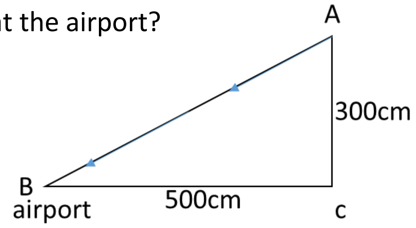
In right angle triangle ADC

$$m(\overline{AC})^2 = m(\overline{AD})^2 + m(\overline{DC})^2$$
$$(13)62 = (AD)^2 + (5)^2$$
$$169 = (AD)^2 + 25$$
$$(AD)^2 = 169 - 25$$
$$(AD)^2 = 144$$
$$AD = \sqrt{144}$$
$$AD = 12cm$$

In right angled triangle ABD

$$(AB)^2 = (AD)^2 + (BD)^2$$
$$(15)^2 = (12)^2 + x^2$$
$$225 = 144 + x^2$$
$$x^2 = 225 - 144$$
$$x^2 = 81$$
$$x = 9cm$$

Q.NO.7. A plane is at height of 300m and is 500m away from the airport as shown in the figure. How much distance will it travel to land at the airport?



Hence A be the position of plane and B be the position of airport.

$$m\overline{AC} = 500m$$
$$m\overline{BC} = 300m$$
$$m\overline{AB} = ?$$

$applying\ pythagoras\ theorem\ on\ right\ triangle\ ABC$

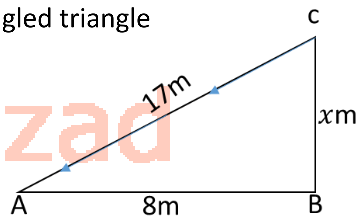
$$|\overline{AB}|^2 = |\overline{AC}|^2 + |\overline{BC}|^2$$
$$= (500)^2 + (300)^2$$
$$= 250000 + 90000$$
$$= 34000$$
$$|AB|^2 = 34 \times 10000$$
$$so\ \ |AB| = \sqrt{34 \times 100 \times 100}$$
$$= 100\sqrt{34}m$$

$so\ \ required\ distance\ is\ 100\sqrt{34}m$

Q.NO.8. A ladder $17m$ long rests against a vertical wall. The foot of the ladder is 8m away from the base of the wall. How high up the wall will the ladder reach?

Answer: Let the height of ladder $= x\ m$

In right angled triangle



$$(Hyp)^2 = (Prep)^2 + (Base)^2$$
$$(17)^2 = (x)^2 + (8)^2$$
$$289 = x^2 + 64$$
$$x^2 = 289 - 64$$
$$x^2 = 225$$
$$x = \sqrt{225} = 15m$$

Q.NO.9. A standard travels to his school by the route as shown in the figure. Find $m\overline{AD}$ the direct distance from his house to school.

According to figure.

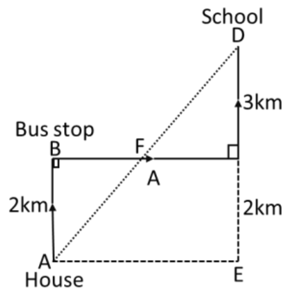
$$m\overline{AB} = 2km$$
$$m\overline{BC} = 6km$$
$$m\overline{CD} = 3km$$

Here $m\overline{AB}\ \ and\ m\overline{CD}\ \ are\ \perp$

$$perpendicular = \overline{AB} + \overline{CD}$$
$$= 2 + 3$$
$$= 5km$$

According to Pythagoras theorem

$$(H)^2 = P^2 + B^2$$



$$(m\overline{AD})^2 = (5)^2 + (6)^2$$
$$= 25 + 36$$
$$(m\overline{AD})^2 = 61$$
$$m\overline{AD} = \sqrt{61}$$
$$m\overline{AD} = \sqrt{61} \text{ km}$$

Pythagoras Theorem

In a right angled, triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

Formula:

$c^2 = a^2 + b^2$

Converse of Pythagoras Theorem

If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right angled triangle.

Results:

Let c be the longest of the sides *a, b* and *c* of a triangle.
* If $a^2 + b^2 = c^2$, then the triangle is right.
* If $a^2 + b^2 > c^2$, then the triangle is acute.
* If $a^2 + b^2 < c^2$, then the triangle is obtuse.

EXERCISE 15

1. Verify that the Δs having the following measures of sides are right
- angled.

(i) $a = 5\ cm, b = 12\ cm, c = 13\ cm$
Sol:
As given $a = 5\ cm, b = 12\ cm, c = 13\ cm$
Using Pythagoras Theorem
$$c^2 = a^2 + b^2$$
$$(13)^2 = (5)^2 + (12)^2$$
$$169 = 25 + 144$$
$$169 = 169$$
Hence, the given sides are the sides of right angle triangle.

(ii) $a = 1.5\ cm, b = 2\ cm, c = 2.5\ cm$
Sol:
As given $a = 1.5\ cm, b = 2\ cm, c = 2.5\ cm$
Using Pythagoras Theorem
$c^2 = a^2 + b^2$
$(2.5)^2 = (1.5)^2 + (2)^2$
$6.25 = 2.5 + 4$
$6.25 = 6.25$
Hence, the given sides are the sides of right angle triangle.

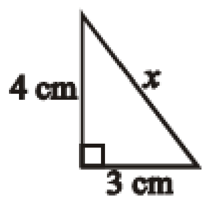
(iii) $a = 9\ cm, b = 12\ cm, c = 15\ cm$
Sol:
As given $a = 9\ cm, b = 12\ cm, c = 15\ cm$
Using Pythagoras Theorem
$c^2 = a^2 + b^2$
$(15)^2 = (9)^2 + (12)^2$
$225 = 81 + 144$
$225 = 225$
Hence, the given sides are the sides of right angle triangle.

(iv) $a = 16\ cm, b = 30\ cm, c = 34\ cm$
Sol:
As given $a = 16\ cm, b = 30\ cm, c = 34\ cm$
Using Pythagoras Theorem
$$c^2 = a^2 + b^2$$
$$(34)^2 = (16)^2 + (30)^2$$
$$1156 = 256 + 900$$
$$1156 = 1156$$
Hence, the given sides are the sides of right angle triangle.

REVIEW EXERCISE 15

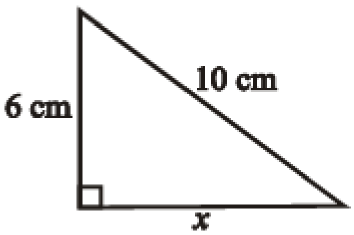
1. Which of the following are true and which are false?
(i) In a right angled triangle greater angle is of 90^o. ... T ...
(ii) In a right angled triangle right angle is of 60^o. F......
(iii) In a right triangle hypotenuse is a side opposite to right angle. ... T ...
(iv) If a, b, c are sides of right angled triangle with c as longer side, then
$c^2 = a^2 + b^2$ ... T ...
(v) If 3 cm and 4 cm are two sides of a right angled triangle, then hypotenuse is 5 cm. ...T...
(vi) If hypotenuse of an isosceles right triangle is $\sqrt{2}cm$, then each of other side is of length $2cm$. ... F ...

2. Find the unknown value in each of the following figures.
(i)



Sol: Let $a = 3\ cm, b = 4\ cm, c = x\ cm$
Using Pythagoras Theorem
$$c^2 = a^2 + b^2$$
$$(x)^2 = (3)^2 + (4)^2$$
$$x^2 = 9 + 16$$
$$x^2 = 25$$
$$\sqrt{x^2} = \sqrt{25}$$
$$x = 5cm$$

(ii)

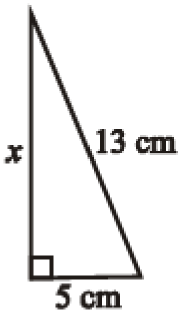


Sol: Let $a = x\ cm, b = 6\ cm, c = 10\ cm$

Using Pythagoras Theorem

$$c^2 = a^2 + b^2$$
$$(10)^2 = (x)^2 + (6)^2$$
$$100 = x^2 + 36$$
$$x^2 = 100 - 36$$
$$x^2 = 64$$
$$\sqrt{x^2} = \sqrt{64}$$
$$x = 8cm$$

(iii)

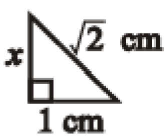


Sol: Let $a = 5\ cm, b = x\ cm, c = 13\ cm$

Using Pythagoras Theorem

$$c^2 = a^2 + b^2$$
$$(13)^2 = (5)^2 + (x)^2$$
$$169 = 25 + x^2$$
$$x^2 = 169 - 25$$
$$x^2 = 144$$
$$\sqrt{x^2} = \sqrt{144}$$
$$x = 12cm$$

(iv)



Sol: Let $a = 1\ cm, b = x\ cm, c = \sqrt{2}\ cm$

Using Pythagoras Theorem

$$c^2 = a^2 + b^2$$
$$\left(\sqrt{2}\right)^2 = (1)^2 + (x)^2$$
$$2 = 1 + x^2$$
$$x^2 = 1 - 1$$
$$x^2 = 1$$
$$\sqrt{x^2} = \sqrt{1}$$
$$x = 1\ cm$$
