**Rational number:**
A number which can be written in the form of $\frac{p}{q}$ $, where\ p, q \in Z \wedge \neq 0$ is called a rational number.
e.g. $\frac{3}{4}, \frac{22}{7}, \frac{2}{6}$.

**Irrational number:**
A real number which cannot be written in the form of $\frac{p}{q}$ $, where\ p, q \in Z \wedge \neq 0$ is called an irrational number.
e.g. $\sqrt{2}$ $, \sqrt{5}$

**Real number:**
The field of all rational and irrational numbers is called the real numbers, or simply the "reals," and denoted $\mathbb{R}$ .

**Terminating decimal:**
A decimal which has only a finite number of digits in its decimal part, is called terminating decimal.
e.g. 202.04, 0.25, 0.5 example of terminating decimal.

**Recurring decimal:**
A decimal in which one or more digits repeats indefinitely is called recurring decimal or periodic decimal.
e.g. 0.33333 , 21.134134 … …

# Exercise 2.1

**Question.1. Identify which of the following are rational and irrational numbers**

**(i).** $\sqrt{3}$
**Solution.**
**Is an irrational number.**

**(ii).** $\frac{1}{6}$
**Solution.**
**Is a rational number.**

**(iii).** $\pi$
**Solution.**
**Is an irrational number.**

**(iv).** $\frac{15}{7}$
**Solution.**
**Is a rational number.**

**(v). 7.25**
**Solution.**
**Is a rational number.**

**(vi).** $\sqrt{29}$
**Solution.**
**Is an irrational number.**

**Question.2. Convert the following fractions into decimal fraction.**

**(i)** $\frac{17}{25}$
Solution.
$$0.68$$

**(ii)** $\frac{19}{4}$
Solution.
$$4.75$$

**(iii)** $\frac{57}{8}$
Solution.
$$7.125$$

**(iv)** $\frac{205}{18}$
Solution.
$$11.3889$$

**(v)** $\frac{5}{8}$
Solution.
$$0.625$$

**(vi)** $\frac{25}{38}$
Solution.
$$0.65789$$

**Question.3. Which of the following statements are true and which are false?**

**(i).** $\frac{2}{3}$ *is an irrational number.*
**Solution.**
**False.**

**(ii).** $\pi$ *is an irrational number.*
**Solution.**
**True.**

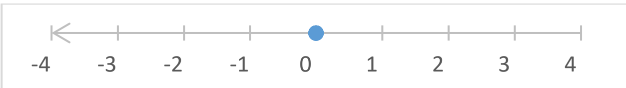
**(iii).** $\frac{1}{9}$ *is a terminating fraction.*
**Solution.**
**False.**

**(iv).** $\frac{3}{4}$ *is terminating fraction.*
**Solution.**
**True.**

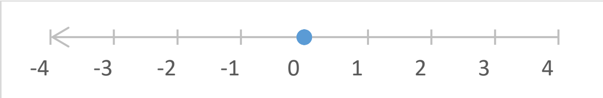
**(v).** $\frac{4}{5}$ *is a recurring fraction..*
**Solution.**
**False.**

**Question.4. Represent the following numbers on the number line**

**(i)** $\frac{2}{3}$
**Solution.**

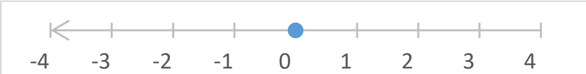



**(ii).** $-\frac{4}{5}$
**Solution.**



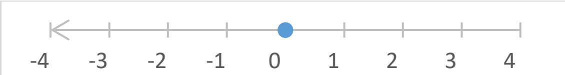

(iii). $1\frac{3}{4}$

**Solution.**



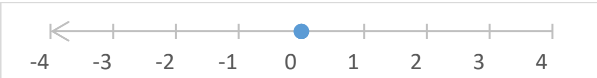

(iv). $-2\frac{5}{8}$

**Solution.**




(v). $\sqrt{5}$

**Solution.**




**Question.5.** Give a rational number between $\frac{3}{4}$ and $\frac{5}{9}$.

**Solution.**

The mean of the numbers is between given numbers. Therefore

$$required\ number = \frac{\frac{3}{4}+\frac{5}{9}}{2}$$

$$= \frac{\frac{27+20}{36}}{2}$$

$$= \frac{\frac{47}{36}}{2}$$

$$= \frac{47}{36\times 2}$$

$$= \frac{47}{72}$$

***Question.6.*** Express the following recurring decimals as the rational number $\frac{p}{q}$, where p, q are integers and $q \neq 0$.

(i). $0.\overline{5}$

**Solution.**

Let

$$x = 0.\overline{5}$$

That is

$$x = 0.5555 \ldots \to (i)$$

Only one digit 5 is being repeated, multiply by 10 on both sides of $(i)$, we have

$$10x = (0.5555 \ldots) \times 10$$

$$10x = 5.5555 \ldots \to (ii)$$

Subtracting $(i)$ *from* $(ii)$, *we have*

$$10x - x = 5.5555 \ldots . - 0.5555 \ldots$$

$$9x = 5$$

$$x = \frac{5}{9}$$

$$0.\overline{5} = \frac{5}{9}$$

*Answer.*

(ii). $0.\overline{13}$

**Solution.**

Let

$$x = 0.\overline{13}$$

That is

$$x = 0.13131313 \ldots \to (i)$$

Only two digits 13 is being repeated, multiply by 100 on both sides of $(i)$, we have

$$100x = (0.13131313 \ldots) \times 100$$

$$100x = 13.13131313 \ldots \to (ii)$$

Subtracting $(i)$ *from* $(ii)$, *we have*

$$100x - x$$

$$= 13.13131313 \ldots . - 0.13131313 \ldots$$

$$99x = 13$$

$$x = \frac{13}{99}$$

$$0.\overline{13} = \frac{13}{99}$$

*Answer.*

(iii). $0.\overline{67}$

**Solution.**

Let

$$x = 0.\overline{67}$$

That is

$$x = 0.67676767 \ldots \to (i)$$

Only two digits 67 is being repeated, multiply by 100 on both sides of $(i)$, we have

$$100x = (0.67676767 \ldots) \times 100$$

$$100x = 67.67676767 \ldots \to (ii)$$

Subtracting $(i)$ *from* $(ii)$, *we have*

$$100x - x$$

$$= 67.67676767 \ldots . - 0.67676767 \ldots$$

$$99x = 67$$

$$x = \frac{67}{99}$$

$$0.\overline{67} = \frac{67}{99}$$

*Answer.*

## Properties of Real Numbers:

### Binary Operations:

A binary operation in a set A is a rule usually denoted by $*$ that assigns to any pair of elements of A to another element of A. e.g. two important binary operations are addition and multiplication in a set of real numbers. ( $\forall$ *stands for all.*)

### Addition Laws:

**Closure Law of Addition:**

$$\forall\, a,b \in \Re \text{ then } a+b \in \Re$$

**Associative Law of Addition:**

$$\forall\, a,b,c \in \Re \text{ then } a+(b+c) = (a+b)+c.$$

**Additive Identity:**

$$\forall\, a \in \Re, \exists\, 0 \in \Re \text{ such that } a+0 = 0+a = a.$$

$\exists$ *Stands for there exist and* 0 *is called the iditive identity.*

**Additive Inverse:**

$$\forall\, a \in \Re, \exists -a \in \Re \text{ such that } a+(-a) = -a+a = 0.$$

$-a$ *and* $a$ *are called the idetive inverse of each other.*

**Commutative Law for Addition:**

$$\forall\, a,b \in \Re \text{ then } a+b = b+a.$$

### Multiplication Laws:

**Closure Law of Multiplication:**

$$\forall\, a,b \in \Re \text{ then } ab \in \Re \qquad \forall \text{ stands for all.}$$

**Associative Law of Multiplication:**

$$\forall\, a,b,c \in \Re \text{ then } a(bc) = (ab)c.$$

**Multiplicative Identity:**

$$\forall\, a \in \Re, \exists\, 1 \in \Re \text{ such that } a.1 = 1.a = a.$$

$\exists$ *Stands for there exist and* 1 *is called the iditive identity.*

**Multiplicative Inverse:**

$$\forall\, a \in \Re, \exists\, a' = \frac{1}{a} \in \Re \text{ such that } a.\frac{1}{a} = \frac{1}{a}.a = 1.$$

$a$ *and* $\frac{1}{a}$ *are called the idetive inverse of each other.*

**Commutative Law for Multiplication:**

$$\forall\, a,b \in \Re \text{ then } ab = ba.$$

### Properties of Equality:

**Reflexive property:**

$$\forall\, a \in \Re \text{ then } a = a$$

**Symmetric Property:**

$$\forall\, a,b \in \Re \text{ and if } a = b \text{ then } b = a.$$

**Transitive Property:**

$$\forall\, a,b,c \in \Re, \text{ if } a = b \text{ and } b = c \text{ then } a = c.$$

**Additive Property:**

$$\forall\, a,b,c \in \Re, a = b \text{ then } a+c = b+c.$$

**Multiplicative Property:**

$$\forall\, a,b,c \in \Re, a = b \text{ then } ac = bc.$$

**Cancellation Property w.r.t. addition:**

$$\forall\, a,b,c \in \Re, a+c = b+c \text{ then } a = b.$$

**Cancellation Property w.r.t. Multiplication:**

$$\forall\, a,b,c \in \Re, ac = bc \text{ then } a = b.$$

**Distributive property of multiplication over addition.**

$$a(b+c) = ab+ac$$

**Distributive property of multiplication over Subtraction.**

$$a(b-c) = ab-ac$$

### Properties of Inequalities (Order properties):

**Trichotomy Property:**

$$\forall\, a,b \in \Re$$
$$\text{either } a = b \text{ or } a > b \text{ or } a < b.$$

**Transitive Property:**

$$\forall\, a,b \in \Re$$

(i). $\text{if } a > b \text{ and } b > c \text{ then } a > c.$

(ii). $\text{if } a < b \text{ and } b < c \text{ then } a < c.$

**Additive Property:**

$$\forall\, a,b \in \Re$$

(i). $\text{if } a > b \text{ then } a+c > b+c.$

(ii). $\text{if } a < b \text{ and then } a+c < b+c.$

**Multiplicative Properties:**

$$\forall\, a,b,c \in \Re$$

$If\ c > 0$
(i). $if\ a > b\ then\ \ ac > bc.$
(ii). $if\ a < b\ and\ then\ ac < bc.$
$If\ c < 0$
(iii). $if\ a > b\ then\ \ ac < bc.$
(iv). $if\ a < b\ and\ then\ ac > bc.$

# Exercise 2.2

**Question.1. Identify the property used in the following.**

**(i). $a + b = b + a$**
**Solution.**
$Commutative\ property\ w.r.t\ Addition.$

**(ii). $(ab)c = a(bc)$**
**Solution.**
$Associative\ property\ w.r.t\ Multiplication.$

**(iii). $7 \times 1 = 7$**
**Solution.**
$Multiplicative\ identity.$

**(iv). $x > y\ or\ x = y\ or\ x < y$**
**Solution.**
$Trichotromy\ Property.$

**(v). $ab = ba$**
**Solution.**
$Commutative\ property\ w.r.t\ Multiplication.$

**(vi). $a + b = b + c => a = b$**
**Solution.**
$Cancellation\ Law\ w.r.t\ Addition.$

**(vii). $5 + (-5) = 0$**
**Solution.**
$Additive\ Inverse.$

**(viii). $7 \times \frac{1}{7}$**
**Solution.**
$Multiplicative\ Inverse.$

**(ix). $a > b =>\ \ ac > bc\ (\ c > 0)$**
**Solution.**
$Multplicative\ propert.$

**Question.2. Fill in the following blanks by stating the properties of real numbers used.**

$$3x + 3(y - x)$$

**Solution.**
Given
$3x + 3(y - x) = 3x + 3y - 3x$
$Distributive\ property\ w.r.t\ muliplication\ over\ subtraction.$
$= 3x - 3x$
$+ 3y \quad commutative\ property\ w.r.t\ addition.$
$= 0 + 3y \quad additive\ inverse\ property.$
$= 3y \quad additive\ identity.$
Answer.

**Question.3. Give the name of property used in the following.**

**(i). $\sqrt{24} + 0 = \sqrt{24}$**
**Solution.**
$Additive\ identity.$

**(ii). $-\frac{2}{3}\left(5 + \frac{7}{2}\right) = \left(-\frac{2}{3}\right)(5) + \left(-\frac{2}{3}\right)\left(\frac{7}{2}\right)$**
**Solution.**
$Distributive\ property\ of\ multiplication\ over\ addition.$

**(iii). $\pi + (-\pi) = 0$**
**Solution.**
$Additive\ Inverse.$

**(iv). $\sqrt{3}.\sqrt{3}\ is\ a\ real\ number.$**
**Solution.**
$Closure\ law\ for\ multiplication.$

**(v). $\left(-\frac{5}{8}\right)\left(-\frac{8}{5}\right) = 1$**
**Solution.**
$Multiplicative\ Inverse.$

**Radicals and Radicands:**

If $n$ is a positive integer greater than 1 and $a$ is a real number , then any real number $x$ such that $x^n = a$ is called the nth root of $a\ , and\ in\ symbols\ is\ written\ as$

$$x = \sqrt[n]{a} \quad or\ x = (a)^{\frac{1}{n}}$$

And $\sqrt[n]{a}$ is called radical, the symbol $\sqrt{\quad}$ is called the radical sign , n is called the index of the radical and the real number $a$ under the radical sign is called the radicand or base.

**Radical and Exponent Form:**

$x = \sqrt[n]{a}\ is\ called\ redical\ form\ and\ a = x^{\frac{1}{n}}\ is\ called\ exponent\ form.$

**Some Properties of Radicals:**

(i). $\sqrt[n]{ab} = \sqrt[n]{a}\ \sqrt[n]{b}$

(ii). $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(iii). $\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a}$

(iv). $\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

(v). $\sqrt[n]{a^n} = a$

# Exercise # 2.3

**Question.1. Write each radical expression in exponential notation and each exponential expression in radical notation. Do not simplify.**

(i). $\sqrt[3]{-64} = (-64)^{\frac{1}{3}}$

**Solution.**

$$\sqrt[3]{-64} = (-64)^{\frac{1}{3}}$$

(ii). $2^{\frac{3}{5}}$

**Solution.**

$$2^{\frac{3}{5}} = \sqrt[5]{2^3}$$

(iii). $-7^{\frac{1}{3}}$

**Solution.**

$$-7^{\frac{1}{3}} = -\sqrt[3]{7}$$

(iv). $y^{-\frac{2}{3}}$

**Solution.**

$$y^{-\frac{2}{3}} = \sqrt[3]{y^{-2}}$$

**Question.2. Tell whether the following statements are true or false?**

(i). $5^{\frac{1}{5}} = \sqrt{5}$

**Solution.**

$$False\ because\ 5^{\frac{1}{5}} = \sqrt[5]{5}\ is\ true.$$

(ii). $2^{\frac{2}{3}} = \sqrt[3]{4}$

**Solution.**

$$True\ because\ 2^{\frac{2}{3}} = \sqrt[3]{2^2} = \sqrt[3]{4}\ is\ true.$$

(iii). $\sqrt{49} = \sqrt{7}$

**Solution.**

$$False\ because\ \sqrt{49} = \sqrt{7^2} = 7 is\ true.$$

(iv). $\sqrt[3]{x^{27}} = x^3$

**Solution.**

$$False\ because\ \sqrt[3]{x^{27}} = x^{\frac{27}{3}} = x^9\ is\ true.$$

**Question.3. Simplify the following radical expressions.**

(i). $\sqrt[3]{-125}$

**Solution.**

$$\sqrt[3]{-125} = \sqrt[3]{-5^3}$$
$$= (-5)^{3\times\frac{1}{3}}$$
$$= -5$$

Answer.

(ii). $\sqrt[4]{32}$

**Solution.**

$$\sqrt[4]{32} = \sqrt[4]{2^4 \times 2}$$
$$= (2^4 \times 2)^{\frac{1}{4}}$$
$$= (2^4)^{\frac{1}{4}} \times 2^{\frac{1}{4}}$$
$$= 2 \times \sqrt[4]{2}$$
$$= 2\sqrt[4]{2}$$

Answer.

(iii). $\sqrt[5]{\frac{3}{32}}$

**Solution.**

$$\sqrt[5]{\frac{3}{32}} = \left(\frac{3}{32}\right)^{\frac{1}{5}}$$
$$= \left(\frac{3}{2^5}\right)^{\frac{1}{5}}$$
$$= \frac{3^{\frac{1}{5}}}{2^{5\times\frac{1}{5}}}$$
$$= \frac{\sqrt[5]{3}}{2}$$

Answer.

(iv). $\sqrt[3]{\frac{-8}{27}}$

**Solution.**

$$\sqrt[3]{\frac{-8}{27}} = \left(\frac{-2^3}{3^3}\right)^{\frac{1}{3}}$$
$$= \frac{-2^{3\times\frac{1}{3}}}{3^{3\times\frac{1}{3}}}$$
$$= \frac{-2}{3}$$

Answer.

**Base and Exponents:**

In the exponential form

$a^n$ ( *read as a to the nth power*) we call $''a''$ as the base and $''n''$ as the exponent or power.

**Laws of Exponents:**

If $a, b \in$

$R\ and\ m\ , n\ are\ positive\ integers, then$

(i). $a^m\ .\ a^n = a^{m+n}$

(ii). $(a^m)^n = a^{mn}$

(iii). $(ab)^n = a^n b^n$

(iv). $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

(v). $\frac{a^m}{a^n} = a^{m-n}$

(vi). $a^0 = 1\ , where\ a \neq 0$

(vii). $a^{-n} = \frac{1}{a^n}\ , where\ a \neq 0$

# Exercise # 2.4

**Question.1. Use laws of exponents to simplify**

(i). $\frac{(243)^{-\frac{2}{3}}(32)^{-\frac{1}{5}}}{\sqrt{(196)^{-1}}}$

**Solution.**

$$\frac{(243)^{-\frac{2}{3}}(32)^{-\frac{1}{5}}}{\sqrt{(196)^{-1}}} = \frac{(3^5)^{-\frac{2}{3}}(2^5)^{-\frac{1}{5}}}{(14^2)^{-1\times\frac{1}{2}}}$$
$$= \frac{3^{-\frac{10}{3}}\,2^{-1}}{14^{-1}}$$
$$= \frac{3^{-\frac{10}{3}}\,2^{-1}}{(2\times 7)^{-1}}$$
$$= \frac{3^{-\frac{10}{3}}\,2^{-1}}{2^{-1}\times 7^{-1}}$$
$$= \frac{3^{-\frac{10}{3}}}{7^{-1}}$$
$$= \frac{7}{3^{\frac{9+1}{3}}}$$
$$= \frac{7}{3^{\frac{9}{3}+\frac{1}{3}}}$$
$$= \frac{7}{3^{\frac{9}{3}}\times 3^{\frac{1}{3}}}$$
$$= \frac{7}{3^3\times\sqrt[3]{3}}$$
$$= \frac{7}{27\sqrt[3]{3}}$$

Answer.

(ii). $(2x^5y^{-4})(-8x^{-3}y^2)$

**Solution.**

$$(2x^5y^{-4})(-8x^{-3}y^2) = (2)(-8)x^5.y^{-4}.x^{-3}y^2$$
$$= -16\,x^{5-3}.y^{-4+2}$$
$$= -16\,x^2.y^{-2}$$
$$= -\frac{16x^2}{y^2}$$

Answer.

(iii). $\left(\frac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0}\right)^{-3}$

**Solution.**

$$\left(\frac{x^{-2}y^{-1}z^{-4}}{x^4y^{-3}z^0}\right)^{-3} = \left(\frac{y^{-1+3}}{x^{4+2}z^{0+4}}\right)^{-3}$$
$$= \left(\frac{y^2}{x^6z^4}\right)^{-3}$$
$$= \left(\frac{x^6z^4}{y^2}\right)^{3}$$
$$= \frac{x^{6\times 3}z^{4\times 3}}{y^{2\times 3}}$$
$$= \frac{x^{18}z^{12}}{y^6}$$

Answer.

(iv). $\frac{(81)^n.3^5-(3)^{4n-1}(243)}{(9)^{2n}.3^3}$

**Solution.**

$$\frac{(81)^n.3^5-(3)^{4n-1}(243)}{(9)^{2n}.3^3}$$
$$= \frac{(3^4)^n.3^5-(3)^{4n-1}(3)^5}{(3^2)^{2n}.3^3}$$
$$= \frac{(3)^{4n}.3^5-(3)^{4n-1}(3)^5}{(3)^{4n}.3^3}$$
$$= \frac{3^{4n+5}-3^{4n-1+5}}{3^{4n+3}}$$
$$= \frac{3^{4n+5}-3^{4n+4}}{3^{4n+3}}$$
$$= \frac{3^{4n+4}(3^1-1)}{3^{4n+3}}$$
$$= 3^{4n+4-4n-3}(2)$$
$$= 3^1(2)$$
$$= 6$$

Answer

**Question.2. Show that**

$$\left(\frac{x^a}{x^b}\right)^{a+b}\times\left(\frac{x^b}{x^c}\right)^{b+c}\times\left(\frac{x^c}{x^a}\right)^{c+a} = 1$$

**Solution.**

$$L.H.S = \left(\frac{x^a}{x^b}\right)^{a+b}\times\left(\frac{x^b}{x^c}\right)^{b+c}\times\left(\frac{x^c}{x^a}\right)^{c+a}$$
$$L.H.S = (x^{a-b})^{a+b}\times(x^{b-c})^{b+c}\times(x^{c-a})^{c+a}$$
$$L.H.S = x^{(a-b)(a+b)}\times x^{(b-c)(b+c)}\times x^{(c-a)(c+a)}$$
$$L.H.S = x^{a^2-b^2}\times x^{b^2-c^2}\times x^{c^2-a^2}$$
$$L.H.S = x^{a^2-b^2+b^2-c^2+c^2-a^2}$$
$$L.H.S = x^0 = 1$$

Hence Proved.

**Question.3. Simplify**

(i). $\frac{2^{\frac{1}{3}}(27)^{\frac{1}{3}}(60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}}(4)^{-\frac{1}{3}}9^{\frac{1}{4}}}$

**Solution.**

$$\frac{2^{\frac{1}{3}}(27)^{\frac{1}{3}}(60)^{\frac{1}{2}}}{(180)^{\frac{1}{2}}(4)^{-\frac{1}{3}}9^{\frac{1}{4}}} = \frac{2^{\frac{1}{3}}(3^3)^{\frac{1}{3}}(2^2.3.5)^{\frac{1}{2}}}{(2^2.3^2.5)^{\frac{1}{2}}(2^2)^{-\frac{1}{3}}(3^2)^{\frac{1}{4}}}$$
$$= \frac{2^{\frac{1}{3}}3^1.2^{2\times\frac{1}{2}}.3^{\frac{1}{2}}.5^{\frac{1}{2}}}{2^{2\times\frac{1}{2}}.3^{2\times\frac{1}{2}}.5^{\frac{1}{2}}.2^{-\frac{2}{3}}3^{2\times\frac{1}{4}}}$$
$$= \frac{2^{\frac{1}{3}}3^1.2^1.3^{\frac{1}{2}}.5^{\frac{1}{2}}}{2^1.3^1.5^{\frac{1}{2}}.2^{-\frac{2}{3}}3^{\frac{1}{2}}}$$

$$=\frac{2^{\frac{1}{3}}}{2^{-\frac{2}{3}}}$$
$$=2^{\frac{1}{3}+\frac{2}{3}}$$
$$=2^{\frac{1+2}{3}}$$
$$=2^{\frac{3}{3}}$$
$$=2$$
Answer.

(ii). $\sqrt{\frac{(216)^{\frac{2}{3}}(25)^{\frac{1}{2}}}{(0.04)^{-\frac{1}{2}}}}$

**Solution.**

$$\sqrt{\frac{(216)^{\frac{2}{3}}(25)^{\frac{1}{2}}}{(0.04)^{-\frac{1}{2}}}}=\sqrt{\frac{(6^3)^{\frac{2}{3}}(5^2)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{-\frac{1}{2}}}}$$
$$=\sqrt{\frac{6^2\,.\,5^1}{\left(\frac{1}{25}\right)^{-\frac{1}{2}}}}$$
$$=\sqrt{\frac{6^2\,.\,5^1}{(25)^{\frac{1}{2}}}}$$
$$=\sqrt{\frac{6^2\,.5}{(5^2)^{\frac{1}{2}}}}$$
$$=\sqrt{\frac{6^2.5}{5}}$$
$$=\sqrt{36}$$
$$=6$$
Answer.

(iii). $5^{2^3}\div(5^2)^3$

**Solution.**

$$5^{2^3}\div(5^2)^3=\frac{5^8}{5^6}$$
$$=5^{8-6}$$
$$=5^2$$
$$=25$$
Answer.

(iv). $(x^3)^2\div x^{3^2}$

**Solution.**

$$(x^3)^2\div x^{3^2}=\frac{x^6}{x^8}$$
$$=\frac{1}{x^{8-6}}$$
$$=\frac{1}{x^2}$$
Answer.

**Complex Numbers:**

The numbers of the form $x+iy$ , where $x,y\in\Re$ ,are called **complex numbers,** here $x$ is called **real part** and y is called **imaginary part** of the complex number.

**Remarks:**

1. Every real number is a complex number with 0 as its imaginary part.

**Conjugate Complex Numbers:**

$if\ Z=a+ib\ be\ a\ complex\ number\ then\ \bar{Z}=a-ib$ is the conjugate of the complex number $Z=a+ib$.

**Remarks:**

1. A real number is self-Conjugate.

**Equality of Two Complex Numbers:**

Two complex numbers $a+bi\ and\ c+di\ are\ said\ to\ be\ equal\ if\ a=c\ and\ b=d.$

That is

$$a+ib=c+id\ \ \Rightarrow a=b\ and\ c=d.$$

# Exercise # 2.5

**Question.1. Evaluate**

**(i).** $\boldsymbol{i^7}$

**Solution.**

$$\boldsymbol{i^7=i^6.i}$$
$$\boldsymbol{=(i^2)^3.i}$$
$$\boldsymbol{=(-1)^3.i}$$
$$\boldsymbol{=(-1).i}$$
$$\boldsymbol{=-i}$$
***Answer.***

**(ii).** $\boldsymbol{i^{50}}$

***Solution.***

$$\boldsymbol{i^{50}=(i^2)^{25}}$$
$$\boldsymbol{=(-1)^{25}}$$
$$\boldsymbol{=-1}$$
***Answer.***

***(iii).*** $\boldsymbol{i^{12}}$

***Solution.***

$$\boldsymbol{i^{12}=(i^2)^6}$$
$$\boldsymbol{=(-1)^6}$$
$$\boldsymbol{=1}$$
**Answer.**

**(iv).** $(-i)^8$

**Solution.**

$$\boldsymbol{(-i)^8=i^8}$$
$$\boldsymbol{=(i^2)^4}$$
$$\boldsymbol{=(-1)^4}$$
$$\boldsymbol{=1}$$
**Answer.**

**(v).** $(-i)^5$

**Solution.**

$$\begin{aligned}(-i)^5 &= -i^5\\ &= -i^4.i\\ &= -(i^2)^2.i\\ &= -(-1)^2.i\\ &= -(1).i\\ &= -i\end{aligned}$$

**Answer.**

**(vi).** $i^{27}$

**Solution.**

$$\begin{aligned}i^{27} &= i^{26}.i\\ &= (i^2)^{13}.i\\ &= (-1)^{13}.i\\ &= (-1).i\\ &= -1\end{aligned}$$

**Answer.**

**Question.2. Write the conjugate of the following numbers.**

**(i).** $2+3i$

**Solution.**

$$Suppose\ \ Z = 2+3i$$
$$\bar{Z} = \overline{2+3i} = 2-3i$$

Answer.

**(ii).** $3-5i$

**Solution.**

$$Suppose\ \ Z = 3-5i$$
$$\bar{Z} = \overline{3-5i} = 3+5i$$

Answer.

**(iii).** $-i$

**Solution.**

$$Suppose\ \ Z = -i$$
$$\bar{Z} = \overline{-i} = +i$$

Answer.

**(iv).** $-3+4i$

**Solution.**

$$Suppose\ \ Z = -3+4i$$
$$\bar{Z} = \overline{-3+4i} = -3-4i$$

Answer.

**(v).** $-4-i$

**Solution.**

$$Suppose\ \ Z = -4-i$$
$$\bar{Z} = \overline{-4-i} = -4+i$$

Answer.

**Question.3. Write the real and imaginary part of the following numbers.**

**(i).** $1+i$

**Solution.**

$$Suppose\ \ Z = 1+i$$
$$Re(Z) = 1 \quad , \quad Im(Z) = 1$$

Answer.

**(ii).** $-1+2i$

**Solution.**

$$Suppose\ \ Z = -1+2i$$
$$Re(Z) = -1 \quad , \quad Im(Z) = 2$$

Answer.

**(iii).** $-3i+2$

**Solution.**

$$Suppose\ \ Z = -3i+2 = 2-3i$$
$$Re(Z) = 2 \quad , \quad Im(Z) = -3$$

Answer.

**(iv).** $-2-2i$

**Solution.**

$$Suppose\ \ Z = -2i-2 = -2-2i$$
$$Re(Z) = -2 \quad , \quad Im(Z) = -2$$

Answer.

**(v).** $-3i$

**Solution.**

$$Suppose\ \ Z = -3i = 0-3i$$
$$Re(Z) = 0 \quad , \quad Im(Z) = -3$$

Answer.

**(vi).** $2+0i$

**Solution.**

$$Suppose\ \ Z = 2+0i$$
$$Re(Z) = 2 \quad , \quad Im(Z) = 0$$

Answer.

**Question.4. Find the value of $x$ and $y$ if**

$$x+iy+1 = 4-3i$$

**Solution.**

**Given that**

$$x+1+iy = 4-3i$$

**Separating real and imaginary parts**

$$\begin{aligned}x+1 &= 4 \quad , \quad y = -3\\ x &= 4-1 \quad , \quad y = -3\\ x &= 3 \quad , \quad y = -3\end{aligned}$$

**Answer.**

## Operations on Complex Numbers:

The symbols *a,b,c,d,k*, where used, represent real numbers

Addition of Two Complex Numbers:

$$(a+ib)+(c+id) = (a+b)+i(c+d).$$

**Scalar Multiplication:**

$$k(a+ib) = ka+ikb.$$

**Subtraction of Two Complex Numbers:**

$$(a+ib)-(c+id) = (a-b)+i(c-d).$$

Multiplication of Two Complex Numbers:

$$(a+ib)(c+id) = (ac-bd)+i(ad+bc).$$

**Division of two Complex Numbers:**

$$\frac{(a+ib)}{(c+id)} = \frac{ac-bd}{c^2+d^2} + i\frac{bc-ad}{c^2+d^2}$$

**Exercise # 2.6**

**Question.1. Identify the following statements as true or false.**

(i). $\sqrt{-3}\times\sqrt{-3} = 3$

**Solution.**

$$False\ because\ \sqrt{-3}\times\sqrt{-3} = \sqrt{3}i\times\sqrt{3}i$$
$$= \left(\sqrt{3}\right)^2 i^2 = -3$$

(ii). $i^{73} = -i$

**Solution.**

$$False\ because\ i^{73} = i^{72}.i = (i^2)^{36}.i$$
$$= (-1)^{36}.i = i$$

(iii). $i^{10} = -1$

**Solution.**

$$True\ because\ i^{10} = (i^2)^5 = (-1)^5 = -1$$

(iv). $Complex\ conjugate\ of\ (-6i + i^2)\ is\ (-1+6i)$

**Solution.**

$$True\ because\ \overline{-6\imath + \imath^2} = \overline{-6\imath - 1} = -1 + 6i$$

(v). Difference of a complex number $z = a + bi\ and\ its\ conjugate\ is\ a\ real\ number.$

**Solution.**

$$False\ because\ Z - \overline{Z} = (a+bi) - (a-bi)$$
$$= a + bi - a + bi = 2bi$$

(vi). If $(a-1) - (b+3)i = 5 + 8i\ then\ a = 6\ and\ \ b = -11.$

**Solution.**

True because Comparing real and imaginary parts in given equation

$$a - 1 = 5 \quad , \quad -(b+3) = 8$$
$$a = 5 + 1 \quad , \quad b + 3 = -8$$
$$a = 6 \quad , \quad b = -8 - 3$$
$$a = 6 \quad , \quad b = -11$$

**(vii)** Product of a complex number and its conjugate is always a non-negative real number.

**Solution.**

$$\boldsymbol{True\ because\ for\ a\ complex\ numer\ Z = a + bi}$$
$$\boldsymbol{Z.\overline{Z} = (a+bi).(a-bi) = a^2 - (bi)^2}$$
$$\boldsymbol{= a^2 + b^2}$$

**Is a real number.**

**Question.2. Express each complex number in the standard form $a + bi$ where 'a' and 'b' are real numbers.**

(i). $(2+3i) + (7-2i)$

**Solution.**

$$(2+3i) + (7-2i) = 2 + 3i + 7 - 2i$$
$$= \boldsymbol{9 + i}$$

**Answer.**

**(ii). $2(5+4i) - 3(7+4i)$**

**Solution.**

$$\boldsymbol{2(5+4i) - 3(7+4i) = 10 + 8i - 21 - 12i}$$
$$\boldsymbol{= -11 - 3i}$$

**Answer.**

**(iii). $-1(-3+5i) - (4+9i)$**

**Solution.**

$$\boldsymbol{-1(-3+5i) - (4+9i) = 3 - 5i - 4 - 9i}$$
$$\boldsymbol{= -1 - 14i}$$

**Answer.**

**(iv). $2i^2 + 6i^3 + 3i^{16} - 6i^{19} + 4i^{25}$**

**Solution.**

$$\boldsymbol{2i^2 + 6i^3 + 3i^{16} - 6i^{19} + 4i^{25}}$$
$$\boldsymbol{= 2(-1) + 6i^2 i + 3i^{16} - 6i^{18}i + 4i^{24}i}$$
$$\boldsymbol{= 2(-1) + 6(-1)i + 3(i^2)^8 - 6(i^2)^9 i + 4(i^2)^{12} i}$$
$$\boldsymbol{= -2 - 6i + 3(-1)^8 - 6(-1)^9 i + 4(-1)^{12} i}$$
$$\boldsymbol{= -2 - 6i + 3(1) - 6(-1)i + 4(1)i}$$
$$\boldsymbol{= -2 - 6i + 3 + 6i + 4i}$$
$$\boldsymbol{= 1 + 4i}$$

**Question.3. Simplify and write your answer in the form $a + bi$.**

**(i). $(-7+3i)(-3+2i)$**

**Solution.**

$$\boldsymbol{(-7+3i)(-3+2i) = 21 - 14i - 9i + 6i^2}$$
$$\boldsymbol{= 21 - 14i - 9i - 6}$$
$$\boldsymbol{= 15 - 23i}$$

**Answer.**

**(ii). $(2-\sqrt{-4})(3-\sqrt{-4})$**

**Solution.**

$$\boldsymbol{(2-\sqrt{-4})(3-\sqrt{-4}) = (2-2i)(3-2i)}$$
$$\boldsymbol{= 2(3-2i) - 2i(3-2i)}$$
$$\boldsymbol{= 6 - 4i - 6i + 4i^2}$$
$$\boldsymbol{= 6 - 10i - 4}$$
$$\boldsymbol{= 2 - 10i}$$

**Answer.**

**(iii). $\left(\sqrt{5} - 3i\right)^2$**

**Solution.**

$$\boldsymbol{\left(\sqrt{5}-3i\right)^2 = \left(\sqrt{5}\right)^2 + (3i)^2 - 2(\sqrt{5})(3i)}$$
$$\boldsymbol{= 5 + 9i^2 - 6\sqrt{5}i}$$
$$\boldsymbol{= 5 - 9 - 6\sqrt{5}i}$$
$$\boldsymbol{= -4 - 6\sqrt{5}i}$$

**Answer.**

**(iv). $(2-3i)(\overline{3-2\imath})$**

**Solution.**

$$\boldsymbol{(2-3i)(\overline{3-2\imath}) = (2-3i)(3+2i)}$$
$$\boldsymbol{= 2(3+2i) - 3i(3+2i)}$$

$$= 6 + 4i - 9i - 6i^2$$
$$= 6 - 5i + 6$$
$$= 12 - 5i$$

**Answer.**

**Question.4. Simplify and write your answer in the form of $a + bi$.**

**(i).** $-\frac{2}{1+i}$

**Solution.**

$$-\frac{2}{1+i} = \frac{-2}{1+i} \times \frac{1-i}{1-i}$$
$$= \frac{-2+2i}{1^2 - i^2}$$
$$= \frac{-2+2i}{1+1}$$
$$= \frac{-2+2i}{2}$$
$$= -\frac{2}{2} + \frac{2i}{2}$$
$$= -1 + i$$

**Answer.**

**(ii).** $\frac{2+3i}{4-i}$

**Solution.**

$$\frac{2+3i}{4-i} = \frac{2+3i}{4-i} \times \frac{4+i}{4+i}$$
$$= \frac{2(4+i) + 3i(4+i)}{4^2 - i^2}$$
$$= \frac{8 + 2i + 12i + 3i^2}{16+1}$$
$$= \frac{8 + 14i - 3}{17}$$
$$= \frac{4 + 14i}{17}$$
$$= \frac{4}{17} + \frac{14}{17}i$$

**Answer.**

**(iii).** $\frac{9-7i}{3+i}$

**Solution.**

$$\frac{9-7i}{3+i} = \frac{9-7i}{3+i} \times \frac{3-i}{3-i}$$
$$= \frac{9(3-i) - 7i(3-i)}{3^2 - i^2}$$
$$= \frac{27 - 9i - 21i + 7i^2}{9+1}$$
$$= \frac{27 - 30i - 7}{10}$$
$$= \frac{20 - 30i}{10}$$
$$= \frac{20}{10} - \frac{30}{10}i$$
$$= 2 - 3i$$

**Answer.**

**(iv).** $\frac{2-6i}{3+i} - \frac{4+i}{3+i}$

**Solution.**

$$\frac{2-6i}{3+i} - \frac{4+i}{3+i} = \frac{2-6i}{3+i} \times \frac{3-i}{3-i} - \frac{4+i}{3+i} \times \frac{3-i}{3-i}$$
$$= \frac{2(3-i) - 6i(3-i)}{3^2 - i^2} - \frac{4(3-i) + i(3-i)}{3^2 - i^2}$$
$$= \frac{6 - 2i - 18i + 6i^2}{9+1}$$
$$- \frac{12 - 4i + 3i - i^2}{9+1}$$
$$= \frac{6 - 20i - 6}{10} - \frac{12 - i + 1}{10}$$
$$= \frac{-20i}{10} - \frac{13 - i}{10}$$
$$= \frac{-20i - 13 + i}{10}$$
$$= \frac{-20i - 13 + i}{10}$$
$$= \frac{-13 - 19i}{10}$$
$$= -\frac{13}{10} - \frac{19}{10}i$$

**Answer.**

**(v).** $\left(\frac{1+i}{1-i}\right)^2$

**Solution.**

$$\left(\frac{1+i}{1-i}\right)^2 = \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^2$$
$$= \left(\frac{1(1+i) + i(1+i)}{1^2 - i^2}\right)^2$$
$$= \left(\frac{1 + i + i + i^2}{1+1}\right)^2$$
$$= \left(\frac{1 + 2i - 1}{2}\right)^2$$
$$= \left(\frac{2i}{2}\right)^2$$
$$= i^2$$
$$= -1$$

**Answer.**

**(vi).** $\frac{1}{(2+3i)(1-i)}$

**Solution.**

$$\frac{1}{(2+3i)(1-i)} = \frac{1}{2(1-i) + 3i(1-i)}$$
$$= \frac{1}{2 - 2i + 3i - 3i^2}$$
$$= \frac{1}{2 + i + 3}$$
$$= \frac{1}{5 + i}$$

$$=\frac{1}{5+i}\times\frac{5-i}{5-i}$$
$$=\frac{5-i}{5^2-(i)^2}$$
$$=\frac{5-i}{25+1}$$
$$=\frac{5-i}{26}$$
$$=\frac{5}{26}-\frac{i}{26}$$

**Answer.**

**Question.5.**

**Calculate (a) $\overline{Z}$ $(b) Z+\overline{Z}$ $(c)\ Z-\overline{Z}$ $(d)\ Z\overline{Z}$ for each of the following**

**(i).** $Z=-i$

**Solution.**

(a). $\overline{Z}=\overline{-i}=i$

(b). $Z+\overline{Z}=-i+i=0$

(c). $Z-\overline{Z}=(-i)-(i)=-i-i=-2i$

(d). $Z\overline{Z}=(-i)(i)=-i^2=1$

**(ii).** $Z=2+i$

**Solution.**

(a). $\overline{Z}=\overline{2+i}=2-i$

(b). $Z+\overline{Z}=2+i+2-i=4$

(c). $Z-\overline{Z}=(2+i)-(2-i)=2+i-2+i=2i$

(d). $Z\overline{Z}=(2+i)(2-i)=2^2-i^2=4+1=5$

**(iii).** $Z=\frac{1+i}{1-i}$

**Solution.**

$$Z=\frac{1+i}{1-i}$$
$$Z=\frac{1+i}{1-i}\times\frac{1+i}{1+i}$$
$$Z=\frac{1+i+i+i^2}{1+1}$$
$$Z=\frac{1+2i-1}{2}$$
$$Z=\frac{2i}{2}$$
$$Z=i$$

(a). $\overline{Z}=\overline{i}=-i$

(b). $Z+\overline{Z}=i-i=0$

(c). $Z-\overline{Z}=(i)-(-i)=i+i=2i$

(d). $Z\overline{Z}=(i)(-i)=-i^2=1$

**(iv).** $Z=\frac{4-3i}{2+4i}$

**Solution.**

$$Z=\frac{4-3i}{2+4i}$$
$$Z=\frac{4-3i}{2+4i}\times\frac{2-4i}{2-4i}$$
$$Z=\frac{8-16i-6i+12i^2}{2^2-(4i)^2}$$
$$Z=\frac{8-22i-12}{4-16i^2}$$
$$Z=\frac{-4-22i}{4+16}$$
$$Z=\frac{-4-22i}{20}$$
$$Z=-\frac{4}{20}-\frac{22}{20}i$$
$$Z=-\frac{1}{5}-\frac{11}{10}i$$

(a). $\overline{Z}=\overline{-\frac{1}{5}-\frac{11}{10}i}=-\frac{1}{5}+\frac{11}{10}i$

(b). $Z+\overline{Z}=-\frac{1}{5}-\frac{11}{10}i+-\frac{1}{5}+\frac{11}{10}i$

$$Z+\overline{Z}=-\frac{1}{5}-\frac{1}{5}=\frac{-1-1}{5}=-\frac{2}{5}=-\frac{2}{5}$$
$$Z+\overline{Z}=-\frac{2}{5}$$

(c). $Z-\overline{Z}=\left(-\frac{1}{5}-\frac{11}{10}i\right)-\left(-\frac{1}{5}+\frac{11}{10}i\right)$

$$Z-\overline{Z}=-\frac{1}{5}-\frac{11}{10}i+\frac{1}{5}-\frac{11}{10}i$$
$$Z-\overline{Z}=-\frac{11}{10}i-\frac{11}{10}i$$
$$Z-\overline{Z}=\frac{-11-11}{10}i$$
$$Z-\overline{Z}=-\frac{22}{10}i$$
$$Z-\overline{Z}=-\frac{11}{5}i$$

(d). $Z\overline{Z}=\left(-\frac{1}{5}-\frac{11}{10}i\right)\left(-\frac{1}{5}+\frac{11}{10}i\right)$

$$Z\overline{Z}=\left(-\frac{1}{5}\right)^2-\left(\frac{11}{10}i\right)^2$$
$$Z\overline{Z}=\frac{1}{25}-\frac{121}{100}i^2$$
$$Z\overline{Z}=\frac{1}{25}+\frac{121}{100}$$
$$Z\overline{Z}=\frac{4+121}{100}$$
$$Z\overline{Z}=\frac{125}{100}$$
$$Z\overline{Z}=\frac{5}{4}$$

**Answer.**

**Question.6. If $z=2+3i\,, w=5-4i\,,$ *show that***

**(i).** $\overline{z+w}=\overline{z}+\overline{w}$

**Solution.**

$$L.H.S=\overline{z+w}$$
$$L.H.S=\overline{2+3i+5-4i}$$
$$L.H.S=\overline{8-i}$$

$$L.H.S = 8 + i - - - (1)$$
$$R.H.S = \bar{z} + \bar{w}$$
$$R.H.S = \overline{2+3\imath} + \overline{5-4\imath}$$
$$R.H.S = 2 - 3i + 5 + 4i$$
$$R.H.S = 8 + i - - - (2)$$

**From (1) and (2) , we have**

$$L.H.S = R.H.S$$

**Hence Proved.**

**(ii).** $\overline{z - w} = \bar{z} - \bar{w}$

**Solution.**

$$L.H.S = \overline{z - w}$$
$$L.H.S = \overline{(2+3\imath) - (5-4\imath)}$$
$$L.H.S = \overline{2 + 3\imath - 5 + 4\imath}$$
$$L.H.S = \overline{-3 + 7\imath}$$
$$L.H.S = -3 - 7i - - - (1)$$
$$R.H.S = \bar{z} - \bar{w}$$
$$R.H.S = (\overline{2+3\imath}) - (\overline{5-4\imath})$$
$$R.H.S = (2 - 3i) - (5 + 4i)$$
$$R.H.S = 2 - 3i - 5 - 4i$$
$$R.H.S = -3 - 7i - - - (2)$$

**From (1) and (2) , we have**

$$L.H.S = R.H.S$$

**Hence Proved.**

**(iii).** $\overline{zw} = \bar{z}\,\bar{w}$

**Solution.**

$$L.H.S = \overline{zw}$$
$$L.H.S = \overline{(2+3\imath)(5-4\imath)}$$
$$L.H.S = \overline{10 - 8\imath + 15\imath - 12\imath^2}$$
$$L.H.S = \overline{10 + 7\imath + 12}$$
$$L.H.S = \overline{22 + 7\imath}$$
$$L.H.S = 22 - 7i - - - (1)$$
$$R.H.S = \bar{z}\bar{w}$$
$$R.H.S = (\overline{2+3\imath})(\overline{5-4\imath})$$
$$R.H.S = (2 - 3i)(5 + 4i)$$
$$R.H.S = 10 + 8i - 15i - 12i^2$$
$$R.H.S = 10 - 7i + 12$$
$$R.H.S = 22 - 7i - - - (2)$$

**From (1) and (2) , we have**

$$L.H.S = R.H.S$$

**Hence Proved.**

**(iv).** $\overline{\left(\frac{z}{w}\right)} = \frac{\bar{z}}{\bar{w}}$

**Solution.**

$$L.H.S = \overline{\left(\frac{z}{w}\right)}$$
$$L.H.S = \overline{\left(\frac{2+3\imath}{5-4\imath}\right)}$$
$$L.H.S = \overline{\left(\frac{2+3\imath}{5-4\imath} \times \frac{5+4\imath}{5+4\imath}\right)}$$
$$L.H.S = \overline{\left(\frac{10 + 8\imath + 15\imath + 12\imath^2}{5^2 - (4\imath)^2}\right)}$$
$$L.H.S = \overline{\left(\frac{10 + 23\imath - 12}{25 - 16\imath^2}\right)}$$
$$L.H.S = \overline{\left(\frac{-2 + 23\imath}{25 + 16}\right)}$$
$$L.H.S = \overline{\left(\frac{-2 + 23\imath}{41}\right)}$$
$$L.H.S = \overline{\left(-\frac{2}{41} + \frac{23}{41}\imath\right)}$$
$$L.H.S = -\frac{2}{41} - \frac{23}{41}i - - - (1)$$
$$R.H.S = \frac{\bar{z}}{\bar{w}}$$
$$R.H.S = \frac{(\overline{2+3\imath})}{\overline{5-4\imath}}$$
$$R.H.S = \frac{2 - 3i}{5 + 4i}$$
$$R.H.S = \frac{2 - 3i}{5 + 4i} \times \frac{5 - 4i}{5 - 4i}$$
$$R.H.S = \frac{10 - 8i - 15i + 12i^2}{5^2 - (4i)^2}$$
$$R.H.S = \frac{10 - 23i - 12}{25 - 16i^2}$$
$$R.H.S = \frac{-2 - 23i}{25 + 16}$$
$$R.H.S = \frac{-2 - 23i}{41}$$
$$R.H.S = -\frac{2}{41} - \frac{23}{41}i - - - (2)$$

**From (1) and (2) , we have**

$$L.H.S = R.H.S$$

**Hence Proved.**

**(v).** $\frac{1}{2}(z + \bar{z})$ *is a real part of z.*

**Solution.**

$$\frac{1}{2}(z + \bar{z}) = \frac{1}{2}\,(2 + 3i + \overline{2 + 3\imath})$$
$$= \frac{1}{2}\,(2 + 3i + 2 - 3i)$$
$$= \frac{1}{2}\,(4)$$
$$= 2 \text{ which is real part of } z.$$

**Hence Proved.**

**(vi).** $\frac{1}{2i}(z - \bar{z})$ *is a imaginary part of z.*

**Solution.**

$$\frac{1}{2i}(z - \bar{z}) = \frac{1}{2i}\,\left((2 + 3i) + (\overline{2 + 3\imath})\right)$$
$$= \frac{1}{2i}\,\left((2 + 3i) - (2 - 3i)\right)$$
$$= \frac{1}{2i}\,(2 + 3i - 2 + 3i)$$

$$= \frac{1}{2i}(6i)$$

$= 3$ *which is imaginary part of z.*

**Hence Proved.**

**Question.7. Solve the following equations for real $x$ and $y$.**

**(i).** $(2-3i)(x+iy) = 4+i$

**Solution.** Given that

$$(2-3i)(x+iy) = 4+i$$
$$2(x+iy) - 3i(x+iy) = 4+i$$
$$2x+2iy-3ix-3i^2y = 4+i$$
$$2x+2iy-3ix+3y = 4+i$$
$$2x+3y+(2y-3x)i = 4+i$$

Comparing real and imaginary parts, we have

$$2x+3y = 4 --- (i)\ ,\quad 2y-3x = 1 --- (ii)$$

$3\times(i)+2\times(ii)$, *we have*

$$3(2x+3y)+2(2y-3x) = 3(4)+2(1)$$
$$6x+9y+4y-6x = 12+2$$
$$13y = 14$$
$$y = \frac{14}{13}$$

Using value of $y$ *in equation* $(i)$, *we have*

$$2x+3\left(\frac{14}{13}\right) = 4$$
$$2x+\frac{42}{13} = 4$$
$$2x = 4-\frac{42}{13}$$
$$2x = \frac{4\times 13-42}{13}$$
$$2x = \frac{52-42}{13}$$
$$2x = \frac{10}{13}$$
$$x = \frac{10}{13\times 2}$$
$$x = \frac{5}{13}$$

Hence required $x = \frac{5}{13}$ *and* $y = \frac{14}{13}$.

**(ii).** $(3-2i)(x+iy) = 2(x-2yi)+2i-1$

**Solution.** Given that

$$(3-2i)(x+iy) = 2(x-2yi)+2i-1$$
$$3(x+iy)-2i(x+iy) = 2x-4yi+2i-1$$
$$3x+3iy-2ix-2i^2y = 2x-1+2i-4yi$$
$$3x+3iy-2ix+2y = 2x-1+(2-4y)i$$
$$3x+2y+(3y-2x)i = 2x-1+(2-4y)i$$

Comparing real and imaginary parts, we have

$$3x+2y = 2x-1\ ,(3y-2x) = 2-4y$$
$$3x-2x+2y = -1\ ,-2x+3y+4y = 2$$
$$x+2y = -1 --- (i)\ ,-2x+7y = 2 --- (ii)$$

$2\times(i)+(ii)$, *we have*

$$2(x+2y)+(-2x+7y) = 2(-1)+2$$
$$2x+4y-2x+7y = -2+2$$
$$11y = 0$$
$$y = 0$$

Using value of $y$ *in equation* $(i)$, *we have*

$$x+2(0) = -1$$
$$x = -1$$

Hence required $x = -1$ *and* $y = 0$.

**(iii).** $(3+4i)^2 - 2(x-iy) = x+yi$

**Solution.** Given that

$$(3+4i)^2 - 2(x-iy) = x+yi$$
$$(3)^2+(4i)^2+2(3)(4i)-2x+2iy = x+yi$$
$$9+16i^2+12i-2x+2iy = x+yi$$
$$9-16+12i-2x+2iy = x+yi$$
$$-7-2x+(12+2y)i = x+yi$$

Comparing real and imaginary parts, we have

$$-7-2x = x\ ,12+2y = y$$
$$x+2x = -7 = \ ,2y-y = 12$$
$$3x = -7\ ,y = 12$$
$$x = -\frac{7}{3}\ ,y = 12$$

Hence required $x = -\frac{7}{3}$ *and* $y = 12$.

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