1. If $a:b = c:d, (a,b,c,d \neq 0)$ then show that

(i).

$$\frac{4a-9b}{4a+9b} = \frac{4c-9d}{4c+9d}$$

Solution:

$a:b = c:d$

Let $\frac{a}{b} = \frac{c}{d} = k$

Then, $\frac{a}{b} = k\,;\frac{c}{d} = k$

Or, $a = bk\,; c = dk$

$$L.H.S = \frac{4a-9b}{4a+9b}$$

Putting $a = bk$ in above equation

$$= \frac{4bk-9b}{4bk+9b}$$

$$= \frac{b(4k-9)}{b(4k+9)}$$

$$L.H.S = \frac{4k-9}{4k+9} \qquad \_\_\_\_\_\_\_(i)$$

$$R.H.S = \frac{4c-9d}{4c+9d}$$

Putting $c = dk$ in above equation

$$R.H.S = \frac{4dk-9d}{4dk+9d}$$

$$R.H.S = \frac{d(4k-9)}{d(4k+9)}$$

$$R.H.S = \frac{4k-9}{4k+9} \qquad \_\_\_\_\_\_\_(ii)$$

From eq(i) & eq(ii)

$L.H.S = R.H.S$

Hence proved that

$$\frac{4a-9b}{4a+9b}=\frac{4c-9d}{4c+9d}$$

(ii).

$$\boldsymbol{\frac{6a-5b}{6a+5b}=\frac{6c-5d}{6c+5d}}$$

Solution:

$a:b = c:d$

Let $\frac{a}{b}=\frac{c}{d}=k$

Then, $\frac{a}{b}=k\,;\frac{c}{d}=k$

Or, $a = bk\,; c = dk$

$$L.H.S=\frac{6a-5b}{6a+5b}$$

Putting $a = bk$ in above equation

$$L.H.S=\frac{6bk-5b}{6bk+5b}$$

$$L.H.S=\frac{b(6k-5)}{b(6b+5)}$$

$$L.H.S=\frac{6k-5}{6k+5} \qquad \_\_\_\_\_\_\_(i)$$

$$R.H.S=\frac{6c-5d}{6c+5d}$$

Putting $c = dk$ in above equation

$$R.H.S=\frac{6dk-5d}{6dk+5}$$

$$R.H.S=\frac{d(6k-5)}{d(6k+5)}$$

$R.H.S = \frac{6k-5}{6k+5}$ ________ (ii)

From eq(i) & eq(ii)

$L.H.S = R.H.S$

Hence proved that

$$\frac{6a-5b}{6a+5b} = \frac{6c-5d}{6c+5d}$$

(iii).

$$\boldsymbol{\frac{a}{b} = \sqrt{\frac{a^2+c^2}{b^2+d^2}}}$$

Solution:

$a : b = c : d$

Let $\frac{a}{b} = \frac{c}{d} = k$

Then, $\frac{a}{b} = k\,; \frac{c}{d} = k$

Or, $a = bk\,; c = dk$

$L.H.S = \frac{a}{b}$

Putting $a = bk$ & $c = dk$ in above equation

$L.H.S = \frac{bk}{b}$

$L.H.S = k$ ________ (i)

$R.H.S = \sqrt{\frac{a^2+c^2}{b^2+d^2}}$

Putting $a = bk$ & $c = dk$ in above equation

$R.H.S = \sqrt{\frac{(bk)^2+(dk)^2}{b^2+d^2}}$

$$R.H.S=\sqrt{\frac{b^2k^2+d^2k^2}{b^2+d^2}}$$

$$R.H.S=\sqrt{\frac{k^2(b^2+d^2)}{b^2+d^2}}$$

$$R.H.S=\sqrt{k^2}$$

$$R.H.S=k \qquad \_\_\_\_\_\_\_\_(ii)$$

From eq(i) & eq(ii)

$$L.H.S=R.H.S$$

Hence proved that

$$\frac{a}{b}=\sqrt{\frac{a^2+c^2}{b^2+d^2}}$$

(iv). $\boldsymbol{a^6+c^6:b^6+d^6=a^3c^3:b^3d^3}$

Solution:

$$a^6+c^6:b^6+d^6=a^3c^3:b^3d^3$$

$$\frac{a^6+c^6}{b^6+d^6}=\frac{a^3c^3}{b^3d^3}$$

$$a:b=c:d$$

Let $\frac{a}{b}=\frac{c}{d}=k$

Then, $\frac{a}{b}=k\,;\frac{c}{d}=k$

Or, $a=bk\,;c=dk$

$$L.H.S=\frac{a^6+c^6}{b^6+d^6}$$

Putting $a=bk$ & $c=dk$ in above equation

$$= \frac{(bk)^6 + (dk)^6}{b^6 + d^6}$$

$$= \frac{b^6k^6 + d^6k^6}{b^6 + d^6}$$

$$= \frac{k^6(b^6 + d^6)}{b^6 + d^6}$$

$$= k^6 \qquad \_\_\_\_\_\_(i)$$

$$R.H.S = \frac{a^3c^3}{b^3d^3}$$

Putting $a = bk$ & $c = dk$ in above equation

$$= \frac{(bk)^3(dk)^3}{b^3d^3}$$

$$= \frac{b^3d^3k^3k^3}{b^3d^3}$$

$$= k^{3+3}$$

$$= k^6 \qquad \_\_\_\_\_\_(ii)$$

From eq(i) & eq(ii)

$L.H.S = R.H.S$

Hence proved that

$$\frac{a^6 + c^6}{b^6 + d^6} = \frac{a^3c^3}{b^3d^3}$$

Or, $a^6 + c^6 : b^6 + d^6 = a^3c^3 : b^3d^3$

(v). $\boldsymbol{p(a + b) + qb : p(c + d) + qd = a : c}$

Solution:

$p(a + b) + qb : p(c + d) + qd = a : c$

$$\frac{p(a + b) + qb}{p(c + d) + qd} = \frac{a}{c}$$

$a:b = c:d$

Let $\frac{a}{b} = \frac{c}{d} = k$

Then, $\frac{a}{b} = k \,; \frac{c}{d} = k$

Or, $a = bk \,; c = dk$

$$L.H.S = \frac{p(a+b)+qb}{p(c+d)+qd}$$

Putting $a = bk$ & $c = dk$ in above equation

$$= \frac{p(bk+b)+qb}{p(dk+d)+qd}$$

$$= \frac{pb(k+1)+qb}{pd(k+1)+qd}$$

$$= \frac{b(p(k+1)+q)}{d(p(k+1)+q)}$$

$$= \frac{b}{d} \qquad \_\_\_\_\_\_(i)$$

$$R.H.S = \frac{a}{c}$$

Putting $a = bk$ & $c = dk$ in above equation

$$= \frac{bk}{dk}$$

$$= \frac{b}{d} \qquad \_\_\_\_\_\_(ii)$$

From eq(i) & eq(ii)

$L.H.S = R.H.S$

Hence proved that

$$\frac{p(a+b)+qb}{p(c+d)+qd} = \frac{a}{c}$$

Or, $p(a+b)+qb : p(c+d)+qd = a:c$

(vi). $\boldsymbol{a^2+b^2:\frac{a^2}{a+b}=c^2+d^2:\frac{c^2}{c+d}}$

Solution:

$$a^2+b^2:\frac{a^2}{a+b}=c^2+d^2:\frac{c^2}{c+d}$$

$$\frac{a^2+b^2}{\frac{a^2}{a+b}}=\frac{c^2+d^2}{\frac{c^2}{c+d}}$$

$$\frac{(a^2+b^2)(a+b)}{a^2}=\frac{(c^2+d^2)(c+d)}{c^2}$$

$a:b=c:d$

Let $\frac{a}{b}=\frac{c}{d}=k$

Then, $\frac{a}{b}=k\,;\frac{c}{d}=k$

Or, $a=bk\,;c=dk$

$$L.H.S=\frac{(a^2+b^2)(a+b)}{a^2}$$

Putting $a=bk$ in above equation

$$=\frac{((bk)^2+b^2)((bk)+b)}{(bk)^2}$$

$$=\frac{(b^2k^2+b^2)(bk+b)}{b^2k^2}$$

$$=\frac{b^2(k^2+1)b(k+1)}{b^2k^2}$$

$$=\frac{b(k^2+1)(k+1)}{k^2} \qquad \text{\_\_\_\_\_\_\_}(i)$$

$$R.H.S=\frac{(c^2+d^2)(c+d)}{c^2}$$

Putting $c=dk$ in above equation

$$=\frac{((dk)^2+d^2)((dk)+d)}{(dk)^2}$$

$$=\frac{(d^2k^2+d^2)(dk+d)}{d^2k^2}$$

$$=\frac{d^2(k^2+1)d(k+1)}{d^2k^2}$$

$$=\frac{d(k^2+1)(k+1)}{k^2}$$

If $b=d$

$$=\frac{b(k^2+1)(k+1)}{k^2} \qquad \_\_\_\_\_\_\_(ii)$$

From eq(i) & eq(ii)

$L.H.S=R.H.S$

Hence proved that

$$\frac{(a^2+b^2)(a+b)}{a^2}=\frac{(c^2+d^2)(c+d)}{c^2}$$

Or, $a^2+b^2:\frac{a^2}{a+b}=c^2+d^2:\frac{c^2}{c+d}$

(vii).

$$\boldsymbol{\frac{a}{a-b}:\frac{a+b}{b}=\frac{c}{c-d}:\frac{c+d}{d}}$$

Solution:

$$\frac{a}{a-b}:\frac{a+b}{b}=\frac{c}{c-d}:\frac{c+d}{d}$$

$$\frac{\frac{a}{a-b}}{\frac{a+b}{b}}=\frac{\frac{c}{c-d}}{\frac{c+d}{d}}$$

$$\frac{ab}{(a+b)(a-b)}=\frac{cd}{(c-d)(c+d)}$$

$$\frac{ab}{a^2-b^2}=\frac{cd}{c^2-d^2}$$

$a:b=c:d$

Let $\frac{a}{b}=\frac{c}{d}=k$

Then, $\frac{a}{b}=k\,;\frac{c}{d}=k$

Or, $a=bk\,;c=dk$

$$L.H.S=\frac{ab}{a^2-b^2}$$

Putting $a=bk$ in above equation

$$L.H.S=\frac{(bk)(b)}{(bk)^2-b^2}$$

$$=\frac{b^2k}{b^2k-b^2}$$

$$=\frac{b^2k}{b^2(k-1)}$$

$$=\frac{k}{k-1}\qquad\_\_\_\_\_\_(i)$$

$$R.H.S=\frac{cd}{c^2-d^2}$$

Putting $c=dk$ in above equation

$$=\frac{(dk)(d)}{(dk)^2-d^2}$$

$$=\frac{d^2k}{d^2k-d^2}$$

$$=\frac{d^2k}{d^2(k-1)}$$

$$=\frac{k}{k-1}\qquad\_\_\_\_\_\_(ii)$$

From eq(i) & eq(ii)

$L.H.S = R.H.S$

Hence proved that

$$\frac{a}{a-b}:\frac{a+b}{b}=\frac{c}{c-d}:\frac{c+d}{d}$$

2. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}(a,b,c,d,e,f\neq 0)$, then show that

(i).

$$\frac{a}{b}=\sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}$$

Solution:

Let $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=k$

Then, $\frac{a}{b}=k\,;\frac{c}{d}=k;\frac{e}{f}=k$

Or, $a=bk\,;c=dk;e=fk$

$L.H.S=\frac{a}{b}$

Putting $a=bk$ in above equation

$$=\frac{bk}{b}$$

$$=k \qquad \_\_\_\_\_\_\_(i)$$

$$R.H.S=\sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}$$

Putting $a=bk\,,c=dk\,\&e=fk$ in above equation

$$=\sqrt{\frac{(bk)^2+(dk)^2+(fk)^2}{b^2+d^2+f^2}}$$

$$= \sqrt{\frac{b^2k^2 + d^2k^2 + f^2k^2}{b^2 + d^2 + f^2}}$$

$$= \sqrt{\frac{k^2(b^2 + d^2 + f^2)}{b^2 + d^2 + f^2}}$$

$$= \sqrt{k^2}$$

$$= k \qquad \_\_\_\_\_\_(ii)$$

From eq(i) & eq(ii)

$L.H.S = R.H.S$

Hence proved that

$$\frac{a}{b} = \sqrt{\frac{a^2 + c^2 + e^2}{b^2 + d^2 + f^2}}$$

(ii).

$$\boldsymbol{\frac{ac + ce + ea}{bd + df + fb} = \left[\frac{ace}{bdf}\right]^{2/3}}$$

Solution:

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

Then, $\frac{a}{b} = k\,; \frac{c}{d} = k; \frac{e}{f} = k$

Or, $a = bk\,; c = dk; e = fk$

$$L.H.S = \frac{ac + ce + ea}{bd + df + fb}$$

Putting $a = bk\,, c = dk\; \& e = fk$ in above equation

$$= \frac{(bk)(dk) + (dk)(fk) + (fk)(bk)}{bd + df + fb}$$

$$= \frac{bdk^2 + dfk^2 + fbk^2}{bd + df + fb}$$

$$= \frac{k^2(bd + df + fb)}{bd + df + fb}$$

$$= k^2 \qquad \_\_\_\_\_\_\_(i)$$

$$R.H.S = \left[\frac{ace}{bdf}\right]^{2/3}$$

Putting $a = bk$, $c = dk$ &$e = fk$ in above equation

$$= \left[\frac{(bk)(dk)(ek)}{bdf}\right]^{2/3}$$

$$= \left[\frac{bdfk^3}{bdf}\right]^{2/3}$$

$$= [k^3]^{2/3}$$

$$= k^{\frac{(3)(2)}{3}}$$

$$= k^2 \qquad \_\_\_\_\_\_\_(ii)$$

From eq(i) & eq(ii)

$$L.H.S = R.H.S$$

Hence proved that

$$\frac{ac + ce + ea}{bd + df + fb} = \left[\frac{ace}{bdf}\right]^{2/3}$$

(iii).

$$\boldsymbol{\frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{fb} = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}}$$

Solution:

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

Then, $\frac{a}{b} = k$; $\frac{c}{d} = k$; $\frac{e}{f} = k$

Or, $a = bk$; $c = dk$; $e = fk$

$L.H.S = \frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{fb}$

Putting $a = bk$, $c = dk$ & $e = fk$ in above equation

$$= \frac{(bk)(dk)}{bd} + \frac{(dk)(fk)}{df} + \frac{(fk)(bk)}{fb}$$

$$= \frac{bdk^2}{bd} + \frac{dfk^2}{df} + \frac{fbk^2}{fb}$$

$$= k^2 + k^2 + k^2$$

$$= 3k^2 \qquad \_\_\_\_\_\_(i)$$

$R.H.S = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$

Putting $a = bk$, $c = dk$ & $e = fk$ in above equation

$$= \frac{(bk)^2}{b^2} + \frac{(dk)^2}{d^2} + \frac{(fk)^2}{f^2}$$

$$= \frac{b^2k^2}{b^2} + \frac{d^2k^2}{d^2} + \frac{f^2k^2}{f^2}$$

$$= k^2 + k^2 + k^2$$

$$= 3k^2 \qquad \_\_\_\_\_\_(ii)$$

From eq(i) & eq(ii)

$L.H.S = R.H.S$

Hence proved that

$$\frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{fb} = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$$