1. Prove that $a:b = c:d$, if

(i).

$$\frac{4a+5b}{4a-5b} = \frac{4c+5d}{4c-5d}$$

Solution:

$$\frac{4a+5b}{4a-5b} = \frac{4c+5d}{4c-5d}$$

By componendo – dividendo theorem,

$$\frac{4a+5b+(4a-5b)}{(4a+5b)-(4a-5b)} = \frac{4c+5d+(4c-5d)}{4c+5d-(4c-5d)}$$

$$\frac{4a+5b+4a-5b}{4a+5b-4a+5b} = \frac{4c+5d+4c-5d}{4c+5d-4c+5d}$$

$$\frac{8a}{10b} = \frac{8c}{10d}$$

Multiplying by $\frac{10}{8}$ on both sides

$$\frac{a}{b} = \frac{c}{d}$$

$$a:b = c:d$$

Hence proved.

(ii).

$$\frac{2a+9b}{2a-9b} = \frac{2c+9d}{2c-9d}$$

Solution:

$$\frac{2a+9b}{2a-9b} = \frac{2c+9d}{2c-9d}$$

By componendo – dividendo theorem,

$$\frac{2a+9b+(2a-9b)}{2a+9b-(2a-9b)} = \frac{2c+9d+(2c-9d)}{2c+9d-(2c-9d)}$$

$$\frac{2a+9b+2a-9b}{2a+9b-2a+9b}=\frac{2c+9d+2c-9d}{2c+9d-2c+9d}$$

$$\frac{4a}{18b}=\frac{4c}{18d}$$

Multiplying by $\frac{18}{4}$ on both sides

$$\frac{a}{b}=\frac{c}{d}$$

$$a:b=c:d$$

Hence proved.

(iii).

$$\boldsymbol{\frac{ac^2+bd^2}{ac^2-bd^2}=\frac{c^3+d^3}{c^3-d^3}}$$

Solution:

$$\frac{ac^2+bd^2}{ac^2-bd^2}=\frac{c^3+d^3}{c^3-d^3}$$

By componendo – dividendo theorem,

$$\frac{ac^2+bd^2+(ac^2-bd^2)}{ac^2+bd^2-(ac^2-bd^2)}=\frac{c^3+d^3+(c^3-d^3)}{c^3+d^3-(c^3-d^3)}$$

$$\frac{ac^2+bd^2+ac^2-bd^2}{ac^2+bd^2-ac^2+bd^2}=\frac{c^3+d^3+c^3-d^3}{c^3+d^3-c^3+d^3}$$

$$\frac{2ac^2}{2bd^2}=\frac{2c^3}{2d^3}$$

$$\frac{ac^2}{bd^2}=\frac{c^3}{2d^3}$$

Multiplying by $\frac{d^2}{c^2}$ on both sides

$$\frac{a}{b}=\frac{c}{d}$$

$$a:b=c:d$$

Hence proved.

(iv).

$$\frac{a^2c+b^2d}{a^2c-b^2d}=\frac{ac^2+bd^2}{ac^2-bd^2}$$

Solution:

$$\frac{a^2c+b^2d}{a^2c-b^2d}=\frac{ac^2+bd^2}{ac^2-bd^2}$$

By componendo – dividendo theorem,

$$\frac{a^2c+b^2d+(a^2c-b^2d)}{a^2c+b^2d-(a^2c-b^2d)}=\frac{ac^2+bd^2+(ac^2-bd^2)}{ac^2+bd^2-(ac^2-bd^2)}$$

$$\frac{a^2c+b^2d+a^2c-b^2d}{a^2c+b^2d-a^2c+b^2d}=\frac{ac^2+bd^2+ac^2-bd^2}{ac^2+bd^2-ac^2+bd^2}$$

$$\frac{2a^2c}{2b^2d}=\frac{2ac^2}{2bd^2}$$

$$\frac{a^2c}{b^2d}=\frac{ac^2}{bd^2}$$

Multiplying by $\frac{bd}{ac}$ on both sides

$$\frac{a}{b}=\frac{c}{d}$$

$a:b=c:d$

Hence proved.

(v).

$$\frac{pa+qb}{pa-qb}=\frac{pc+qd}{pc-qd}$$

Solution:

$$\frac{pa+qb}{pa-qb}=\frac{pc+qd}{pc-qd}$$

By componendo – dividendo theorem,

$$\frac{pa+qb+(pa-qb)}{pa+qb-(pa-qb)}=\frac{pc+qd+(pc-qd)}{pc+qd-pc+qd}$$

$$\frac{pa+qb+pa-qb}{pa+qb-pa+qb}=\frac{pc+qd+pc-qd}{pc+qd-pc+qd}$$

$$\frac{2pa}{2qb}=\frac{2pc}{2qd}$$

$$\frac{pa}{qb}=\frac{pc}{qd}$$

Multiplying by $\frac{q}{p}$ on both sides

$$\frac{a}{b}=\frac{c}{d}$$

$$a:b=c:d$$

Hence proved.

(vi).

$$\boldsymbol{\frac{a+b+c+d}{a+b-c-d}=\frac{a-b+c-d}{a-b-c+d}}$$

Solution:

$$\frac{a+b+c+d}{a+b-c-d}=\frac{a-b+c-d}{a-b-c+d}$$

By componendo – dividendo theorem,

$$\frac{a+b+c+d+(a+b-c-d)}{a+b+c+d-(a+b-c-d)}=\frac{a-b+c-d+(a-b-c+d)}{a-b+c-d-(a-b-c+d)}$$

$$\frac{a+b+c+d+a+b-c-d}{a+b+c+d-a-b+c+d}=\frac{a-b-c-d+a-b-c+d}{a-b-c-d-a+b+c-d}$$

$$\frac{2a+2b}{2c+2d}=\frac{2a-2b}{2c-2d}$$

$$\frac{2(a+b)}{2(c+d)}=\frac{2(a-b)}{2(c-d)}$$

$$\frac{a+b}{c+d}=\frac{a-b}{c-d}$$

Or,

$$\frac{a+b}{a-b}=\frac{c+d}{c-d}$$

By componendo – dividendo theorem,

$$\frac{a+b+(a-b)}{a+b+(a-b)}=\frac{c+d+(c-d)}{c+d-(c-d)}$$

$$\frac{a+b+a-b}{a+b-a+b}=\frac{c+d+c-d}{c+d-c+d}$$

$$\frac{2a}{2b}=\frac{2c}{2d}$$

$$\frac{a}{b}=\frac{c}{d}$$

$$a:b=c:d$$

Hence proved.

(vii).

$$\boldsymbol{\frac{2a+3b+2c+3d}{2a+3b-2c-3d}=\frac{2a-3b+2c-3d}{2a-3b-2c+3d}}$$

Solution:

$$\frac{2a+3b+2c+3d}{2a+3b-2c-3d}=\frac{2a-3b+2c-3d}{2a-3b-2c+3d}$$

By componendo – dividendo theorem,

$$\frac{2a+3b+2c+3d+(2a+3b-2c-3d)}{2a+3b+2c+3d-(2a+3b-2c-3d)}=\frac{2a-3b+2c-2d+(2a-3b-2c+3d)}{2a-3b+2c-3d-(2a-3b-2c+3d)}$$

$$\frac{2a+3b+2c+3d+2a+3b-2c-3d}{2a+3b+2c+3d-2a-3b+2c+3d}=\frac{2a-3b-2c-3d+2a-3b-2c+3d}{2a-3b-2c-3d-2a+3b+2c-3d}$$

$$\frac{4a+6b}{4c+6d}=\frac{4a-6b}{4c-6d}$$

$$\frac{2(2a+3b)}{2(2c+3d)}=\frac{2(2a-3b)}{2(2c-3d)}$$

$$\frac{(2a+3b)}{(2c+3d)}=\frac{(2a-3b)}{(2c-3d)}$$

Or,

$$\frac{2a+3b}{2a-3b}=\frac{2c+3d}{2c-3d}$$

By componendo – dividendo theorem,

$$\frac{2a+3b+(2a-3b)}{2a+3b+(2a-3b)}=\frac{2c+3d+(2c-3d)}{2c+3d-(2c-3d)}$$

$$\frac{2a+3b+2a-3b}{2a+3b-2a+3b}=\frac{2c+3d+2c-3d}{2c+3d-2c+3d}$$

$$\frac{4a}{6b}=\frac{4c}{6d}$$

Multiplying by $\frac{6}{4}$ on both sides

$$\frac{a}{b}=\frac{c}{d}$$

$$a:b=c:d$$

Hence proved.

(viii).
$$\boldsymbol{\frac{a^2+b^2}{a^2-b^2}=\frac{ac+bd}{ac-bd}}$$

Solution:

$$\frac{a^2+b^2}{a^2-b^2}=\frac{ac+bd}{ac-bd}$$

By componendo – dividendo theorem,

$$\frac{a^2+b^2+(a^2-b^2)}{a^2+b^2-(a^2-b^2)}=\frac{ac+bd+(ac-bd)}{ac+bd-(ac-bd)}$$

$$\frac{a^2+b^2+a^2-b^2}{a^2+b^2-a^2+b^2}=\frac{ac+bd+ac-bd}{ac+bd-ac+bd}$$

$$\frac{2a^2}{2b^2}=\frac{2ac}{2bd}$$

$$\frac{a^2}{b^2} = \frac{ac}{bd}$$

Multiplying by $\frac{b}{a}$ on both sides

$$\frac{a}{b} = \frac{c}{d}$$

$$a:b = c:d$$

Hence proved.

2. Using theorem of componendo-dividendo

(i). Find the value of $\frac{x+2y}{x-2y} + \frac{x+2z}{x-2z}$, $if\ x = \frac{4yz}{y+z}$

Solution:

Since,

$$x = \frac{4yz}{y+z} \qquad \text{\_\_\_\_\_\_}(i)$$

$$x = \frac{(2y)(2z)}{y+z}$$

$$\frac{x}{2y} = \frac{2z}{y+z}$$

By componendo – dividendo theorem,

$$\frac{x+2y}{x-2y} = \frac{2z+(y+z)}{2z-(y+z)}$$

$$\frac{x+2y}{x-2y} = \frac{2z+y+z}{2z-y-z}$$

$$\frac{x+2y}{x-2y} = \frac{3z+y}{z-y} \qquad \text{\_\_\_\_\_\_}(ii)$$

Again from eq(i), we have

$$\frac{x}{2z} = \frac{2y}{y+z}$$

$$\frac{x+2z}{x-2z}=\frac{2y+(y+z)}{2y-(y+z)}$$

$$\frac{x+2z}{x-2z}=\frac{2y+y+z}{2y-y+z}$$

$$\frac{x+2z}{x-2z}=\frac{3y+z}{y+z} \qquad \_\_\_\_\_\_(iii)$$

Adding eq(ii) & eq(iii)

$$\frac{x+2y}{x-2y}+\frac{x+2z}{x-2z}=\frac{3z+y}{z-y}+\frac{3y+z}{y+z}$$

$$\frac{x+2y}{x-2y}+\frac{x+2z}{x-2z}=\frac{3z+y}{z-y}+\frac{3y+z}{-(z-y)}$$

$$\frac{x+2y}{x-2y}+\frac{x+2z}{x-2z}=\frac{3z+y}{z-y}-\frac{3y+z}{z-y}$$

$$\frac{x+2y}{x-2y}+\frac{x+2z}{x-2z}=\frac{3z+y-3y-z}{z-y}$$

$$\frac{x+2y}{x-2y}+\frac{x+2z}{x-2z}=\frac{2z-2y}{z-y}$$

$$\frac{x+2y}{x-2y}+\frac{x+2z}{x-2z}=\frac{2(z-y)}{z-y}$$

$$\frac{x+2y}{x-2y}+\frac{x+2z}{x-2z}=2$$

(ii). Find the value of $\frac{m+5n}{m-5n}+\frac{m+5p}{m-5p}, \text{if } m=\frac{10np}{n+p}$

Solution:

Since,

$$m=\frac{10np}{n+p} \qquad \_\_\_\_\_\_(i)$$

$$m=\frac{(5n)(2p)}{n+p}$$

$$\frac{m}{5n}=\frac{2p}{n+p}$$

By componendo – dividendo theorem,

$$\frac{m+5n}{m-5n}=\frac{\left(2p+(n+p)\right)}{2p-(n+p)}$$

$$\frac{m+5n}{m-5n}=\frac{2p+n+p}{2p-n-p}$$

$$\frac{m+5n}{m-5n}=\frac{3p+n}{p-n} \qquad \text{\_\_\_\_\_\_}(ii)$$

Again from eq(i), we have

$$\frac{m}{5p}=\frac{2n}{n+p}$$

By componendo – dividendo theorem,

$$\frac{m+5p}{m-5p}=\frac{2n+(n+p)}{2n-(n+p)}$$

$$\frac{m+5p}{m-5p}=\frac{2n+n+p}{2n-n-p}$$

$$\frac{m+5p}{m-5p}=\frac{3n+p}{n-p} \qquad \text{\_\_\_\_\_\_}(iii)$$

Adding eq(ii) & eq(iii)

$$\frac{m+5n}{m-5n}+\frac{m+5p}{m-5p}=\frac{3p+n}{p-n}+\frac{3n+p}{n-p}$$

$$=\frac{3p+n}{p-n}+\frac{3n+p}{-(p-n)}$$

$$=\frac{3p+n}{p-n}-\frac{3n+p}{p-n}$$

$$=\frac{3p+n-3n-p}{p-n}$$

$$=\frac{2p-2n}{p-n}$$

$$=\frac{2(p-n)}{p-n}$$

$$\frac{m+5n}{m-5n}+\frac{m+5p}{m-5p}=2$$

(iii). Find the value of $\frac{x-6a}{x+6a}-\frac{x+6b}{x-6b}, if\ x=\frac{12ab}{a-b}$.

Solution:

Since,

$$x=\frac{12ab}{a-b} \qquad \_\_\_\_\_\_(i)$$

$$x=\frac{(2b)(6a)}{a-b}$$

$$\frac{x}{6a}=\frac{2b}{a-b}$$

By componendo – dividendo theorem,

$$\frac{x-6a}{x+6b}=\frac{2b-(a-b)}{2b+(a-b)}$$

$$\frac{x+6a}{x-6b}=\frac{2b-a+b}{2b+a-b}$$

$$\frac{x+6a}{x-6b}=\frac{3b-a}{a+b} \qquad \_\_\_\_\_\_(ii)$$

Again from eq(i)

$$\frac{x}{6b}=\frac{2a}{a-b}$$

$$\frac{x+6b}{x-6b}=\frac{2a+(a-b)}{2a-(a-b)}$$

$$\frac{x+6b}{x-6b}=\frac{2a+a-b}{2a-a+b}$$

$$\frac{x+6b}{x-6b}=\frac{3a-b}{a+b} \qquad \_\_\_\_\_\_(iii)$$

Subtracting eq(ii) & eq(iii)

$$\frac{x+6a}{x-6b}-\frac{x+6b}{x-6b}=\frac{3b-a}{a+b}-\frac{3a+b}{a+b}$$

$$=\frac{3b-a-3a-b}{a+b}$$

$$=\frac{4b-4a}{a+b}$$

$$=\frac{4(b-a)}{a+b}$$

(iv). Find the value of $\frac{x-3y}{x+3y}-\frac{x+3z}{x-3z}, if\ x=\frac{3yz}{y-z}$

Solution:

$$x=\frac{3yz}{y-z} \qquad \_\_\_\_\_\_(i)$$

$$\frac{x}{3y}=\frac{z}{y-z}$$

By componendo – dividendo theorem,

$$\frac{x-3y}{x+3y}=\frac{z-(y-z)}{z+(y-z)}$$

$$\frac{x-3y}{x+3y}=\frac{z-y+z}{z+y-z}$$

$$\frac{x-3y}{x+3y}=\frac{2z-y}{y} \qquad \_\_\_\_\_\_(ii)$$

Again from eq(i),

$$\frac{x}{3z}=\frac{y}{y-z}$$

By componendo – dividendo theorem,

$$\frac{x+3z}{x-3z}=\frac{y+(y-z)}{y-(y-z)}$$

$$\frac{x+3z}{x-3z}=\frac{y+y-z}{y-y+z}$$

$$\frac{x+3z}{x-3z}=\frac{2y-z}{z} \qquad \_\_\_\_\_\_(iii)$$

Subtracting eq(ii) & eq(iii),

$$\frac{x-3y}{x+3y}-\frac{x+3z}{x-3z}=\frac{2z-y}{y}-\frac{2y-z}{z}$$

$$=\frac{z(2z-y)-y(2y-z)}{yz}$$

$$=\frac{2z^2-yz-2y^2+yz}{yz}$$

$$=\frac{2z^2-2y^2}{yz}$$

$$=\frac{2(z^2-y^2)}{yz}$$

(v). Find the value of $\frac{s-3p}{s+3p}+\frac{s+3q}{s-3q}, if\ s=\frac{6pq}{p-q}$.

Solution:

$$s=\frac{6pq}{p-q} \qquad \_\_\_\_\_\_(i)$$

$$s=\frac{(3p)(2q)}{p-q}$$

$$\frac{s}{3p}=\frac{2q}{p-q}$$

By componendo – dividendo theorem,

$$\frac{s-3p}{s+3p}=\frac{2q-(p-q)}{2q+(p-q)}$$

$$\frac{s-3p}{s+3p}=\frac{2q-p+q}{2q+p-q}$$

$$\frac{s-3p}{s+3p}=\frac{3q-p}{p+q} \qquad \_\_\_\_\_\_(ii)$$

Again from eq(ii),

$$\frac{s}{3q}=\frac{2p}{p-q}$$

$$\frac{s+3q}{s-3q}=\frac{2p+(p-q)}{2p-(p-q)}$$

$$\frac{s+3q}{s-3q}=\frac{2p+p-q}{2p-p+q}$$

$$\frac{s+3q}{s-3q}=\frac{3p-q}{p+q} \qquad \_\_\_\_\_\_(iii)$$

Adding eq(i) & eq(ii)

$$\frac{s-3p}{s+3p}+\frac{s+3q}{s-3q}=\frac{3q-p}{p+q}+\frac{3p-q}{p+q}$$

$$=\frac{3q-p+3p-q}{p+q}$$

$$=\frac{2p+2q}{p+q}$$

$$=\frac{2(p+q)}{p+q}$$

$$=2$$

(vi). Solve $\frac{(x-2)^2-(x-4)^2}{(x-2)^2+(x-4)^2}=\frac{12}{13}$

Solution:

$$\frac{(x-2)^2-(x-4)^2}{(x-2)^2+(x-4)^2}=\frac{12}{13}$$

By componendo – dividendo theorem,

$$\frac{(x-2)^2-(x-4)^2+((x-2)^2+(x-4)^2)}{(x-2)^2-(x-4)^2-((x-2)^2+(x-4)^2)}=\frac{12+13}{12-13}$$

$$\frac{(x-2)^2-(x-4)^2+(x-2)^2+(x-4)^2}{(x-2)^2-(x-4)^2-(x-2)^2-(x-4)^2}=\frac{25}{-1}$$

$$\frac{2(x-2)^2}{-2(x-4)^2}=-25$$

$$\frac{(x-2)^2}{(x-4)^2}=25$$

$$\frac{(x-2)^2}{(x-4)^2}=(5)^2$$

Taking square root on both sides

$$\sqrt{\frac{(x-2)^2}{(x-4)^2}}=\sqrt{(5)^2}$$

$$\frac{x-2}{x-4}=\pm 5$$

$\frac{x-2}{x-4}=5$; $\frac{x-2}{x-4}=-5$

$x-2=5(x-4)$; $x-2=-5(x-4)$

$x-2=5x-20$; $x-2=-5x+20$

$x-5x=-20+2$; $x+5x=20+2$

$-4x=-18$; $6x=22$

$x=\frac{-18}{-4}$; $x=\frac{22}{6}$

$x=\frac{9}{2}$; $x=\frac{11}{3}$

So, solution set is $\left\{\frac{9}{2},\frac{11}{3}\right\}$

(vii). Solve $\frac{\sqrt{x^2+2}+\sqrt{x^2-2}}{\sqrt{x^2+2}-\sqrt{x^2-2}}=2$

Solution:

$$\frac{\sqrt{x^2+2}+\sqrt{x^2-2}}{\sqrt{x^2+2}-\sqrt{x^2-2}}=2$$

By componendo – dividendo theorem,

$$\frac{\sqrt{x^2+2}+\sqrt{x^2-2}+(\sqrt{x^2+2}-\sqrt{x^2-2})}{\sqrt{x^2+2}+\sqrt{x^2-2}-(\sqrt{x^2+2}-\sqrt{x^2-2})}=\frac{2+1}{2-1}$$

$$\frac{\sqrt{x^2+2}+\sqrt{x^2-2}+\sqrt{x^2+2}-\sqrt{x^2-2}}{\sqrt{x^2+2}+\sqrt{x^2-2}-\sqrt{x^2+2}+\sqrt{x^2-2}}=\frac{3}{1}$$

$$\frac{2\sqrt{x^2+2}}{2\sqrt{x^2-2}}=3$$

$$\frac{\sqrt{x^2+2}}{\sqrt{x^2-2}}=3$$

Taking square on both sides

$$\left(\frac{\sqrt{x^2+2}}{\sqrt{x^2-2}}\right)^2=(3)^2$$

$$\frac{x^2+2}{x^2-2}=9$$

$$x^2+2=9(x^2-2)$$

$$x^2+2=9x^2-18$$

$$x^2-9x^2=-18-2$$

$$-8x^2=-20$$

$$x^2=\frac{-20}{-8}$$

$$x^2=\frac{5}{2}$$

Taking square root on both sides

$$\sqrt{x^2} = \sqrt{\frac{5}{2}}$$

$$x = \pm\sqrt{\frac{5}{2}}$$

Since solution $x = \pm\sqrt{\frac{5}{2}}$ does not satisfy the given equation, so $x = \pm\sqrt{\frac{5}{2}}$ is an extraneous root.

Solution set is $\{x = \pm\sqrt{\frac{5}{2}}$ Extraneous root, $\emptyset\}$

(viii). Solve $\frac{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}}{\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}} = \frac{1}{3}$

Solution:

$$\frac{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}}{\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}} = \frac{1}{3}$$

By componendo – dividendo theorem,

$$\frac{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}+(\sqrt{x^2+8p^2}+\sqrt{x^2-p^2})}{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}-(\sqrt{x^2+8p^2}+\sqrt{x^2-p^2})} = \frac{1}{3}$$

$$\frac{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}+\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}}{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}-\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}} = \frac{1+3}{1-3}$$

$$\frac{2\sqrt{x^2+8p^2}}{-2\sqrt{x^2-p^2}} = \frac{4}{-2}$$

$$\frac{\sqrt{x^2+8p^2}}{\sqrt{x^2-p^2}} = 2$$

Taking square on both sides

$$\left(\frac{\sqrt{x^2+8p^2}}{\sqrt{x^2-p^2}}\right)^2 = (2)^2$$

$$\frac{x^2+8p^2}{x^2-p^2}=4$$

$$x^2+8p^2=4(x^2-p^2)$$

$$x^2+8p^2=4x^2-4p^2$$

$$x^2-4x^2=-4p^2-8p^2$$

$$-3x^2=-12p^2$$

$$x^2=\frac{-12p^2}{-3}$$

$$x^2=4p^2$$

$$x^2=(2p)^2$$

Taking square root on both sides

$$\sqrt{x^2}=\sqrt{(2p)^2}$$

$$x=\pm 2p$$

So, solution set is $\{2p,-2p\}$

(ix). Solve $\frac{(x+5)^3-(x-3)^3}{(x+5)^3+(x-3)^3}=\frac{13}{14}$

Solution:

$$\frac{(x+5)^3-(x-3)^3+((x+5)^3+(x-3)^3)}{(x+5)^3-(x-3)^3-((x+5)^3+(x-3)^3)}=\frac{13+14}{13-14}$$

$$\frac{(x+5)^3-(x-3)^3+(x+5)^3+(x-3)^3}{(x+5)^3-(x-3)^3-(x+5)^3-(x-3)^3}=\frac{27}{-1}$$

$$\frac{2(x+5)^3}{-2(x-3)^3}=\frac{27}{-1}$$

$$\frac{(x+5)^3}{(x-3)^3}=27$$

$$\frac{(x+5)^3}{(x-3)^3}=(3)^3$$

Taking cube root on both sides

$$\sqrt[3]{\frac{(x+5)^3}{(x-3)^3}} = \sqrt[3]{(3)^3}$$

$$\frac{x+5}{x-3} = 3$$

$$x+5 = 3(x-3)$$

$$x+5 = 3x-9$$

$$x-3x = -9+5$$

$$-2x = -14$$

$$x = \frac{-14}{-2}$$

$$x = 7$$

So, solution set is $\{7\}$