Resolve into partial fractions.

Question N0.1

$$\frac{7x-9}{(x+1)(x-3)}$$

Solution:

Let $\frac{7x-9}{(x+1)(x-3)} = \frac{A}{(x+1)} + \frac{B}{(x-3)} \to (i)$

Multiplying equation (i) by $(x+1)(x-3)$

$7x-9 = A(x-3) + B(x+1) \to (ii)$

For determine the value of A and B

⇨ $x-3=0 \Rightarrow x=3$ and $x+1=0 \Rightarrow x=-1$

putting $x=3$ and $x=-1$ put in eq(ii) we get

For $x=3$

$7(3)-9 = B(3+1)$

$21-9=4B$

$12=4B$

$\Rightarrow \boxed{B=3}$

For $x=-1$

$7(-1)-9 = A(-1-3)$

$-7-9=-4A$

$-16=-4A$

$\Rightarrow \boxed{A=4}$

Putting the value of A and B in equation (i) We get the required partial fractions

$$\frac{4}{(x+1)} + \frac{3}{(x-3)}$$

Thus

$$\frac{7x-9}{(x+1)(x-3)} = \frac{4}{(x+1)} + \frac{3}{(x-3)}$$

Question No.2

$$\frac{x-11}{(x-4)(x+3)}$$

Solution:

Let $\frac{x-11}{(x-4)(x+3)} = \frac{A}{(x-4)} + \frac{B}{(x+3)} \to (i)$

Multiplying equation (i) by $(x-4)(x+3)$ on Both sides, we get

$x-11 = A(x+3) + B(x-4) \to (ii)$

$\Rightarrow x+3=0 \Rightarrow x=-3$ and $x-4=0 \Rightarrow x=4$

putting $x=-3$ and $x=4$ put in eq(ii) we get

For $x=-3$

$-3-11 = B(-3-4)$

$-14=-7B$

$\Rightarrow \boxed{B=2}$

For $x=4$

$4-11 = A(4+3)$

$-7=7A$

$\Rightarrow \boxed{A=-1}$

Putting the value of A and B in equation (i) We get the required partial fractions

$$\frac{-1}{(x-4)} + \frac{2}{(x+3)}$$

Hence the required partial fraction are

$$\frac{x-11}{(x-4)(x+3)} = \frac{-1}{(x-4)} + \frac{2}{(x+3)}$$

Question N0.3

$$\frac{3x-1}{x^2-1}$$

Solution:

$$\frac{3x-1}{x^2-1} = \frac{3x-1}{(x-1)(x+1)}$$

Let $\frac{3x-1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} \to (i)$

Multiplying both sides by $(x-1)(x+1)$

we get $3x-1 = A(x+1) + B(x-1) \to (ii)$

Let $x+1=0$ i.e $x=-1$ and $x-1=0$ i.e $x=1$

putting $x=-1$ and $x=1$ put in eq(ii) we get

For $x=1$

$3(1)-1 = A(1+1)$

$3-1=2A$

$2=2A$

$\Rightarrow \boxed{A=1}$

For $x=-1$

$3(-1)-1 = B(-1-1)$

$-3-1=-2B$

$-4=-2B$

$\Rightarrow \boxed{B=2}$

Hence the required partial fraction are

$$\frac{3x-1}{(x-1)(x+1)} = \frac{1}{x-1} + \frac{2}{x+1}$$

Question No.4

$$\frac{x-5}{x^2+2x-3}$$

Solution:

$$\frac{x-5}{x^2+2x-3} = \frac{x-5}{x^2+3x-x-3}$$

$$= \frac{x-5}{x(x+3)-1(x+3)} = \frac{x-5}{(x-1)(x+3)}$$

$$\frac{x-5}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3} \to (ii)$$

Multiplying both sides by $(x-1)(x+3)$, *we get*

$x-5 = A(x+3) + B(x-1) \to (ii)$

Let $x+3=0 \Rightarrow x=-3$ and $x-1=0 \Rightarrow x=1$

putting $x=-3$ and $x=1$ in equation(ii) we get

For $x=-3$

$-3-5 = +B(-3-1)$

$-8=-4B$

$B = \frac{-8}{-4}$

$\Rightarrow \boxed{B=2}$

For $x=1$

$1-5 = A(1+3)$

$-4=4A$

$A = \frac{-4}{4}$

$\Rightarrow \boxed{A=-1}$

Hence the required partial fractions are

$$\frac{x-5}{(x-1)(x+3)} = \frac{-1}{x-1} + \frac{2}{x+3}$$

Question No.5

$$\frac{3x+3}{(x-1)(x+2)}$$

Solution:

$$Let\ \frac{3x+3}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2} \quad \rightarrow (i)$$

Multiplying both sides by $(x-1)(x+2)$ *we get*

$$3x+3 = A(x+2) + B(x-1) \rightarrow (ii)$$

Let $x-1=0$ *i.e* $x=1$ *and* $x+2=0$ *i.e* $x=-2$

Putting $x=1$ *and* $x=-2$ *in equation (ii)*

For $x = 1$	For $x = -2$
$3(1)+3 = A(1+2)$	$3(-2)+3 = B(-2-1)$
$3+3 = 3A$	$-6+3 = B(-2)$
$6 = 3A$	$-6+3 = -3B$
$\frac{6}{3} = A$	$B = \frac{-3}{-3}$
$\Rightarrow$ $\boxed{A = 2}$	$\Rightarrow$ $\boxed{A = 1}$

Hence the required partial fractions are

$$\frac{3x+3}{(x-1)(x+2)} = \frac{2}{x-1} + \frac{1}{x+2}$$

Question N0.6

$$\frac{7x-25}{(x-4)(x-3)}$$

Solution:

$$Let\ \frac{7x-25}{(x-4)(x-3)} = \frac{A}{x-4} + \frac{B}{x-3}$$

Multiplying both sides by $x-4)(x-3)$, *we get*

$$7x-25 = A(x-3) + B(x-4) \rightarrow (ii)$$

Let $x-3=0$ *i.e* $x=3$ *and* $x-4=0$ *i.e* $x=4$

Putting $x=3$ *and* $x=4$ *in equation (ii)we get*

For $x = 3$	For $x = 4$
$7(3)-25 = B(3-4)$	$7(4)-25 = A(4-3)$
$21-25 = -B$	$28-25 = 1A$
$-4 = -B$	$3 = A$
$\Rightarrow$ $\boxed{B = 4}$	$\Rightarrow$ $\boxed{A = 3}$

Hence the required partial fractions are

$$\frac{7x-25}{(x-4)(x-3)} = \frac{3}{x-4} + \frac{4}{x-3}$$

Question No.7

$$\boldsymbol{\frac{x^2+2x+1}{(x-2)(x+3)}}$$

Solution:

$\frac{x^2+2x+1}{(x-2)(x+3}$ *is an important fraction.*

First we resolve it into proper fraction.

By log division we get

$$\begin{array}{r} 1 \\ x^2+x-6\overline{)\,x^2+2x+1} \\ \underline{\pm x^2 \pm x \mp 1} \\ x+7 \end{array}$$

We have $\frac{x^2+2x+1}{x62+x-6} = 1 + \frac{x+7}{x^2+x-6}$

$$Let\ \frac{x+7}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3} \rightarrow (i)$$

Multiplying both sides by $(x-2)(x+3)$ *we get*

$$x+7 = A(x+3) + B(x-2) \rightarrow (ii)$$

Let $x+3=0$ *i.e* $x=-3$

and $x-2=0$ *i.e* $x=2$

For $x = -3$	For $x = 2$
$-3+7 = B(-3-2)$	$2+7 = A(2+3)$
$4 = -5B$	$9 = 5A$
$\Rightarrow$ $\boxed{B = -\frac{4}{5}}$	$\Rightarrow$ $\boxed{A = \frac{9}{5}}$

Hence the required partial fractions are

$$\frac{x^2+2x+1}{x62+x-6} = 1 + \frac{9}{5(x-2)} - \frac{4}{5(x+3)}$$

Question No.8

$$\boldsymbol{\frac{6x^3+5x^2-7}{3x^2-2x-1}}$$

Solution:

$\frac{6x^3+5x^2-7}{3x^2-2x-1}$ is an improper fraction.

First we resolve in to proper fraction.

$$\begin{array}{r} 2x+3 \\ 3x^2-2x-1\overline{)\,6x^3+5x^2-7} \\ \underline{\pm 6x^3 \pm 4x^2 \mp 2x} \\ 9x^2+2x-7 \\ \underline{\pm 9x^2 \mp 6x \mp 3} \\ 8x-4 \end{array}$$

$$\frac{6x^3+5x^2-7}{3x^2-2x-1} = (2x+3) + \frac{8x-4}{(3x+1)(x-1)}$$

Now, Let $\frac{8x-4}{(3x+1)(x-1)} = \frac{A}{3x+1} + \frac{B}{x-1}$

Multiplying both sides by $(3x+1)(x-1$, *we get*

$$8x-4 = A(x-1) + B(3x+1) \rightarrow (ii)$$

Let $x-1=0$ *i.e* $x=1$

and $3x+1=0$ *i.e* $x=-\frac{1}{3}$

putting $x=1$ *and* $x=-\frac{1}{3}$ *in equation (ii)*

we get

For $x = 1$	For $x = -\frac{1}{3}$
$8(1)-4 = B[3(1)+1]$	$8\left(-\frac{1}{3}\right)-4 = A\left(-\frac{1}{3}-1\right)$
$8-4 = 4B$	$-\frac{8}{3}-4 = A\left(\frac{-1-3}{3}\right)$
$4 = 4B$	$\frac{-8-12}{3} = \frac{A(-4)}{3}$
⇨ $4B = 4$	$-\frac{20}{3} = \frac{A(-4)}{3}$
$B = \frac{4}{4}$	
$\Rightarrow$ $\boxed{B = 1}$	$\Rightarrow$ $\boxed{A = 5}$

Hence the required of a fraction when $D(x)$ *consists*

Of repeated linear factors are

$$\frac{6x^3+5x^2-7}{3x^2-2x-1} = 2x+3 + \frac{5}{3x+1} + \frac{1}{x-1}$$

Exercise 4.2

Resolve into partial fractions:

Question No.1

$$\frac{\boldsymbol{x^2-3x+1}}{\boldsymbol{(x-1)^2(x-2)}}$$

Solution:

$$Let\ \frac{x^2-3x+1}{(x-1)^2(x-2)}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x-2}\to(i)$$

Multiplying both sides by $(x-1)^2(x-2)$ $we\ get$

$$x^2-3x+1=A(x-1)(x-2)+B(x-2)+C(x-1)^2\to(ii)$$

$$x^2-3x+1=A(x^2-3x+2)+B(x-2)+C(x^2-2x+1)$$

$$Putting\ x-1=0\ i.e x=1\ in\ (ii) we\ get$$

$$(1)^2-3(1)+1=(1-2)$$

$$1-3+1=B(-1)$$

$$-1=-B$$

$$\Rightarrow \boxed{B=1}$$

$$Putting\ x-2=0\ i.e\ x=2\ in\ (ii) we\ get$$

$$(2)^2-3+1=C(2-1)^2$$

$$4-6+1=C$$

$$-1=C\Rightarrow -1$$

$$\Rightarrow \boxed{C=-1}$$

Equating the coefficient of x^2 in (ii) we get

$$1=A+C$$

$$1=A-1$$

$$\Rightarrow A=1+1$$

$$\Rightarrow \boxed{A=2}$$

Hence the required partial fractions are

$$\frac{x^2-3x+1}{(x-1)^2(x-2)}=\frac{2}{x-1}+\frac{1}{(x-1)^2}+\frac{1}{x-2}$$

Question No.2

$$\frac{\boldsymbol{x^2+7x+11}}{\boldsymbol{(x+2)^2(x+3)}}$$

Solution:

$$Let\ \frac{x^2+7x+11}{(x+2)^2(x+3)}=\frac{A}{x+2}+\frac{B}{(x+2)^2}+\frac{C}{x+3}\to(i)$$

Multiplying both sides by $(x+2)^2(x+3)$

$$\Rightarrow x^2+7x+11=A(x+2)(x+3)+B(x+3)+C(x+2)^2$$

$$\Rightarrow x^2+7x+11=A(x^2+5x+6)+B(x+3)+C(x^2+4x+4)\to(ii)$$

$$Putting\ x+2=0\ i.e x=-2\ in(ii) we\ get$$

$$(-2)^2+7(-2)+11=B(-2+3)$$

$$4-14+11=B$$

$$\Rightarrow \boxed{B=1}$$

Putting $x+3=0\ i.e\ x=-3\ in(ii) we\ get$

$$(-3)^2+7(-3)+11=C(-3+2)^2$$

$$9-21+11=C(-1)^2$$

$$20-21=C(1)$$

$$-1=C$$

$$\Rightarrow \boxed{C=-1}$$

Equating coefficient of x^2 in (ii) we get

$$A+C=1$$

$$A-1=1$$

$$A=1+1$$

$$\Rightarrow \boxed{A=2}$$

Hence the required partial fractions are

$$\frac{x^2+7x+11}{(x+2)^2(x+3)}=\frac{2}{x+2}+\frac{1}{(x+2)^2}-\frac{1}{x+3}$$

Question No.3

$$\frac{\boldsymbol{9}}{\boldsymbol{(x-1)(x+2)^2}}$$

Solution:

$$Let\ \frac{9}{(x-1)(x+2)^2}=\frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{(x+2)^2}\to(i)$$

Multiplying both sides by $(x-1)(x+2)^2$ $we\ get$

$$9=A(x+2)^2+B(x-1)(x+2)+C(x-1)\to(ii)$$

$$Putting\ x-1=0\ i.e\ x=1\ in(ii) we\ get$$

$$9=A(1+2)^2$$

$$9=A(3)^2$$

$$9=9A$$

$$\Rightarrow \boxed{A=1}$$

$$Putting\ x+2=0\ i.e\ x=-2\ in\ (ii) we\ get$$

$$9=C(-2-1)$$

$$9=-3C$$

$$\Rightarrow \boxed{C=-3}$$

Equating the coefficient of x^2 in (ii) we get

$$A+B=0$$

$$B=-A$$

$$\Rightarrow \boxed{B=-1}$$

Hence the partial fractions are

$$\frac{9}{(x-1)(x+2)^2}=\frac{1}{x-1}-\frac{1}{x+2}+\frac{3}{(x+2)^2}$$

Question No.4

$$\frac{\boldsymbol{x^4+1}}{\boldsymbol{x^2(x-1)}}$$

Solution:

$$\frac{x^4+1}{x^2(x-1)}=\frac{x^4+1}{x^3-x^2}\ is\ an\ improper\ fraction.$$

$$First\ we\ resolve\ it\ into\ proper\ fraction.$$

$$x^3-x^2\ \overline{\big)\,x^4+1}$$

$$\underline{\pm x^4\quad \mp x^3}$$

$$x^3+1$$

$$\underline{\pm x^3\quad \mp x^2}$$

$$x^3+1$$

$$\frac{x^4+1}{x^2(x-1)}=(x+1)+\frac{x^2+1}{x^2(x-1)}\to(i)$$

$$Let\ \frac{x^2+1}{x^2(x-1)}=(x+1)+\frac{x^2+1}{x^2(x-1)}\to(ii)$$

Multiplying both sides by $x^2(x-1)$we get

$$x^2+1=A(x)(x-1)+B(x-1)+cx^2\to(iii)$$

Puttting $x = 0$ in (iii) we get

$$0 + 1 = B(0 - 1)$$
$$1 = -B$$
$$\Rightarrow \boxed{B = 1}$$

Putting $x - 1 = 0$ *i.e* $x = 1$ *in (iii) we get*

$$(1)^2 + 1 = C(1)^2$$
$$1 + 1 = C(1)$$
$$2 = C$$
$$\Rightarrow \boxed{C = 2}$$

Equating the coefficient of x^2 *in (iii)* we get

$$A + C = 1$$
$$A + 2 = 1$$
$$A = 1 - 2$$
$$\Rightarrow \boxed{A = -1}$$

Putting the value of A, B, C *in equation (ii)*

Thus required partial fraction are

$$\frac{x^4 + 1}{x^2(x - 1)} = (x + 1) - \frac{1}{x} - \frac{1}{x^2} + \frac{2}{x - 1}$$

Question No.5

$$\frac{\mathbf{7x + 4}}{\mathbf{(3x + 2)(x + 1)^2}}$$

Solution:

$$Let\ \frac{7x + 4}{(3x + 2)(x + 1)^2} = \frac{A}{3x + 2} + \frac{B}{x + 1} + \frac{C}{(x + 1)^2} \rightarrow (i)$$

Multiplying both sides by $(3x + 2)(x + 1)^2$ *we get*

$$7x + 4 = A(x + 1)^2 + B(3x + 2)(x + 1) + C(3x + 2) \rightarrow (ii)$$

Putting $3x + 2 = 0$ i.e $x = -\frac{2}{3}$ in (ii) we get

$$7\left(-\frac{2}{3}\right) + 4 = A\left(-\frac{2}{3} + 1\right)^2$$
$$-\frac{14}{3} + 4 = A\left(\frac{-2 + 3}{3}\right)^2$$
$$\frac{-14 + 12}{3} = A\left(\frac{1}{3}\right)^2$$
$$-\frac{2}{3} = \frac{1}{9}A$$
$$-18 = 3A$$
$$A = -\frac{18}{3}$$
$$\Rightarrow \boxed{A = -6}$$

Putting $x + 1 = 0$ *i.e* $x = -1$ *in (ii) we get*

$$7(-1) + 4 = C(3(-1) + (+2)$$
$$-7 + 4 = -C$$
$$\Rightarrow\ -3 = -C$$
$$\Rightarrow \boxed{C = 3}$$

Equating the value of A, B and C in equation (i) we get required partial fractions.

$$\frac{7x + 4}{(3x + 2)(x + 1)^2} = \frac{-6}{3x + 2} + \frac{2}{x + 1} + \frac{3}{(x + 1)^2}$$

Question No.6

$$\frac{\mathbf{1}}{\mathbf{(x - 1)^2(x + 1)}}$$

Solution:

$$Let\ \frac{1}{(x - 1)^2(x + 1)} = \frac{A}{x - 1} + \frac{B}{(x - 1)^2} + \frac{C}{x + 1} \rightarrow (ii)$$

Multiplying both sides by $(x - 1)(x - 1)^2$ *we get*

$$1 = A(x - 1)(x + 1) + B(x + 1) + C(x - 1)^2 \rightarrow (ii)$$

Putting $x - 1 = 0$ i.s $x = 1$ in (ii) we get

$$1 = B(1 + 1)$$
$$1 = 2B$$
$$\Rightarrow \boxed{B = \frac{1}{2}}$$

Putting $x + 1 = 0$ i.e $x = -1$ in (ii) we get

$$1 = C(-1 - 1)^2$$
$$1 = C(-1 - 1)^2$$
$$1 = C(-2)^2$$
$$1 = 4C$$
$$\Rightarrow \boxed{C = \frac{1}{4}}$$

Equating the coefficient of x^2 *in (ii) we get*

$$A + C = 0$$
$$A = -C$$
$$A = -\left(\frac{1}{4}\right)$$
$$\Rightarrow \boxed{A = -\frac{1}{4}}$$

Putting the value of

$A, B,$ and C in equation (i) we got required partial fractions

$$\frac{1}{(x - 1)^2(x + 1)} = \frac{-1}{4(x - 1)} + \frac{1}{2(x - 1)^2} + \frac{1}{4(x + 1)}$$

Question No.7

$$\frac{\mathbf{3x^2 + 15x + 16}}{\mathbf{(x + 2)^2}}$$

Solution:

$$\frac{3x^2 + 15x + 16}{(x + 2)^2} = \frac{3x^2 + 15x + 16}{x^2 + 4x + 4}$$

The given fraction is improper fraction.

By long division,

$$\begin{array}{r} 3 \\ x^2 + 4x + 4\ \overline{)\ 3x^2 + 15x + 16} \\ \pm 3x^2 \pm 12x \pm 12 \\ \hline 3x + 4 \end{array}$$

$$\frac{3x^2 + 15x + 16}{(x + 2)^2} = 3 + \frac{3x + 4}{x^2 + 4x + 4} \rightarrow (i)$$

$$Let\ \frac{3x + 4}{(x + 2)^2} = \frac{A}{x + 2} + \frac{B}{(x + 2)^2} \rightarrow (ii)$$

Multiplying both sides by $(x + 2)^2$ *we get*

$$3x + 4 = A(x + 2) + B \rightarrow (iii)$$

Putting $x + 2 = 0$ i.e $x = -2$ in (iii) we get

$$3(-2) + 4 = B \Rightarrow -6 + 4$$

$$\Rightarrow \boxed{B = -2}$$

Equating the coefficient of "x" we get

$$3 = A$$

$$\Rightarrow \boxed{A = 3}$$

Putting the value of A and B *in equation* (ii) and *using* eq.(i) *we get required partial fractions.*

$$\frac{3x^2+15x+16}{(x+2)^2} = 3 + \frac{3}{x+2} - \frac{2}{(x+2)^2}$$

Question No.8

$$\frac{\mathbf{1}}{(\boldsymbol{x^2} - \mathbf{1})(\boldsymbol{x} + \mathbf{1})}$$

Solution:

$$\frac{1}{(x^2-1)(x+1)} = \frac{1}{(x-1)(x+1)(x+1)}$$

$$= \frac{1}{(x-1)(x+1)^2}$$

$$Let\ \frac{1}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

Multiplying both sides by $(x-1)(x+1)^2$ *we get*

$$1 = A(x+1)^2 + B(x+1)(x-1) + C(x-1) \rightarrow (ii)$$

Putting $x - 1 = 0$ *i.e* $x = 1$ *in* (ii) *we get*

$$1 = A(1+1)^2$$

$$1 = A(2)^2$$

$$1 = 4A$$

$$\Rightarrow \boxed{A = \frac{1}{4}}$$

Putting $x + 1 = o$ *i.e* $x = -1$ *in* (ii) *we get*

$$1 = C(-1-1)$$

$$1 = -2C$$

$$\Rightarrow \boxed{C = \frac{-1}{2}}$$

Equating the coefficient of x^2 *in equation* (ii)

We get $A + B = 0$

$$B = -A$$

$$B = -\left(\frac{1}{4}\right)$$

$$\Rightarrow \boxed{B = -\frac{1}{4}}$$

Putting the value of A *and* B *in equation* (ii)

We get required partial fractions.

$$\frac{1}{(x-1)(x+1)^2} = \frac{1}{4(x-1)} - \frac{1}{4(x+1)} - \frac{1}{2(x+1)^2}$$

Exercise 4.3

Resolve into partial fraction.

Question No.1

$$\frac{\mathbf{3x-11}}{\mathbf{(x+3)(x^2+1)}}$$

Solution:

$$\frac{\mathbf{3x-11}}{\mathbf{(x+3)(x^2+1)}}$$

$$Let\ \frac{\mathbf{3x-11}}{\mathbf{(x+3)(x^2+1)}}=\frac{\mathbf{A}}{\mathbf{x+3}}+\frac{\mathbf{Bx+C}}{x^2+1}$$

Multiplying both sides $(x-3)(x^2+1), we\ get$

$$3x-11=A(x^2+1)+(Bx+C)(x+3)\rightarrow(i)$$

$$3x-11=A(x^2+1)+Bx(x+3)+C(x+3)\rightarrow(ii)$$

$$Putting\ x+3=0\ \ i.e\ x=-3, we\ get$$

$$3(-3)-11=A[(-3)^2+1]$$

$$-9-11=A(9+1)$$

$$-20=10A$$

$$A=\frac{-20}{10}$$

$$\Rightarrow\ \boxed{A=-2}$$

Now equating the coefficient of x^2 and x we get from equation (iii)

$$A+B=0$$
$$-2+B=0$$
$$B=2$$
$$\Rightarrow\ \boxed{B=1}$$

$$3B+C=3$$
$$3(2)+C=3$$
$$6+C=3$$
$$C=3-6$$
$$\Rightarrow\ \boxed{C=-3}$$

Putting the value $of\ A,B\ and\ C\ in\ equation\ (i)we\ get$ Required partial fractions.

$$\frac{3x-11}{(x+3)(x^2+1)}=\frac{-2}{x+3}+\frac{2x-3}{x^2+1}$$

Question No.2

$$\frac{\mathbf{3x+7}}{\mathbf{(x^2+1)(x+3)}}$$

Solution:

$$Let\ \frac{\mathbf{3x+7}}{\mathbf{(x^2+1)(x+3)}}=\frac{\mathbf{Ax+B}}{\mathbf{x^2+1}}+\frac{\mathbf{C}}{\mathbf{x+3}}\rightarrow\mathbf{(ii)}$$

Multiplying both sides by $x^2+1)(x+3)$

$$3x+7=(Ax+B)(x+3)+C(x^2+1)$$

$$3x+7=Ax(x+3)+B(x+3)+C(x^2+1)\rightarrow(ii)$$

$$Putting\ x+3=0\ \ i.e\ x=-3\ in\ (ii)we\ get$$

$$3(-3)+7=C[(-3)^2+1]$$

$$-9+7=C(9+1)$$

$$-2=10C$$

$$\Rightarrow\ C=-\frac{2}{10}$$

$$\Rightarrow\ \boxed{C=\frac{-1}{5}}$$

Now equation the coefficient of $x^2\ and\ x$ in equation (iii) we get.

$$A+C=0$$
$$A+\left(\frac{-1}{5}\right)=0$$
$$A-\frac{1}{5}=0$$
$$\Rightarrow\ \boxed{A=\frac{1}{5}}$$

$$3A+B=3$$
$$3\left(\frac{1}{5}\right)+B=3$$
$$B=3-\frac{3}{5}$$
$$B=3-\frac{3}{5}$$
$$B=\frac{15-3}{5}$$
$$B=\frac{12}{5}$$
$$\Rightarrow\ \boxed{C=-3}$$

Putting the value of $A,B\ and\ C\ in\ equation\ (i)\ we\ get\ required\ \ partial\ fraction.$

$$\frac{3x+7}{(x^2+1)(x+3)}=\frac{x+12}{5(x^2+1)}-\frac{1}{5(x+3)}$$

Question No.3

$$\frac{\mathbf{1}}{\mathbf{(x+1)(x^2+1)}}$$

Solution:

$$Let\ \frac{1}{(x+1)(x^2+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2+1}\rightarrow(ii)$$

Multiplying both sides by $(x+1)(x^2+1)we\ get$

$$1=A(x^2+1)+(Bx+C)(x+1)$$

$$1=A(x^2+1)+Bx(x+1)+C(x+1)\rightarrow(ii)$$

$$putting\ \ x+1=0\ i.e\ x=-1\ in\ (ii)we\ get$$

$$1=A[(-1)^2+1]$$

$$1=A(1+1)$$

$$1=2A$$

$$\Rightarrow\ \boxed{A=\frac{1}{2}}$$

Equation the coefficients of $x^2\ and\ x\ in\ equation\ (ii)$ We get

$$A+B=0$$
$$\frac{1}{2}+B=0$$
$$B=-\frac{1}{2}$$

$$B+C=0$$
$$-\frac{1}{2}+C=0$$
$$C=\frac{1}{2}$$

$\Rightarrow$ $\boxed{B = -\frac{1}{2}}$ $\quad\Rightarrow$ $\boxed{C = \frac{1}{2}}$

Putting the value of A, B and C in equations (i) we
Get required partial fractions.

$$\frac{1}{(x+1)(x^2+1)} = \frac{1}{2(x+1)} + \frac{x-1}{2(x^2+1)}$$

Question No.4

$$\mathbf{\frac{9x-7}{(x+3)(x^2+1)}}$$

Solution:

$$Let\ \frac{9x-7}{(x+3)(x^2+1)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+1} \rightarrow (i)$$

Multiplying both sides by $(x+3)(x^2+1)$ we get

$$9x-7 = A(x^2+1) + (Bx+C)(x^2+1)$$
$$9x-7 = A(x^2+1) + Bx(x+3) + C(x+3) \rightarrow (ii)$$

Putting $x+3=0$ i.e $x=-3$ in (ii) we get

$$9(-3)-7 = A[(-3)^2+1]$$
$$-27-7 = A(9+1)$$
$$-34 = 10A$$

$\Rightarrow$ $A = \frac{-34}{10}$

$\Rightarrow$ $\boxed{A = \frac{-17}{5}}$

Equating coefficient of x^2 and x in equation (ii) we get

$$A+B=0$$
$$\frac{-17}{5}+B=0$$
$$B=\frac{17}{5}$$

$\Rightarrow$ $\boxed{B = \frac{-17}{5}}$

$$3B+C=9$$
$$3\left(\frac{17}{2}\right)+C=9$$
$$\frac{51}{5}+C=9$$
$$C=9-\frac{51}{5}$$
$$C=\frac{45-51}{5}$$

$\Rightarrow$ $\boxed{C = \frac{-6}{5}}$

Putting the value of A, B and C in equation (i) we get required partial fraction.

$$\frac{9x-7}{(x+3)(x^2+1)} = \frac{-17}{5(x+3)} + \frac{17x-6}{5(x^2+1)}$$

Question No.5

$$\mathbf{\frac{3x+7}{(x+3)(x^2+4)}}$$

Solution:

$$Let\ \frac{3x+7}{(x+3)(x^2+4)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+4} \rightarrow (i)$$

Multiplying both sides by $(x+3)(x^2+4)$ we get

$$3x+7 = A(x^2+4) + (Bx+C)(x+3)$$
$$3x+7 = A(x^2+4) + Bx(x+3) + C(x+3) \rightarrow (ii)$$

Putting $x+3=0$ i.e $x=-3$ in (ii) we get

$$3(-3)+7 = A((-3)^2+4)$$
$$-9+7 = A(9+4)$$
$$-2 = 13A$$

$\Rightarrow$ $\boxed{A = \frac{-2}{13}}$

Equating the coefficient of x^2 and x in equation (ii) we get

$$A+B=0$$
$$\frac{-2}{13}+B=0$$
$$B=\frac{2}{13}$$

$\Rightarrow$ $\boxed{B = \frac{2}{13}}$

$$3B+C=3$$
$$3\left(\frac{2}{13}\right)+C=3$$
$$\frac{6}{13}+C=9$$
$$C=3-\frac{6}{13}$$
$$C=\frac{39-6}{13}$$

$\Rightarrow$ $\boxed{C = \frac{33}{13}}$

Putting the value of A, B and C in equation (i) we get
Required partial fractions.

$$\frac{3x+7}{(x+3)(x^2+4)} = \frac{-2}{13(x+3)} + \frac{2x+33}{13(x^2+4)}$$

Question No.6

$$\mathbf{\frac{x^2}{(x+2)(x^2+4)}}$$

Solution:

$$Let \frac{x^2}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+4} \rightarrow (i)$$

Multiplying both sides by $(x+2)(x^2+4)$ we get

$$x^2 = A(x^2+4) + (Bx+C)(x+2)$$
$$x^2 = A(x^2+4) + Bx(x+2) + C(x+2) \rightarrow (ii)$$

Putting $x+2=0$ i.e $x=-2$ in (ii) we get

$$(-2)^2 = A[(-2)^2+4]$$
$$4 = A(4+A)$$
$$4 = 8+A$$

$\Rightarrow$ $\boxed{A = \frac{1}{2}}$

Equating the coefficients of x^3 $and\ x\ in\ equation\ (ii)$
We get

$$A+B=1 \qquad\qquad 2B+C=0$$
$$\frac{1}{2}+B=1 \qquad\qquad 2\left(\frac{1}{2}\right)+C=0$$
$$B=1-\frac{1}{2} \qquad\qquad 1+C=0$$
$$C=0-1=-1$$
$$\Rightarrow \boxed{B=\frac{1}{2}} \qquad \Rightarrow \boxed{C=-1}$$

Putting the value of $A, B\ and\ C\ in\ equation\ (i) we\ get$
Required partial fractions.

Putting $x+1=0\ \ i.e\ x=-1\ in\ (ii) we\ get$

$$1=A[(-1)^2-(-1)+1]$$
$$1=A[(-1)^2-1(-1)+1]$$
$$1=A(1+1+1)$$
$$1=3A$$
$$\Rightarrow \boxed{A=\frac{1}{3}}$$

Comparing the coefficients of
$x^2\ and\ x\ in\ equation\ (ii) we\ get$

$$A+B=0 \qquad\qquad -A+B+C=0$$
$$\frac{1}{3}+B=0 \qquad\qquad \left(-\frac{1}{3}\right)-\frac{1}{3}+C=0$$
$$B=\frac{-1}{3} \qquad\qquad -\frac{2}{3}+C=0$$
$$\Rightarrow \boxed{B=\frac{-1}{3}} \qquad \Rightarrow \boxed{C=\frac{2}{3}}$$

Putting the value of $A, and\ B\ and\ C\ in\ equation\ (i) we\ get$
Required partial fractions.

$$\frac{1}{(x+1)(x^2-x+1)}=\frac{1}{3(x+1)}-\frac{x-2}{3(x^2-x+1)}$$

Question No.8

$$\frac{\boldsymbol{x^2+1}}{\boldsymbol{x^3+1}}$$

Solution:

$$\frac{x^2+1}{[x^3+1}=\frac{x^2+1}{(x+1)(x^2-x+1)}$$

$$\frac{x^2}{(x+2)(x^2+4)}=\frac{1}{2(x+2)}+\frac{x-2}{2(x^2+4)}$$

Question No.7

$$\frac{\boldsymbol{1}}{\boldsymbol{x^3+1}} \qquad \left[\boldsymbol{Hint}: \frac{\boldsymbol{1}}{\boldsymbol{x^3+1}}=\frac{\boldsymbol{1}}{\boldsymbol{(x+1)(x^2-x+1)}}\right]$$

Solution:

$$Let\ \frac{1}{(x+1)(x^2-x+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2-x+1}\rightarrow(i)$$

Multiplying both sides by $(x-1)(x^2-x+1), we\ get$

$$1=A(x^2-x+1)+(Bx+C)(x+1)$$
$$1=A(x^2-x+1)+Bx(x+1)+c(x+1)\rightarrow(ii)$$

$$Let\ \frac{x^2+1}{(x+1)(x^2-x+1)}=\frac{A}{x+1}+\frac{Bx+C}{x^2-x+1}\rightarrow(i)$$

Multiplying both sides by $(x+1)(x^2-x+1), we\ get$

$$x^2+1=A(x^2-x+1)+(Bx+C)(x+1)$$
$$x^2+1=A(x^2-x+1)+Bx(x+1)+C(x+1)\rightarrow(ii)$$

Putting $x+1=0\ i.e\ x=-1\ in\ (ii) we\ get$

$$(-1)^2+1=A[(-1)^2-(-1)+1]$$
$$1+1=A(1+1+1)$$
$$2=3A$$
$$\Rightarrow \boxed{A=\frac{2}{3}}$$

Equating the coefficients of
$x^2\ and\ x\ in\ equation\ (ii) we\ get$

$$A+B=1 \qquad\qquad -A+B+C=0$$
$$\frac{2}{3}+B=1 \qquad\qquad \left(\frac{-2}{3}\right)+\frac{1}{3}+C=0$$
$$B=1-\frac{2}{3} \qquad\qquad \frac{-1}{3}+C=0$$
$$\Rightarrow \boxed{B=\frac{1}{3}} \qquad \Rightarrow \boxed{C=\frac{1}{3}}$$

Putting the value of $A, B\ and\ C\ in\ equation\ (i)\ we\ get$
$required\ partial\ fractions.$

$$\frac{x^2+1}{(x+1)(x^2-x+1)}=\frac{2}{3(x+1)}+\frac{x+1}{3(x^2-x+1)}$$

Exercise 4.4

Resolve into Partial Fractions.

Question No.1

$$\frac{\boldsymbol{x^3}}{\boldsymbol{(x^2+4)^2}}$$

Solution:

$$Let\ \frac{x^3}{(x^2+4)^2}=\frac{Ax+B}{x^2+4}+\frac{Cx+D}{(x^2+4)^2}\rightarrow(i)$$

$$Multiplying\ bothsides\ by\ (x^2+4)^2, we\ get$$

$$x^3=(Ax+B)(x^2+4)+(Cx+D)$$

$$x^3=Ax(x^2+4)+B(x^2+4)+(Cx+D)\rightarrow(iii)$$

Equating the coefficients of $x^3, x^2, x\ and\ constant,$ we get

Coefficients $of\ \ x^3$: $A=1$

Coefficients of x^2: $B=0$

Coefficients of x: $4A+C=0$

⇨ $C=-4$

Constants:$4B+D=0$

$$4(0)+D=0$$

⇨ $D=0$

Putting the value of $A, B, and\ C\ in\ equation\ (i)$

We get required partial fractions.

$$\frac{x^3}{(x^2+4)^2}=\frac{x}{x^2+4}-\frac{4x}{(x^2+4)^2}$$

Question No.2

$$\frac{\boldsymbol{x^4+3x^2+x+1}}{\boldsymbol{(x+1)(x^2+1)^2}}$$

Solution:

$$Let\ \frac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2}=\frac{A}{x+1}+\frac{Bx+C}{x^2+1}+\frac{Dx+E}{(x^2+1)^2}\rightarrow(\boldsymbol{i})$$

Multiplying both sides by $(\boldsymbol{x}+\boldsymbol{1})(\boldsymbol{x^2}+\boldsymbol{1})^2$ *we get*

$$x^4+3x^2+x+1=A(x^2+1)^2+(Bx+C)(x+1)(x^2+1)+(Dx+E)(x+1)\rightarrow(ii)$$

$$x^4+3x^2+x+1=A(x^4+2x^2+1)$$
$$+Bx(x^3+x^2+x+1)+C(x^3+x^2+x+1)$$
$$+Dx(x+1)+E(x+1)$$

$$x^4+3x^2+x+1=A(x^4+2x^2+1)$$
$$+B(x^4+x^3+x^2+x)+C(x^3+x^2+x+1)$$
$$D(x^2+x)+E(x+1)\rightarrow(iii)$$

Putting $x+1=0\ i.e\ x=-1\ in\ eq(ii) we\ get$

$$(-1)^4+3(-1)^2+(-1)+1=A[(-1)^2+1]^2$$

$$1+3(1)-1+1=A(1+1)^2$$

$$4=4A$$

⇨ $A=1$

Now equating the coefficients of $x^4, x^3, x^2, x\ and$

$$constants, we\ get\ from\ equation\ (iii)$$

$$coefficients\ of\ x^4:\quad A+B=1$$

$$1+B=1$$

$$B=1-1$$

$$\Rightarrow \boxed{B=0}$$

Coefficients of x^3: $B+C=0$

$$0+C=0$$

$$\Rightarrow \boxed{C=0}$$

Coefficients of x^2: $2A+B+C+D=3$

$$2(1)+0+0+D=3$$

$$D=3-2$$

$$D=1$$

$$Coefficients\ of\ x:\ B+C+D+E=1$$

$$0+0+1+E=1$$

$$E=1-1$$

$$\Rightarrow \boxed{E=0}$$

Putting the value $of\ A, B, C\ and\ D\ in\ equation\ (i)$

We get required partial fractions.

$$Let\ \frac{x^4+3x^2+x+1}{(x+1)(x^2+1)^2}=\frac{1}{x+1}+\frac{x}{(x^2+1)^2}$$

Question No.3

$$\frac{\boldsymbol{x^2}}{\boldsymbol{(x+1)(x^2+1)^2}}$$

Solution:

$$Let\ \frac{\boldsymbol{x^2}}{\boldsymbol{(x+1)(x^2+1)^2}}=\frac{A}{x+1}+\frac{Bx+C}{x^2+1}+\frac{Dx+E}{(x^2+1)^2}\rightarrow(\boldsymbol{i})$$

Multiplying both sides by $(\boldsymbol{x}+\boldsymbol{1})(\boldsymbol{x^2}+\boldsymbol{1})^2$ *we get*

$$x^2=A(x^2+1)^2+(Bx+C)(x+1)(x^2+1)+(Dx+E)(x+1)\rightarrow(ii)$$

$$x^2=A(x^4+2x^2+1)+Bx(x^3+x^2+x+1)+$$
$$C(x^3+x^2+x+1)+Dx(x+1)+E(x+1)$$

$$x^2=A(x^4+2x^2+1)+B(x^4+x^3+x^2+x)$$
$$+C(x^3+x^2+x+1)+D(x^2+x)+E(x+1)\rightarrow(iii)$$

Putting $x+1=0\ i.e\ x=-1\ in\ eq(ii) we\ get$

$$(-1)^2=A[(-1)^2+1]^2$$

$$1=A(1+1)^2$$

$$1=4A$$

$$\Rightarrow \boxed{A=\frac{1}{4}}$$

Now equating the coefficients of x^4, x^3, x^2, x *and constants, we get from equation* (iii)

coefficients of x^4: $A + B = 0$

$$\frac{1}{4} + B = 1$$

$$\Rightarrow \boxed{B = -\frac{1}{4}}$$

Coefficients of x^3: $B + C = 0$

$$-\frac{1}{4} + C = 0$$

$$\Rightarrow \boxed{C = \frac{1}{4}}$$

Coefficients of x^2: $2A + B + C + D = 1$

$$2\left(\frac{1}{4}\right) - \frac{1}{4} + \frac{1}{4} + D = 1$$

$$\frac{1}{2} + D = 1$$

$$D = 1 - \frac{1}{2}$$

$$\Rightarrow D = \frac{2-1}{2}$$

$$\Rightarrow \boxed{D = \frac{1}{2}}$$

Coefficients of x: $B + C + D + E = 0$

$$-\frac{1}{4} + \frac{1}{2} + \frac{1}{2} + E = 0$$

$$\frac{1}{2} + E = 0$$

$$\Rightarrow \boxed{E = -\frac{1}{2}}$$

Putting the value *of* A, B, C *and* D *in equation* (i)
We get required partial fractions.

$$\frac{x^2}{(x+1)(x^2+1)^2} = \frac{A}{4(x+1)} + \frac{x-1}{4(x^2+1)} + \frac{x-1}{2(x^2+1)^2}$$

Question No.4

$$\frac{x^2}{(x-1)(x^2+1)^2}$$

Solution:

$$\frac{x^2}{(x-1)(x^2+1)^2} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2} \rightarrow (i)$$

Multiplying both sides by $(x+1)(x^2+1)^2$ *we get*

$$x^2 = A(x^2+1)^2 + (Bx+C)(x-1)(x^2+1) + (Dx+E)(x-1) \rightarrow (ii)$$

$$x^2 = A(x^4+2x^2+1) + Bx(x-1)(x^2+1) + C(x-1)(x^2+1) + Dx(x-1) + E(x-1)$$

$$x^2 = A(x^4+2x^2+1) + B(x^4 - x^3 + x^2 - x)$$

$$+C(x^3 - x^2 + x - 1) + D(x^2 - x) + E(x-1) \rightarrow (iii)$$

Putting $x - 1 = 0$ *i.e* $x = 1$ *in eq*(ii)*we get*

$$(1)^2 = A[(1)^2+1]^2$$

$$1 = A(1+1)^2$$

$$1 = 4A$$

$$\Rightarrow \boxed{A = \frac{1}{4}}$$

Now equating the coefficients of x^4, x^3, x^2, x *and constants, we get from equation* (iii)

coefficients of x^4: $A + B = 0$

$$\frac{1}{4} + B = 1$$

$$\Rightarrow \boxed{B = -\frac{1}{4}}$$

Coefficients of x^3: $B + C = 0$

$$-\left(-\frac{1}{4}\right) + C = 0$$

$$\Rightarrow \boxed{C = -\frac{1}{4}}$$

Coefficients of x^2: $2A + B - C + D = 1$

$$2\left(\frac{1}{4}\right) - \frac{1}{4} - \left(-\frac{1}{4}\right) + D = 1$$

$$\frac{1}{2} - \frac{1}{4} + \frac{1}{4} + D = 1$$

$$D = 1 - \frac{1}{2}$$

$$\Rightarrow D = \frac{2-1}{2}$$

$$\Rightarrow \boxed{D = \frac{1}{2}}$$

Coefficients of x: $-B + C - D + E = 0$

$$-\left(-\frac{1}{4}\right) - \frac{1}{4} - \frac{1}{2} + E = 0$$

$$\frac{1}{4} - \frac{1}{4} - \frac{1}{2} + E = 0$$

$$-\frac{1}{2} + E = 0$$

$$\Rightarrow \boxed{E = \frac{1}{2}}$$

Putting the value *of* A, B, C *and* D *in equation* (i)
We get required partial fractions.

$$\frac{x^2}{(x-1)(x^2+1)^2} = \frac{1}{4(x+1)} + \frac{x+1}{4(x^2+1)} + \frac{x+1}{2(x^2+1)^2}$$

Question No.5

$$\frac{x^4}{(x^2+2)^2}$$

Solution:

$$\frac{x^4}{(x^2+2)^2} = \frac{x^4}{x^4+4x^2+4} \text{ is an improper fraction}$$

First we resolve it into proper fraction

$$\begin{array}{r} 1 \\ x^4+4x^2+4\overline{)\,x^4} \\ \underline{\pm x^4 \pm 4x^2 \pm 4} \\ -4x^2-4 \end{array}$$

$$-4x^2-4 = (Ax+B)(x^2+2)+(Cx+D)$$

$$-4x^2-4 = A(x^3+2x)+B(x^2+2)+Cx+D \rightarrow (ii)$$

Equating the coefficients of x^3, x^2, x and constants

In equation (ii) we get

Coefficients of x^2: $B=-4$

Coefficients of x: $2A+C=0$

$$2(0)+C=0$$

Constants : $2B+D=-4$

$$2(-4)+D=-4$$
$$-8+D=-4$$
$$D=8-4$$
$$D=4$$

Putting the value of A, B, C and D in equation (i) we get required partial fractions.

$$\frac{x^4}{(x^2+2)^2} = 1+\frac{-4}{x^2+2}+\frac{4}{(x^2+2)^2}$$

$$\frac{x^4}{(x^2+2)^2} = 1-\frac{4}{x^2+2}+\frac{4}{(x^2+2)^2}$$

Question No.6

$$\boldsymbol{\frac{x^5}{(x^2+1)^2}}$$

Solution:

$$\frac{x^5}{(x^2+1)^2} = \frac{x^5}{x^4+2x^2+1} \text{ is an improper fraction.}$$

First we resolve it into proper fraction.

$$\frac{x^4}{(x^2+2)^2} = 1+\frac{-4x^2-4}{(x^2+2)^2}$$

$$\text{Let } \frac{-4x^2-4}{(x^2+2)^2} = \frac{Ax+B}{x^2+2}+\frac{Cx+D}{(x^2+2)^2} \rightarrow (i)$$

Multiplying both sides by $(x^2+2)^2$ we get

$$\begin{array}{r} x \\ x^4+2x^2+1\overline{)\,x^5} \\ \underline{\pm x^5 \pm 2x^3 \pm x} \\ -2x^3-x \end{array}$$

$$\frac{x^5}{(x^2+1)^2} = x+\frac{-2x^3-x}{(x^2+1)^2}$$

$$\text{Let } \frac{-2x^3-x}{(x^2+1)^2} = \frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2} \rightarrow (i)$$

Multiplying both sides by $(x^2+1)^2$ we get

$$-2x^3-x = (Ax+B)(x^2+1)+(Cx+D)$$

$$-2x-x = A(x^3+x)+B(x^2+1)+Cx+D$$

Equating the coefficients of x^3, x^2, x and constants

We get

Coefficients of x^3: $A=-2$

Coefficients of x^2: $B=0$

Coefficients of x: $A+C=-1$

$$-2+C=-1$$
$$C=-1+2$$
$$\Rightarrow \boxed{C=1}$$

Constants: $B+D=0$

$$0+D=0$$
$$\Rightarrow \boxed{D=0}$$

Hence the required partial fractions are

$$\frac{x^5}{(x^2+1)^2} = x+\frac{-2x}{x^2+1}+\frac{x}{(x^2+1)^2}$$

$$\frac{x^5}{(x^2+1)^2} = x-\frac{2x}{x^2+1}+\frac{x}{(x^2+1)^2}$$