

Exercise 3.1

Question No.1

Express the following as ratio $a : b$ and as a fraction in its simplest form.

i. Rs.750 , Rs.1250

$$750 : 1250$$

$$\frac{750}{10} : \frac{1250}{10} \quad \text{divide by 10}$$

$$\frac{75}{5} : \frac{125}{5} \quad \text{divide by 5}$$

$$\frac{15}{5} : \frac{25}{5} \quad \text{divide by 5}$$

$$3 : 5$$

$$\frac{3}{5}$$

ii. 450cm, 3m

$$450\text{cm} : 3 \times 100\text{cm}$$

$$\because 1\text{m} = 100\text{cm}$$

$$\frac{450}{10} = \frac{300}{10} \quad \text{divide by 10}$$

$$\frac{45}{5} : \frac{30}{5} \quad \text{divide by 5}$$

$$\frac{9}{3} : \frac{6}{3} \quad \text{divide by 3}$$

iii. 4kg, 2kg 750gm

$$4 \times 1000\text{gm} : 2 \times 1000\text{gm} + 750\text{gm}$$

$$\because 1\text{kg} = 1000\text{gm}$$

$$4000\text{gm} : 2000\text{gm} + 750\text{gm}$$

$$4000\text{gm} : 2750\text{gm}$$

$$2000\text{gm} : 1375\text{gm} \quad \text{divide by 2}$$

$$16\text{gm} : 11\text{gm} \quad \text{divide by 125}$$

$$\frac{16\text{gm}}{11\text{gm}} = \frac{16}{11}$$

iv.

27min. 30 sec, 1 hour

$$\left(\begin{array}{l} 1650\text{sec} : 3600\text{sec} \\ 165\text{sec} : 360\text{sec} \\ 11\text{sec} : 24\text{sec} \\ \frac{11\text{sec}}{24\text{sec}} \end{array} \right) \quad \because$$

$$\frac{11}{24} \text{ Ans.}$$

$$1\text{hour} = 60\text{min}$$

$$1\text{min} = 60\text{sec}$$

$$27\text{min} = 27 \times 60 = 1620\text{s}$$

$$27\text{min}30\text{sec} = 1620 + 30 = 1650$$

v. $75^\circ, 225^\circ$

$$75^\circ : 225^\circ$$

$$3^\circ : 9^\circ \quad \text{Dividing by 25}$$

$$1^\circ : 3^\circ \quad \text{Dividing by 3}$$

$$\frac{1}{3} = 1:3$$

Question No.2

In a class of 60 students, 25 students are girls and remaining students are boys compute the ratio of

$$\text{Total students} = 60$$

$$\text{Girls} = 25$$

$$\text{boys} = 60 - 25 = 35$$

i. Boys to total students

$$\text{Boys} : \text{Total students}$$

$$35 : 60 \quad \text{divide by 5}$$

$$7 : 12$$

ii. Boys to girls

$$\text{Boys} : \text{Girls}$$

$$35 : 25 \quad \text{Divide by 5}$$

$$7 : 5$$

Question No.3

if $3(4x - 5y) = 2x - 7y$, find the ratio $x : y$

Solution:

$$3(4x - 5y) = 2x - 7y$$

$$12x - 15y = 2x - 7y$$

$$\Rightarrow 12x - 2x = -7y + 15y$$

$$\Rightarrow 10x = 8y$$

$$\Rightarrow 5x = 4y \quad \text{divide by 2}$$

$$\frac{x}{y} = \frac{4}{5}$$

$$x : y = 4 : 5$$

Question No.4

Find the value of p , if the ratio $2p + 5 : 3p + 4$ and $3 : 4$ are equal.

Solution:

$$\frac{2p+5}{3p+4} = \frac{3}{4}$$

$$4(2p + 5) = 3(3p + 4)$$

$$8p + 20 = 9p + 12$$

$$20 - 12 = 9p - 8p$$

$$8 = p$$

Question No.5

if the ratio $3x + 1 : 6 + 4x$ and $2 : 5$ are equal. find the value of x .

Solution:

$$\frac{3x + 1}{6 + 4x} = \frac{2}{5}$$

$$5(3x + 1) = 2(6 + 4x)$$

$$15x + 5 = 12 + 8x$$

$$15x - 8x = 12 - 5$$

$$7x = 7$$

$$x = \frac{7}{7}$$

$$x = 1$$

Question No.6

Two numbers are in ratio

$5 : 8$. if 9 is added to each number, we get a new ratio $8 : 11$ find the numbers.

Solution:

Let a number x

According to the given condition

$$\frac{5x+9}{8x+9} = \frac{8}{11}$$

$$\Rightarrow 11(5x + 9) = 8(8x + 9)$$

$$\Rightarrow 55x + 99 = 64x + 72$$

$$\Rightarrow 99 - 72 = 64x - 55x$$

$$\Rightarrow 27 = 9x$$

$$\Rightarrow x = 3$$

$$\text{Now } 5x = 5(3)$$

$$8x = 8(3) = 24$$

Hence required numbers are 15 and 24.

Question No.7

If 10 is added in each number of the ratio

$4 : 13$ we get $1 : 2$.

What are the numbers?

Solution:

Let a number x

According to the given Condition

$$\frac{4x + 10}{13x + 10} = \frac{1}{2}$$

$$\Rightarrow 2(4x + 10) = 1(13x + 10)$$

$$\Rightarrow 8x + 20 = 13x + 10$$

$$\Rightarrow 20 - 10 = 13x - 8x$$

$$10 = 5x$$

$$x = \frac{10}{5} = 2$$

$$x = 2$$

$$\text{Now 1st number} = 4x = 4(2) = 8$$

$$\text{2nd number} = 13x = 13(2) = 26$$

Question No.8

Find the cost of 8kg of mangoes, if 5kg of mangoes cost Rs. 250

Solution:

Let the cost of 8kg of mangoes be x - rupees.

$$8\text{kg} : 5\text{kg} :: \text{Rs. } x : \text{Rs. } 250$$

$$8\text{kg} : 5\text{kg} = \text{Rs. } x : \text{Rs. } 250$$

product of extremes = product of means

$$8 \times 250 = 5x$$

$$\frac{8 \times 250}{5} = x$$

$$x = 400\text{Rs.}$$

Question No.9

If $a : b = 7 : 6$, find the value of $3a + 5b : 7b - 5a$

5a

Solution:

As given that $a : b = 7 : 6$ or

$$\frac{a}{b} = \frac{7}{6}$$

Now

$$3a + 5b : 7b : 5a = \frac{3a + 5b}{7b - 5a}$$

Dividing numerators and denominator by b

$$\frac{3a + 5b}{7b - 5a} = \frac{3\left(\frac{a}{b}\right) + 5\left(\frac{b}{b}\right)}{7\left(\frac{b}{b}\right) - 5\left(\frac{a}{b}\right)} = \frac{3\left(\frac{a}{b}\right) + 5}{7 - 5\left(\frac{a}{b}\right)}$$

$$\because \frac{a}{b} = \frac{7}{6}$$

$$\text{so } \frac{3\left(\frac{7}{6}\right) + 5}{7 - 5\left(\frac{7}{6}\right)} = \frac{\frac{21}{6} + 5}{7 - \frac{35}{6}} = \frac{\frac{21 + 30}{6}}{\frac{42 - 35}{6}}$$

$$\frac{\frac{51}{6}}{\frac{7}{6}} = \frac{51}{6} \times \frac{6}{7} = \frac{51}{7} = 51 : 7$$

Question No.10 Complete the following

i. if $\frac{24}{7} = \frac{6}{x}$ then $4x = 7$

ii. if $\frac{5a}{3x} = \frac{15b}{y}$ then $ay = 9bx$

iii. if $\frac{9pb}{2lm} = \frac{18p}{5m}$, then $5q = 4l$

Question No.11 find x in the following proportions.

i. $3x - 2 : 4 :: 2x + 3 : 7$

Product of extremes

= *Product of means*

$$7(3x - 2) = 4(2x + 3)$$

$$21x - 14 = 8x + 12$$

$$21x - 8x = 12 + 14$$

$$13x = 26$$

$$x = \frac{26}{13} = 2$$

ii. $\frac{3x-1}{7} : \frac{3}{5} :: \frac{2x}{3} : \frac{7}{5}$

Product of extremes

= *Product of means*

$$\left(\frac{3x-1}{7}\right)\frac{7}{5} = \frac{3}{5}\left(\frac{2x}{3}\right)$$

$$\frac{3x-1}{5} = \frac{2x}{5}$$

$$3x - 1 = 2x$$

$$3x - 2x = 1$$

$$x = 1$$

iii. $\frac{x-3}{2} : \frac{5}{x-1} :: \frac{x-1}{3} : \frac{4}{x+4}$

Solution:

Product of extremes

= *product of means*

$$\left(\frac{x-3}{2}\right)\frac{4}{x+4} = \frac{5}{x-1}\left(\frac{x-1}{3}\right)$$

$$\frac{2x-6}{x+4} = \frac{5}{3}$$

$$3(2x - 6) = 5(x + 4)$$

$$6x - 18 = 5x + 20$$

$$6x - 5x = 20 + 18$$

$$x = 38$$

iv. $P^2 + pq + q^2 :: \frac{p^3-q^3}{p+q} : (p-q)^2$

product of extremes

= *product of means*

$$(p^2 + pq + q^2)(p - q)^2$$

$$= x \times \frac{p^3 - q^3}{p + q}$$

$$(p^2 + pq + q^2)(p - q)(p - q)$$

$$= x \times \frac{p^3 - q^3}{p + q}$$

$$(p^3 - q^3)(p - q) = x \times \frac{p^3 - q^3}{p + q}$$

$$(p^3 - q^3)(p - q) \times \frac{p + q}{p^3 - q^3} = x$$

$$x = (p - q)(p + q)$$

$$x = p^2 - q^2$$

v. $8 - x : 11 - x :: 16 - x : 25 - x$

product of extremes

= *product of means*

$$(8 - x)(25 - x) = (11 - x)(16 - x)$$

$$200 - 8x - 25x + x^2$$

$$= 176 - 11x - 16x + x^2$$

$$200 - 33x + x^2 = 176 - 27x + x^2$$

$$-33x + x^2 + 27x - x^2 = 176 - 200$$

$$-6x = -24$$

$$x = 4$$

Exercise 3.2

Question No.1

if y varies directly as x , and $y = 8$ when $x = 2$ Find

i. y in terms of x

$$y \propto x$$

$$y = kx \rightarrow (i)$$

$$8 = k \times 2$$

$$k = \frac{8}{2} = 4$$

So equation (i) becomes

$$y = 4x$$

ii. y when $x = 5$

$$y = 4x$$

$$y = 4 \times 5$$

$$y = 20$$

iii. x when $y = 28$

$$y = 4x$$

$$28 = 4x$$

$$\frac{28}{4} = x$$

$$x = 7$$

Question No2.

if $y \propto x$, and $y = 7$ when $x = 3$ find

i. y in terms of x

$$y \propto x$$

$$y = kx \rightarrow (i)$$

$$7 = k \times 3$$

$$k = \frac{7}{3}$$

So equation (i) becomes

$$y = \frac{7}{3} \times x = \frac{7x}{3}$$

- ii. x when y = 35 and y when x = 18
When y = 35

$$y = \frac{7}{3} \times x$$

$$35 = \frac{7}{3} \times x$$

$$\frac{35 \times 3}{7} = x$$

$$5 \times 3 = x$$

$$15 = x$$

Put x = 18 in (i) we get

$$y = kx$$

$$y = \frac{7}{3} \times 18$$

$$y = 42$$

Question No.3 if $R \propto T$ and $R = 5$ when $T = 8$ find the equation connection R and T . Also find R when $T = 64$ and find T when $R = 21$ solution:

$$R \propto T$$

$$R = kT$$

when $R = 5$ and $T = 8$

$$5 = k8$$

$$k = \frac{5}{8}$$

So,

$$R = \frac{5}{8}T$$

Put $T = 64$

$$R = \frac{5}{8}T$$

Put $T = 64$

$$R = \frac{5}{8} \times 64 = 40$$

$$R = 40$$

when $R = 20$

$$20 = \frac{5}{8} \times T$$

$$20 \times \frac{8}{5} = T$$

$$4 \times 8 = T$$

$$T = 32$$

Question No.5

if $V \propto R^3$, and $V = 5$ when $R = 3$ find when $V = 625$

$$V \propto R^3$$

$$V = kR^3$$

$$5 = k(3)^3$$

$$5 = 27k$$

$$\frac{5}{27} = k$$

So,

$$\text{eq (i) becomes } V = \frac{5}{27}R^3$$

Put $V = 625$

$$625 = \frac{5}{27} \times R^3$$

$$R^3 = 625 \times \frac{27}{5}$$

$$R^3 = 125 \times 27$$

Taking cubes on both sides

$$R^{3 \times \frac{1}{3}} = (5^3)^{\frac{1}{3}} \times (3^3)^{\frac{1}{3}}$$

$$R = 5 \times 3 = 15$$

$$R = 15$$

Question NO.6 if w varies directly as u^3 and $w = 81$ when $u = 3$ Find w when $u = 5$

$$w \propto u^3$$

$$w = ku^3 \rightarrow (i)$$

$$\text{when } w = 81 \text{ and } u = 3$$

so eq (i) becomes

$$81 = k(3)^3$$

$$81 = k27$$

$$\frac{81}{27} = k$$

$$k = 3$$

So,

$$w = 3u^3$$

Put $u = 5$ in (i)

$$w = 3 \times 5^3$$

$$3 \times 125$$

$$w = 375$$

Question NO.7 if y varies inversely as x and $y = 7$ when $x = 2$ find y when $x = 126$

Solution:

$$y \propto \frac{1}{x}$$

$$y = \frac{k}{x} \rightarrow (i)$$

when $y = 7$ and $x = 2$

so (i)eq becomes

$$7 = \frac{k}{2}$$

$$7 \times 2 = k$$

$$14 = k$$

So,

$$y = \frac{14}{x}$$

when $x = 126$

$$y = \frac{14}{126} = \frac{1}{9}$$

$$y = \frac{1}{9}$$

Question No.8 if $y \propto \frac{1}{x}$ and $y = 4$ when $x = 3$, find x when $y = 24$

$$y \propto \frac{1}{x}$$

$$y = \frac{k}{x}$$

when $y = 4$ and $x = 3$

$$4 = \frac{k}{3}$$

$$12 = k$$

or $k = 12$

So,

$$y = \frac{12}{x}$$

When $y = 24$

$$24 = \frac{12}{x}$$

$$x = \frac{12}{24}$$

$$x = \frac{1}{2}$$

Question No.9 if $w \propto \frac{1}{z}$ and $w = 5$ when $z = 7$, find w when $z = \frac{175}{4}$

Solution:

$$w \propto \frac{1}{z}$$

$$w = \frac{k}{z} \rightarrow (i)$$

When $w = 5$ and $z = 7$

so eq (i) becomes

$$5 = \frac{k}{7}$$

$$5 \times 7 = k$$

$$k = 35$$

So,

$$w = \frac{35}{z}$$

when $z = \frac{175}{4}$

$$w = \frac{35}{\frac{175}{4}} = 35 \times \frac{4}{175} = \frac{4}{5}$$

$$w = \frac{4}{5}$$

Question No.10 if $A \propto \frac{1}{r^2}$ and $A = 2$ when $r = 3$ find r when $A = 72$

$$A \propto \frac{1}{r^2}$$

$$A = \frac{k}{r^2}$$

when $A = 2$ and $r = 3$

$$2 = \frac{k}{(3)^2}$$

$$2 = \frac{k}{9}$$

$$2 \times 9 = k$$

$$k = 18$$

So,

$$A = \frac{18}{r^2}$$

When $A = 72$

$$72 = \frac{18}{r^2}$$

$$r^2 = \frac{18}{72}$$

$$r^2 = \frac{1}{4}$$

$$r = \frac{1}{2}$$

Taking squaring root on both sides

$$r = \pm \frac{1}{2}$$

Question No.11 if $a \propto \frac{1}{b^2}$ and $a = 3$ when $b = 4$, find a when $b = 8$

Solution:

$$a \propto \frac{1}{b^2}$$

$$a = \frac{k}{b^2} \rightarrow (i)$$

when $a = 3$ and $b = 4$

$$3 = \frac{k}{(4)^2} = \frac{k}{16}$$

$$3 \times 16 = k$$

$$48 = k$$

$$\text{or } k = 48$$

So, (i) becomes

$$a = \frac{48}{b^2}$$

When $b = 8$

$$a = \frac{k}{b^2}$$

$$a = \frac{48}{8^2}$$

$$a = \frac{48}{64}$$

$$a = \frac{3}{4}$$

Question No.12 if $v \propto \frac{1}{r^3}$ and $V = 5$ when $r = 3$ find V when $r = 6$ and find r when $v = 320$

$$V \propto \frac{1}{r^3}$$

$$V = \frac{k}{r^3}$$

When $V = 5$ and $r = 3$

$$5 = \frac{k}{3^3}$$

$$5 = \frac{k}{27}$$

$$5 \times 27 = k$$

$$135 = k$$

$$\text{or } k = 135$$

When $r = 6$

$$V = \frac{135}{6^3}$$

$$V = \frac{135}{216}$$

$$V = \frac{5}{8}$$

when $V = 320$

When $V = 320$

$$320 = \frac{135}{r^3}$$

$$r^3 = \frac{135}{320}$$

$$r^3 = \frac{27}{64}$$

Taking cube on both sides

$$r^{3 \times \frac{1}{3}} = \frac{(3)^{3 \times \frac{1}{3}}}{(4)^{3 \times \frac{1}{3}}}$$

$$r = \frac{3}{4}$$

Question No.13 if $m \propto \frac{1}{n^3}$ and $m = 2$ when $n = 4$, find m when $n = 6$ and n when $m = 432$

Solution:

$$m \propto \frac{1}{n^3}$$

$$m = \frac{k}{n^3}$$

When $m = 2$ and $n = 4$

$$m = \frac{k}{n^3}$$

When $m = 2$ and $n = 4$

$$m = \frac{k}{n^3}$$

$$2 = \frac{k}{(4)^3}$$

$$2 = \frac{k}{64}$$

$$2 \times 64 = k$$

$$128 = k$$

So,

$$m = \frac{128}{n^3}$$

When $n = 6$

$$m = \frac{128}{6^3}$$

$$m = \frac{128}{216}$$

$$m = \frac{16}{27}$$

When $m = 432$

$$m = \frac{k}{n^3}$$

$$432 = \frac{128}{n^3}$$

$$n^3 = \frac{128}{432}$$

$$n^3 = \frac{8}{27}$$

Taking cube on both sides by

$$n^{3 \times \frac{1}{3}} = \frac{(2)^{3 \times \frac{1}{3}}}{(3)^{3 \times \frac{1}{3}}}$$

$$n = \frac{2}{3}$$

Exercise 3.3

Question No.1 find third proportional to

i. 6,12

let x be the third proportional to

$$6 : 12 :: 12 : x$$

product of Extremes = product of Means

$$6x = 12 \times 12$$

$$x = \frac{12 \times 12}{6}$$

$$x = 24$$

ii. $a^3, 3a^2$

let x be the third proportional to

$$a^3 : 3a^2 :: 3a^2 : x$$

product of Extremes = product of Means

$$a^3 \times x = 3a^2 \times 3a^2$$

$$x = \frac{3a^2 \times 3a^2}{a^3} = \frac{9a^4}{a^3}$$

$$x = 9a$$

iii. $a^2 - b^2, a - b$

let x be the third proportional to

$$a^2 - b^2 : a - b :: a - b : x$$

product of Extremes = product of Means

$$a^2 - b^2 \times x = a - b \times a - b$$

$$x = \frac{a - b \times a - b}{a^2 - b^2}$$

$$x = \frac{a - b \times a - b}{(a - b)(a + b)}$$

$$x = \frac{a - b}{a + b}$$

iv. $(x - y)^2, x^3 - y^3$

let x be the third proportional to

$$(x - y)^2 : x^3 - y^3 :: x^3 - y^3 : c$$

product of Extremes = product of Means

$$(x - y)^2 \times c = x^3 - y^3 \times x^3 - y^3$$

$$c = \frac{x^3 - y^3 \times x^3 - y^3}{(x - y)^2}$$

$$c = \frac{(x - y)(x^2 + xy + y^2)(x - y)(x^2 + xy + y^2)}{(x - y)(x - y)}$$

$$x = (x^2 + xy + y^2)(x^2 + xy + y^2)$$

v. $(x + y)^2, x^3 - xy - 2y^2$

let x be the third proportional to

$$(x + y)^2 : (x^2 - xy - 2y^2) :: (x^2 - xy - 2y^2)$$

$$: c$$

product of Extremes = product of Means

$$(x - y)^2 \times c = (x^2 - xy - 2y^2)$$

$$\times (x^2 - xy - 2y^2)$$

$$c = \frac{(x^2 - xy - 2y^2) \times (x^2 - xy - 2y^2)}{(x + y)^2}$$

$$c = \frac{(x^2 - xy - 2y^2)^2}{(x + y)^2}$$

$$c = \frac{(x^2 - 2xy + xy - 2y^2)^2}{(x + y)^2}$$

$$c = \frac{(x(x - 2y) + y(x - 2y))^2}{(x + y)^2}$$

$$c = \frac{((x + y)(x - 2y))^2}{(x + y)^2}$$

$$c = \frac{(x + y)^2(x - 2y)^2}{(x + y)^2}$$

$$c = (x - 2y)^2$$

$$c = (x - 2y)(x - 2y)$$

vi. $\frac{p^2 - q^2}{p^3 + q^3}, \frac{p - q}{p^2 - pq + q^2}$

let x be the third proportional to

$$\frac{p^2 - q^2}{p^3 + q^3} : \frac{p - q}{p^2 - pq + q^2} :: \frac{p - q}{p^2 - pq + q^2} : c$$

product of Extremes = product of Means

$$\frac{p^2 - q^2}{p^3 + q^3} \times c = \frac{p - q}{p^2 - pq + q^2}$$

$$\times \frac{p - q}{p^2 - pq + q^2}$$

$$c = \frac{p - q}{p^2 - pq + q^2} \times \frac{p - q}{p^2 - pq + q^2}$$

$$\times \frac{p^3 + q^3}{p^2 - q^2}$$

$$c = \frac{p - q}{p^2 - pq + q^2} \times \frac{p - q}{p^2 - pq + q^2}$$

$$\times \frac{(p + q)(p^2 - pq + q^2)}{(p - q)(p + q)}$$

$$c = \frac{p - q}{p^2 - pq + q^2}$$

Question No.2 find a fourth proportional to

i. 5,8,15

let x be the fourth proportional

$$5 : 8 :: 15 : x$$

product of Extremes = Product of means

$$5x = 8 \times 15$$

$$x = \frac{8 \times 15}{5}$$

$$x = 8 \times 3$$

$$x = 24$$

ii. $4x^2, 2x^3, 18x^5$

let x be the fourth proportional

$$4x^2 : 2x^3 :: 18x^5 : c$$

product of Extremes = Product of means

$$4x^4 \times c = 2x^3 \times 18x^5$$

$$c = \frac{2x^3 \times 18x^5}{4x^4}$$

$$c = \frac{36x^8}{4x^4}$$

$$c = 9x^4$$

iii. $15a^5b^6, 10a^2b^5, 21a^3b^3$

let x be the fourth proportional

$$15a^5b^6 : 10a^2b^5 :: 21a^3b^3 : c$$

product of Extremes = Product of means

$$15a^5b^6 \times c = 10a^2b^5 \times 21a^3b^3$$

$$c = \frac{10a^2b^5 \times 21a^3b^3}{15a^5b^6}$$

$$c = \frac{2a^2 b^5 \times 7a^3 b^3}{a^5 b^6}$$

$$c = 14 a^2 b^5 \times a^3 b^3 \times a^{-5} b^{-6}$$

$$c = 14a^{2+3-5} b^{5+3-6}$$

$$c = 14b^2$$

iv. $x^2 - 11x + 24, x - 3, 5x^4 - 40x^3$

let x be the fourth proportional

$$x^2 - 11x + 24 : x - 3 :: 5x^4 - 40x^3$$

: c

product of Extremes = Product of means

$$x^2 - 11x + 24 \times c$$

$$= (x - 3) \times (5x^4 - 40x^3)$$

$$c = \frac{(x - 3) \times (5x^4 - 40x^3)}{x^2 - 11x + 24}$$

$$c = \frac{(x - 3)5x^3(x - 8)}{x^2 - 8x - 3x + 24}$$

$$c = \frac{5x^3(x - 3)(x - 8)}{x(x - 8) - 3(x - 8)}$$

$$c = \frac{5x^3(x - 3)(x - 8)}{(x - 3)(x - 8)}$$

$$c = 5x^3$$

v. $p^3 + q^3, p^2 - q^2, p^2 - pq + q^2$

let x be the fourth proportional

$$p^3 + q^3 : p^2 - q^2 :: p^2 - pq + q^2 : c$$

product of Extremes = Product of means

$$p^3 + q^3 \times c = p^2 - q^2 \times p^2 - pq + q^2$$

$$c = \frac{p^2 - q^2 \times p^2 - pq + q^2}{p^3 + q^3}$$

$$c = \frac{(p - q)(p + q)(p^2 - pq + q^2)}{p^3 + q^3}$$

$$c = \frac{(p - q)(p^3 + q^3)}{p^3 + q^3}$$

$$c = (p - q)$$

vi. $(p^2 - q^2)(p^2 + pq + q^2), p^3 + q^3, p^3 - q^3$

let x be the fourth proportional

$$(p^2 - q^2)(p^2 + pq + q^2) : p^3 + q^3 :$$

: $p^3 - q^3 : c$

$$(p^2 - q^2)(p^2 + pq + q^2) \times c$$

$$= p^3 + q^3 \times p^3 - q^3$$

$$c = \frac{p^3 + q^3 \times p^3 - q^3}{(p^2 - q^2)(p^2 + pq + q^2)}$$

$$c = \frac{(p + q)(p^2 - pq + q^2)(p - q)(p^2 + pq + q^2)}{(p + q)(p - q)(p^2 + pq + q^2)}$$

$$c = p^2 - pq + q^2$$

Question No.3 find a mean proportional between

i. 20,45

let x be the mean proportional

$$20 : x :: x : 45$$

product of means = product of extremes

$$x^2 = 20 \times 45$$

$$x^2 = 900$$

Taking squaring root on both sides

$$\sqrt{x^2} = \sqrt{(30)^2}$$

$$x = \pm 30$$

ii. $20x^3y^5, 5x^7y$

let x be the mean proportional

$$20x^3y^5 : c :: c : 5x^7y$$

product of means = product of extremes

$$c^2 = 20x^3y^5 \times 5x^7y$$

$$c^2 = 100x^{10}y^6$$

Taking squaring root on both sides

$$\sqrt{c^2} = \sqrt{(10)^2x^{10}y^6}$$

$$c = \pm 10x^5y^3$$

iii. $15p^4qr^3, 135q^5r^7$

let x be the mean proportional

$$15p^4qr^3 : x :: x : 135q^5r^7$$

product of means = product of extremes

$$x^2 = 15p^4qr^3 \times 135q^5r^7$$

$$x^2 = 2025p^4q^6r^{10}$$

Taking squaring root on both sides

$$\sqrt{x^2} = \sqrt{(45)^2p^4q^6r^{10}}$$

$$x = \pm 45p^2q^3r^5$$

iv. $x^2 - y^2, \frac{x-y}{x+y}$

let x be the mean proportional

$$x^2 - y^2 : c :: c : \frac{x-y}{x+y}$$

product of means = product of extremes

$$c^2 = (x^2 - y^2) \times \frac{x-y}{x+y}$$

$$c^2 = (x-y)(x+y) \times \frac{x-y}{x+y}$$

$$c^2 = (x-y)(x-y)$$

$$c^2 = (x-y)^2$$

Taking squaring root on both sides

$$\sqrt{c^2} = \sqrt{(x-y)^2}$$

$$c = \pm(x-y)$$

Question No.4 find the values of the letter involved in the following continued proportional

i. 5, p, 45

$$5 : p :: p : 45$$

product of means = product of extremes

$$p^2 = 5 \times 45$$

$$p^2 = 225$$

Squaring root on both sides

$$\sqrt{p^2} = \sqrt{(15)^2}$$

$$p = \pm 15$$

ii. 8, x, 18

$$8 : x :: x : 18$$

product of means = product of extremes

$$x^2 = 8 \times 18$$

$$x^2 = 144$$

Squaring root on both sides

$$\sqrt{x^2} = \sqrt{(12)^2}$$

$$x = \pm 12$$

iii. $12, 3p - 6, 27$

$$12 : 3p - 6 :: 3p - 6 : 27$$

product of means = product of extremes

$$(3p - 6)^2 = 12 \times 27$$

$$(3p - 6)^2 = 324$$

Squaring root on both sides

$$\sqrt{(3p - 6)^2} = \sqrt{(18)^2}$$

$$3p - 6 = \pm 18$$

$$3p - 6 = +18$$

$$3p = 18 + 6$$

$$3p = 24$$

$$p = \frac{24}{3} = 8$$

$$p = 8$$

$$3p - 6 = -18$$

$$3p = -18 + 6$$

$$3p = -12$$

$$p = -\frac{12}{3}$$

$$p = -4$$

iv. $7, m - 3, 28$

$$7 : m - 3 :: 3m - 3 : 28$$

product of means = product of extremes

$$(m - 3)^2 = 7 \times 28$$

$$(m - 3)^2 = 196$$

Squaring root on both sides

$$\sqrt{(m - 3)^2} = \sqrt{(14)^2}$$

$$m - 3 = \pm 14$$

$$m - 3 = +14$$

$$m = 14 + 3$$

$$m = 14$$

$$m - 3 = 14$$

$$m = -14 + 3$$

$$m = -11$$

Exercise 3.4

Question No.1 prove that $a:b = c:d$ if

$$i. \frac{4a+5b}{4a-5b} = \frac{4c+5d}{4c-5d}$$

Solution:

By componendo - dividendo

$$\frac{(4a + 5b) + (4a - 5b)}{(4a + 5b) - (4a - 5b)}$$

$$= \frac{(4c + 5d) + (4c - 5d)}{(4c + 5d) - (4c - 5d)}$$

$$\frac{4a + 5b + 4a - 5b}{4a + 5b - 4a + 5b} = \frac{4c + 5d + 4c - 5d}{4c + 5d - 4c + 5d}$$

$$\frac{4a + 4a}{5b + 5b} = \frac{4c + 4c}{5d + 5d}$$

$$\frac{8a}{10b} = \frac{8c}{10d}$$

Multiplying by $\frac{10}{8}$

$$\frac{a}{b} = \frac{c}{d}$$

$$a:b = c:d$$

$$ii. \frac{2a+9b}{2a-9b} = \frac{2c+9d}{2c-9d}$$

By componendo - dividendo

$$\frac{(2a + 9b) + (2a - 9b)}{(2a + 9b) - (2a - 9b)}$$

$$= \frac{(2c + 9d) + (2c - 9d)}{(2c + 9d) - (2c - 9d)}$$

$$\frac{2a + 9b + 2a - 9b}{2a + 9b - 2a + 9b} = \frac{2c + 9d + 2c - 9d}{2c + 9d - 2c + 9d}$$

$$\frac{2a + 2a}{9b + 9b} = \frac{2c + 2c}{9d + 9d}$$

$$\frac{4a}{18b} = \frac{4c}{18d}$$

Multiplying by $\frac{18}{4}$

$$\frac{a}{b} = \frac{c}{d}$$

$$a:b = c:d$$

$$iii. \frac{ac^2+bd^2}{ac^2-bd^2} = \frac{c^3+d^3}{c^3-d^3}$$

By componendo - dividendo

$$\frac{(ac^2 + bd^2) + (ac^2 - bd^2)}{(ac^2 + bd^2) - (ac^2 - bd^2)}$$

$$= \frac{(c^3 + d^3) + (c^3 - d^3)}{(c^3 + d^3) - (c^3 - d^3)}$$

$$\frac{ac^2 + bd^2 + ac^2 - bd^2}{ac^2 + bd^2 - ac^2 + bd^2} = \frac{c^3 + d^3 + c^3 - d^3}{c^3 + d^3 - c^3 + d^3}$$

$$\frac{2ac^2}{2bd^2} = \frac{2c^3}{2d^3}$$

$$\frac{ac^2}{bd^2} = \frac{c^3}{d^3}$$

Multiplying by $\frac{d^2}{c^2}$

$$\frac{a}{b} = \frac{c}{d}$$

$$a:b = c:d$$

$$iv. \frac{a^2c+b^2d}{a^2c-b^2d} = \frac{ac^2+bd^2}{ac^2-bd^2}$$

By componendo - dividendo

$$\frac{(a^2c + b^2d) + (a^2c - b^2d)}{(a^2c + b^2d) - (a^2c - b^2d)}$$

$$= \frac{(ac^2 + bd^2) + (ac^2 - bd^2)}{(ac^2 + bd^2) - (ac^2 - bd^2)}$$

$$\frac{a^2c + b^2d + a^2c - b^2d}{a^2c + b^2d - a^2c + b^2d}$$

$$= \frac{ac^2 + bd^2 + ac^2 - bd^2}{ac^2 + bd^2 - ac^2 + bd^2}$$

$$\frac{2a^2c}{2b^2d} = \frac{2ac^2}{2bd^2}$$

$$\frac{a^2c}{b^2d} = \frac{ac^2}{bd^2}$$

Multiplying by $\frac{bd}{ac}$

$$v. \frac{pa+qb}{pa-qb} = \frac{pc+qd}{pc-qd}$$

By componendo - dividendo

$$\frac{(pa + qb) + (pa - qb)}{(pa + qb) - (pa - qb)} = \frac{(pc + qd) + (pc - qd)}{(pc + qd) - (pc - qd)}$$

$$\frac{pa + qb + pa - qb}{pa + qb - pa + qb} = \frac{pc + qd + pc - qd}{pc + qd - pc + qd}$$

$$\frac{2pa}{2qb} = \frac{2pc}{2qd}$$

$$\frac{pa}{qb} = \frac{pc}{qd}$$

Multiplying by $\frac{q}{p}$

$$\frac{a}{b} = \frac{c}{d}$$

$$a : b = c : d$$

vi. $\frac{a+b+c+d}{a+b-c-d} = \frac{a-b+c-d}{a-b-c+d}$

By componendo - dividendo

$$\frac{(a + b + c + d) + (a + b - c - d)}{(a + b + c + d) - (a + b - c - d)} = \frac{(a - b + c - d) + (a - b - c + d)}{(a - b + c - d) - (a - b - c + d)}$$

$$\frac{a + b + c + d + a + b - c - d}{a + b + c + d - a - b + c + d} = \frac{a - b + c - d + a - b - c + d}{a - b + c - d - a + b + c + d}$$

$$\frac{2a + 2b}{2c + 2d} = \frac{2a - 2b}{2c - 2d}$$

$$\frac{a + b}{c + d} = \frac{a - b}{c - d}$$

by componendo - dividendo

$$\frac{(a + b) + (a - b)}{(a + b) - (a - b)} = \frac{(c + d) + (c - d)}{(c + d) - (c - d)}$$

$$\frac{2a}{2b} = \frac{2c}{2d}$$

$$\frac{a}{b} = \frac{c}{d}$$

$$a : b = c : d$$

vii. $\frac{2a+3b+2c+3d}{2a+3b-2c-3d} = \frac{2a-3b+2c+3d}{2a-3b-2c+3d}$

Solution:

$$\frac{(2a + 3b + 2c + 3d) + (2a + 3b - 2c - 3d)}{(2a + 3b + 2c + 3d) - (2a + 3b - 2c - 3d)} = \frac{(2a - 3b + 2c + 3d) + (2a - 3b - 2c + 3d)}{(2a - 3b + 2c - 3d) - (2a - 3b - 2c + 3d)}$$

$$\frac{2a + 3b + 2c + 3d + 2a + 3b - 2c - 3d}{2a + 3b + 2c + 3d - 2a + 3b + 2c + 3d} = \frac{2a - 3b + 2c - 3d + 2a - 3b - 2c + 3d}{4a + 6b} = \frac{4c - 6d}{4a + 6b} = \frac{4c - 6d}{4c + 6d} = \frac{4a - 6b}{4c - 6d}$$

by componendo - dividendo

$$\frac{(4a + 6b) + (4a - 6b)}{(4a + 6b) - (4a - 6b)} = \frac{(4c + 6d) + (4c - 6d)}{(4c + 6d) - (4c - 6d)}$$

$$\frac{4a + 6b + 4a - 6b}{4a + 6b - 4a + 6b} = \frac{4c + 6d + 4c - 6d}{4c + 6d - 4c + 6d}$$

$$\frac{8a}{12b} = \frac{8c}{12d}$$

$$\frac{2a}{3b} = \frac{2c}{3d}$$

Multiplying by $\frac{3}{2}$

$$\frac{a}{b} = \frac{c}{d}$$

$$a : b = c : d$$

viii. $\frac{a^2+b^2}{a^2-b^2} = \frac{ac+bd}{ac-bd}$

by componendo - dividendo

$$\frac{(a^2 + b^2) + (a^2 - b^2)}{(a^2 + b^2) - (a^2 - b^2)} = \frac{(ac + bd) + (ac - bd)}{(ac + bd) - (ac - bd)}$$

$$\frac{a^2 + b^2 + a^2 - b^2}{a^2 + b^2 - a^2 + b^2} = \frac{ac + bd + ac - bd}{ac + bd - ac + bd}$$

$$\frac{2a^2}{2b^2} = \frac{2ac}{2bd}$$

$$\frac{a^2}{b^2} = \frac{ac}{bd}$$

Multiplying by $\frac{b}{a}$

$$\frac{a}{b} = \frac{c}{d}$$

$$a : b = c : d$$

Question No.2 using theorem of componendo - dividendo

i. Find the value of $\frac{x+2y}{x-2y} + \frac{x+2z}{x-2z}$ if $x = \frac{4yz}{y+z}$

$$x = \frac{4yz}{y+z} \rightarrow (i)$$

From eq. (i)

$$x = \frac{2y + 2z}{y + z}$$

$$\frac{x}{2y} = \frac{2z}{y+z}$$

By applying componendo – dividendo theorem

$$\frac{x+2y}{x-2y} = \frac{2z+y+z}{2z-y-z}$$

$$\frac{x+2y}{x-2y} = \frac{y+3z}{z-y} \rightarrow \text{(ii)}$$

From eq (i)

$$x = \frac{2y \times 2z}{y+z}$$

$$\frac{x}{2z} = \frac{2y}{y+z}$$

By applying componendo – dividendo theorem

$$\frac{x+2z}{x-2z} = \frac{2y+y+z}{2y-y-z}$$

$$\frac{x+2z}{x-2z} = \frac{z+3y}{y-z} \rightarrow \text{(iii)}$$

Adding equations (ii) and (iii)

$$\frac{x+2y}{x-2y} + \frac{x+2z}{x-2z} = \frac{y+3z}{z-y} + \frac{z+3y}{y-z}$$

$$= -\frac{y+3z}{y-z} + \frac{z+3y}{y-z}$$

$$= \frac{z+3y}{y-z} - \frac{y+3z}{y-z}$$

$$= \frac{z+3y-y-3z}{y-z}$$

$$= \frac{2y-2z}{y-z}$$

$$= \frac{2(y-z)}{y-z} = 2$$

ii. find the value of $\frac{m+5n}{m-5n} + \frac{m+5p}{m-5p}$

$$\text{If } m = \frac{10np}{n+p}$$

Solution:

$$m = \frac{10np}{n+p} \rightarrow \text{(i)}$$

From eq(i)

$$m = \frac{5n \times 2p}{n+p}$$

$$\frac{m}{5n} = \frac{2p}{n+p}$$

By applying componendo – dividendo theorem

$$\frac{m+5n}{m-5n} = \frac{2p+n+p}{2p-n-p}$$

$$\frac{m+5n}{m-5n} = \frac{3p+n}{p-n} \rightarrow \text{(ii)}$$

From eq.(i)

$$m = \frac{2n \times 5p}{n+p}$$

$$\frac{m}{5p} = \frac{2n}{n+p}$$

By applying componendo – dividendo theorem

$$\frac{m+5p}{m-5p} = \frac{2n+n+p}{2n-n-p}$$

$$\frac{m+5p}{m-5p} = \frac{3n+p}{n-p} \rightarrow \text{(iii)}$$

Adding (ii) and (iii)

$$\frac{m+5n}{m-5n} + \frac{m+5p}{m-5p} = \frac{3p+n}{p-n} + \frac{3n+p}{n-p}$$

$$= -\frac{3p+n}{n-p} + \frac{3n+p}{n-p}$$

$$= \frac{3n+p}{n-p} - \frac{3p+n}{n-p}$$

$$= \frac{3n+p-3p-n}{n-p}$$

$$= \frac{2n-2p}{n-p}$$

$$\frac{2(n-p)}{n-p} = 2$$

iii. Find the value of $\frac{x-6a}{x+6a} - \frac{x+6a}{x-6a}$, if $x = \frac{12ab}{a-b}$

Solution:

$$x = \frac{12ab}{a-b} \rightarrow \text{(i)}$$

From equation (i)

$$x = \frac{6a \times 2b}{a-b}$$

$$\frac{x}{6a} = \frac{2b}{a-b}$$

By applying componendo – dividendo theorem

$$\frac{x+6a}{x-6a} = \frac{2b+a-b}{2b+a-b}$$

$$\frac{x + 6a}{x - 6a} = \frac{a + b}{3b - a}$$

$$\frac{x - 6a}{x + 6a} = \frac{3b - a}{a + b} \rightarrow \text{(ii)}$$

From eq.(i)

$$x = \frac{6b \times 2a}{a - b}$$

$$\frac{x}{6b} = \frac{2a}{a - b}$$

By applying componendo – dividendo theorem

$$\frac{x + 6b}{x - 6b} = \frac{2a + a - b}{2a - a + b}$$

$$\frac{x + 6b}{x - 6b} = \frac{3a - b}{a + b} \rightarrow \text{(iii)}$$

Subtracting equation (iii) from (ii)

$$\frac{x - 6a}{x + 6a} - \frac{x + 6b}{x - 6b} = \frac{3b - a}{a + b} - \frac{3a - b}{a + b}$$

$$= \frac{3b - a - 3a + b}{a + b}$$

$$= \frac{-4a + 4b}{a + b}$$

$$= \frac{4(b - a)}{a + b}$$

iv. Find the value of $\frac{x-3y}{x+3y} - \frac{x+3z}{x-3z}$, if $x = \frac{3yz}{y-z}$

$$x = \frac{3yz}{y - z} \rightarrow \text{(i)}$$

From equation (i)

$$x = \frac{3y \times z}{y - z}$$

$$\frac{x}{3y} = \frac{z}{y - z}$$

By applying componendo – dividendo theorem

$$\frac{x + 3y}{x - 3y} = \frac{z + y - z}{z - y + z}$$

$$\frac{x + 3y}{x - 3y} = \frac{y}{2z - y}$$

$$\frac{x - 3y}{x + 3y} = \frac{2z - y}{y} \rightarrow \text{(ii)}$$

From equation (i)

$$x = \frac{3z \times y}{y - z}$$

$$\frac{x}{3z} = \frac{y}{y - z}$$

By applying componendo – dividendo theorem

$$\frac{x + 3z}{x - 3z} = \frac{y + y - z}{y - y + z}$$

$$\frac{x + 3z}{x - 3z} = \frac{2y - z}{z}$$

$$\frac{x + 3z}{x - 3z} = \frac{2y - z}{z} \rightarrow \text{(iii)}$$

Subtracting equation (iii) from eq. (ii)

$$\frac{x - 3y}{x + 3y} - \frac{x + 3z}{x - 3z} = \frac{2z - y}{y} - \frac{2y - z}{z}$$

$$= \frac{z(2z - y) - y(2y - z)}{yz}$$

$$= \frac{2z^2 - zy - 2y^2 + yz}{yz}$$

v. Find the value of $\frac{s-3p}{s+3p} + \frac{s+3q}{s-3q}$, if $s = \frac{6pq}{p-q}$

$$s = \frac{6pq}{p - q} \rightarrow \text{(i)}$$

From eq. (i)

$$s = \frac{3p \times 2p}{p - q}$$

$$\frac{s}{3p} = \frac{2p}{p - q}$$

By applying componendo – dividendo theorem

$$\frac{s + 3p}{s - 3p} = \frac{2p + p - q}{2p - p + q}$$

$$\frac{s + 3p}{s - 3p} = \frac{q + p}{3q - p}$$

$$\frac{s - 3p}{s + 3p} = \frac{3q - p}{p + q} \rightarrow \text{(ii)}$$

From eq.(i)

$$s = \frac{2p \times 3q}{p - q}$$

$$\frac{s}{3q} = \frac{2p}{p - q}$$

$$\frac{s + 3q}{s - 3q} = \frac{2p + p - q}{2p - p + q}$$

$$\frac{s + 3q}{s - 3q} = \frac{3p - q}{p + q} \rightarrow \text{(iii)}$$

Adding equation (ii) and (i)

$$\begin{aligned} \frac{s-3p}{s+3p} + \frac{s+3q}{s-3q} &= \frac{3q-p}{p+q} + \frac{3p-q}{p+q} \\ &= \frac{3q-p+3p-q}{p+q} \\ &= \frac{2q+2p}{p+q} \\ &= 2 \frac{p+q}{p+q} \\ &= 2 \end{aligned}$$

vi. Solve $\frac{(x-2)^2-(x-4)^2}{(x-2)^2+(x-4)^2} = \frac{12}{13}$

$$\frac{(x-2)^2-(x-4)^2}{(x-2)^2+(x-4)^2} = \frac{12}{13}$$

By applying componendo – dividendo theorem

$$\begin{aligned} \frac{(x-2)^2-(x-4)^2+(x-2)^2+(x-4)^2}{(x-2)^2-(x-4)^2-(x-2)^2-(x-4)^2} &= \frac{12+13}{12-13} \\ &= \frac{25}{-1} \\ \frac{2(x-2)^2}{-2(x-4)^2} &= \frac{25}{-1} \\ \frac{(x-2)^2}{(x-4)^2} &= 25 \end{aligned}$$

Taking root on both side

$$\frac{x-2}{x-4} = \pm 5$$

$$\begin{aligned} \frac{x-2}{x-4} &= 5 \\ x-2 &= 5(x-4) \\ x-2 &= 5x-20 \\ x-5x &= -20+2 \\ -4x &= -18 \\ 4x &= 18 \\ x &= \frac{18}{4} \\ x &= \frac{9}{2} \end{aligned}$$

$$\begin{aligned} \frac{x-2}{x-4} &= -5 \\ x-2 &= -5(x-4) \\ x-2 &= -5x+20 \\ x+5x &= 20+2 \\ 6x &= 22 \\ x &= \frac{22}{6} \\ x &= \frac{11}{3} \end{aligned}$$

$$S.S = \left\{ \frac{9}{2}, \frac{11}{3} \right\}$$

vii. Solve $\frac{\sqrt{x^2+2}+\sqrt{x^2-2}}{\sqrt{x^2+2}-\sqrt{x^2-2}} = 2$

Solution:

$$\frac{\sqrt{x^2+2}+\sqrt{x^2-2}}{\sqrt{x^2+2}-\sqrt{x^2-2}} = 2$$

By applying componendo – dividendo theorem

$$\begin{aligned} \frac{\sqrt{x^2+2}+\sqrt{x^2-2}+\sqrt{x^2+2}-\sqrt{x^2-2}}{\sqrt{x^2+2}+\sqrt{x^2-2}-\sqrt{x^2+2}+\sqrt{x^2-2}} &= \frac{2+1}{2-1} \\ &= \frac{2\sqrt{x^2+2}}{2\sqrt{x^2-2}} = \frac{3}{1} \\ \frac{\sqrt{x^2+2}}{\sqrt{x^2-2}} &= 3 \end{aligned}$$

Taking square on both sides

$$\begin{aligned} \frac{x^2+3}{x^2-2} &= 9 \\ x^2+2 &= 9(x^2-2) \\ x^2+2 &= 9x^2-18 \\ 2+18 &= 9x^2-x^2 \\ 20 &= 8x^2 \\ \text{or } 8x^2 &= 20 \\ x^2 &= \frac{20}{8} \\ x^2 &= \frac{5}{2} \\ x &= \pm \sqrt{\frac{5}{2}} \end{aligned}$$

If we check the given equation for this value doesn't satisfy the equation so the given solution is extraneous.

So,

$$S.S = \{ \}$$

viii. $\frac{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}}{\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}} = \frac{1}{3}$

Solution:

$$\frac{\sqrt{x^2+8p^2}-\sqrt{x^2-p^2}}{\sqrt{x^2+8p^2}+\sqrt{x^2-p^2}} = \frac{1}{3}$$

By applying componendo – dividendo theorem

$$\frac{\sqrt{x^2 + 8p^2} - \sqrt{x^2 - p^2} + \sqrt{x^2 + 8p^2} + \sqrt{x^2 - p^2}}{\sqrt{x^2 + 8p^2} - \sqrt{x^2 - p^2} - \sqrt{x^2 + 8p^2} - \sqrt{x^2 - p^2}} = \frac{1+3}{1-3}$$

$$\frac{2\sqrt{x^2 + 8p^2}}{-2\sqrt{x^2 - p^2}} = \frac{4}{-2}$$

$$\frac{\sqrt{x^2 + 8p^2}}{\sqrt{x^2 - p^2}} = 2$$

Taking square on both sides

$$\frac{x^2 + 8p^2}{x^2 - p^2} = 4$$

$$x^2 + 8p^2 = 4(x^2 - p^2)$$

$$x^2 + 8p^2 = 4x^2 - 4p^2$$

$$x^2 - 4x^2 = -4p^2 - 8p^2$$

$$-3x^2 = -12p^2$$

$$x^2 = \frac{12}{3}p^2$$

$$x^2 = 4p^2$$

Taking squaring root both sides

$$x = \pm 2p$$

$$S.S = \{2p, -2p\}$$

ix. Solve $\frac{(x+5)^3 - (x-3)^3}{(x+5)^3 + (x-3)^3} = \frac{13}{14}$

Solution:

$$\frac{(x+5)^3 - (x-3)^3}{(x+5)^3 + (x-3)^3} = \frac{13}{14}$$

By applying componendo – dividendo theorem

$$\frac{(x+5)^3 - (x-3)^3 + (x+5)^3 + (x-3)^3}{(x+5)^3 - (x-3)^3 - (x+5)^3 - (x-3)^3} = \frac{13+14}{13-14}$$

$$\frac{2(x+5)^3}{-2(x-3)^3} = \frac{27}{-1}$$

$$\frac{(x+5)^3}{(x-3)^2} = 27$$

Taking cube roots on both side

$$\frac{x+5}{x-3} = 3$$

$$x+5 = 3(x-3)$$

$$x+5 = 3x-9$$

$$x - 3x = -9 - 5$$

$$-2x = -14$$

$$x = \frac{14}{2}$$

$$x = 7$$

$$S.S = \{7\}$$

Exercise 3.5

Question No.1 if **S** varies directly as u^2 and inversely as v and $s = 7$ when $u = 3, v = 2$. Find the value of **S** when $u=6$

Solution:

$$s \propto u^2$$

$$s \propto \frac{1}{v}$$

$$s = k \frac{u^2}{v} \rightarrow (i)$$

put $s = 7, u = 3, v = 2$

$$7 = k \frac{3^2}{2}$$

$$7 = k \frac{9}{2}$$

$$\frac{7 \times 2}{9} = k$$

$$\frac{14}{9} = k$$

$$\text{or } k = \frac{14}{9}$$

So, equation (i) becomes

$$S = \frac{14u^2}{9v} \rightarrow (ii)$$

Put $u = 6$ and $v = 10$ in equation (ii)

$$S = \frac{14(6)^2}{9(10)}$$

$$S = \frac{14 \times 36}{9 \times 10}$$

$$S = \frac{28}{5}$$

Question No.2 if **w** varies jointly as x, y^2 and z and $w=5$ when $x=2, y=3, z=10$. Find **w** when $x=4, y=7$ and $z=3$.

Solution:

$$w \propto x$$

$$w \propto y^2$$

$$w \propto z$$

$$w = kxy^2z \rightarrow (i)$$

$$\text{Put } w = 5, x = 2, y = 3, z = 10$$

$$5 = k(2)(3)^2(10)$$

$$5 = k(180)$$

$$k = \frac{5}{180}$$

$$k = \frac{1}{36}$$

so, Equation (i) becomes

$$w = \frac{xy^2z}{36} \rightarrow (ii)$$

$$\text{Put } x = 4, y = 7 \text{ and } z = 3 \text{ in equation (ii)}$$

$$S = \frac{4(7)^2(3)}{36}$$

$$S = \frac{49}{3}$$

Question No.3 if Y varies directly as x^3 and inversly as z^3 and t, and $y = 16$

When $x = 4, z = 2, t =$

3. find the value of y when $x = 2, z =$

3 and $t = 4$

Solution:

$$y \propto x^3$$

$$y \propto \frac{1}{z^2}$$

$$y \propto \frac{1}{t}$$

$$y = k \frac{x^3}{z^2t} \rightarrow (i)$$

$$\text{Put } y = 16, x = 4, z = 2, t = 3$$

$$16 = k \frac{4^3}{2^2(3)}$$

$$16 = k \frac{64}{12}$$

$$\frac{16 \times 12}{64} = k$$

$$\frac{16}{4} = k$$

$$3 = k$$

$$\text{or } k = 3$$

So, equation (i) becomes

$$y = \frac{3x^3}{z^2t} \rightarrow (ii)$$

Put $x = 2, z = 3$ and $t = 4$ in equation(ii)

$$y = \frac{3(2)^3}{(3)^2(4)}$$

$$y = \frac{3(8)}{9(4)}$$

Divide by 3

$$y = \frac{8}{12}$$

Divide by 4

$$y = \frac{2}{3}$$

Question No.4 if u varies directly as x^2 and inversly as the product yz^3 , and $u = 2$ when $x=8, y=7, z=2$

Solution:

$$u \propto x^2$$

$$u \propto \frac{1}{yz^3}$$

$$u = k \frac{x^2}{yz^3} \rightarrow (i)$$

$$\text{put } u = 2, x = 8, y = 7, z = 2$$

$$2 = k \frac{8^2}{7(2)^3}$$

$$2 = k \frac{64}{7 \times 8}$$

$$2 = k \frac{64}{56}$$

$$\frac{2 \times 56}{64} = k$$

divide by 8

$$\frac{2 \times 7}{8} = k$$

$$\frac{14}{8} = k$$

Divide by 2

$$\frac{7}{4} = k$$

So, Equation (i)

$$u = \frac{7x^2}{4yz^3} \rightarrow (ii)$$

put $x = 6, y = 3$ and $z = 2$ in equation (ii)

$$u = \frac{7(6)^2}{4(3)(2)^3}$$

$$u = \frac{7 \times 36}{4(3)(8)}$$

$$u = \frac{7 \times 9}{3(8)} \quad (\div \text{ by } 4)$$

$$u = \frac{7 \times 3}{8} \quad (\div \text{ by } 3)$$

$$u = \frac{21}{8}$$

Question No.5 if v varies directly as the product xy^3 and inversely as z^2 and $v = 27$ when

$x = 7, y = 6, z = 7$. find the **value of v** when $x = 6, y = 2, z = 3$

Solution:

$$v \propto xy^3$$

$$v \propto \frac{1}{z^2}$$

$$u = k \frac{xy^3}{z^2} \rightarrow (i)$$

put $v = 27, x = 7, y = 6, z = 7$

$$27 = k \frac{(7)(6)^3}{(7)^2}$$

$$27 = k \frac{216}{7} \quad (\div \text{ by } 7)$$

$$\frac{27 \times 7}{216} = k$$

$$\frac{7}{8}$$

$$= k \quad \text{by } (27 \div 216)$$

$$= 8)$$

$$\text{or } k = \frac{7}{8}$$

so, Equation (i) becomes

$$u = \frac{7xy^3}{8z^2} \rightarrow (ii)$$

put $x = 6, y = 2, z = 3$ in equation (ii)

$$u = \frac{7(6)(2)^3}{8(9)}$$

$$u = \frac{7 \times 6 \times 8}{8 \times 9}$$

$$u = \frac{7 \times 6}{9}$$

$$u = \frac{7 \times 2}{3}$$

$$u = \frac{14}{3}$$

Question No.6 if w varies inversely as the cube of u , and $w=5$ when $u=3$. Find w when $u=6$

Solution:

$$w \propto \frac{1}{u^3}$$

$$w = \frac{k}{u^3} \rightarrow (i)$$

Put $w=5$ and $u=3$

$$5 = \frac{k}{3^3}$$

$$5 = \frac{k}{27}$$

$$(5 \times 27) = k$$

$$135 = k$$

$$\text{or } 135 = k$$

So, equation (i) becomes

$$w = \frac{135}{u^3} \rightarrow (ii)$$

Put $u=6$ in equation (ii)

$$w = \frac{135}{(6)^3}$$

$$w = \frac{135}{216}$$

$$w = \frac{5}{8}$$

Exercise 3.6

Question No1 if $a : b = c :$

d , then show that

$$i. \quad \frac{4a-9b}{4a+9b} = \frac{4c-9d}{4c+9d}$$

Solution:

As $a : b = c : d$

let $\frac{a}{b} = \frac{c}{d} = k$

then $a = bk$ and $c = dk$

$$\frac{4a - 9b}{4a + 9b} = \frac{4c - 9d}{4c + 9d}$$

Putting the valves

$$\frac{4(bk) - 9b}{4(bk) + 9b} = \frac{4(dk) - 9d}{4(dk) + 9d}$$

Putting the valves

$$\begin{aligned} \frac{4bk - 9b}{4bk + 9b} &= \frac{4dk - 9d}{4dk + 9d} \\ \frac{b(4k - 9)}{b(4k + 9)} &= \frac{d(4k - 9)}{d(4k + 9)} \\ \frac{(4k - 9)}{(4k + 9)} &= \frac{(4k - 9)}{(4k + 9)} \\ \text{L. H. S} &= \text{R. H. S} \end{aligned}$$

ii. $\frac{6a-5b}{6a+5b} = \frac{6c-5d}{6c+5d}$

Solution:

As $a : b = c : d$

let $\frac{a}{b} = \frac{c}{d} = k$

then $a = bk$ and $c = dk$

$$\frac{6a - 5b}{6a + 5b} = \frac{6c - 5d}{6c + 5d}$$

Putting these valves

$$\begin{aligned} \frac{6(bk) - 5b}{6(bk) + 5b} &= \frac{6(dk) - 5d}{6(dk) + 5d} \\ \frac{6bk - 5b}{6bk + 5b} &= \frac{6dk - 5d}{6dk + 5d} \\ \frac{b(6k - 5)}{b(6k + 5)} &= \frac{d(6k - 5)}{d(6k + 5)} \\ \frac{6k - 5}{6k + 5} &= \frac{6k - 5}{6k + 5} \\ \text{L. H. S} &= \text{R. H. S} \end{aligned}$$

iii. $\frac{a}{b} = \sqrt{\frac{a^2+c^2}{b^2+d^2}}$

Solution:

As $a : b = c : d$

let $\frac{a}{b} = \frac{c}{d} = k$

then $a = bk$ and $c = dk$

$$\frac{a}{b} = \sqrt{\frac{a^2 + c^2}{b^2 + d^2}}$$

Putting these valves

$$\frac{bk}{b} = \sqrt{\frac{(bk)^2 + (dk)^2}{b^2 + d^2}}$$

$$\frac{bk}{b} = \sqrt{\frac{b^2k^2 + k^2d^2}{b^2 + d^2}}$$

$$k = \sqrt{\frac{k^2(b^2 + d^2)}{b^2 + d^2}}$$

$$k = \sqrt{k^2}$$

$$k = k$$

$$\text{L. H. S} = \text{R. H. S}$$

iv. $a^6 + c^6 : b^6 + d^6 = a^3c^3 : b^3d^3$

$$\frac{a^6 + c^6}{b^6 + d^6} = \frac{a^3c^3}{b^3d^3}$$

As $a : b = c : d$

let $\frac{a}{b} = \frac{c}{d} = k$

then $a = bk$ and $c = dk$

$$\frac{a^6 + c^6}{b^6 + d^6} = \frac{a^3c^3}{b^3d^3}$$

Putting these valves

$$\begin{aligned} \frac{(bk)^6 + (dk)^6}{b^6 + d^6} &= \frac{(bk)^3(dk)^3}{b^3d^3} \\ \frac{b^6k^6 + d^6k^6}{b^6 + d^6} &= \frac{b^3k^3d^3k^3}{b^3d^3} \\ \frac{k^6(b^6 + d^6)}{b^6 + d^6} &= \frac{k^6(b^3d^3)}{b^3d^3} \\ k^6 &= k^6 \end{aligned}$$

$$\text{L. H. S} = \text{R. H. S}$$

v. $p(a + b) + qb : a : c = (c + d) + qd =$

$$\frac{p(a + b) + qb}{p(c + d) + qd} = \frac{a}{c}$$

As $a : b = c : d$

let $\frac{a}{b} = \frac{c}{d} = k$

then $a = bk$ and $c = dk$

putting these valves

$$\begin{aligned} \frac{p(bk + b) + qb}{p(dk + d) + qd} &= \frac{bk}{dk} \\ \frac{(pkb + pb) + qb}{(pdk + pd) + qd} &= \frac{bk}{dk} \\ \frac{b[p(k + 1) + q]}{d[p(k + 1) + q]} &= \frac{b}{d} \\ \frac{b}{d} &= \frac{d}{d} \end{aligned}$$

$$\text{L. H. S} = \text{R. H. S}$$

$$\text{vi. } a^2 + b^2 : \frac{a^3}{a+b} = c^2 + d^2 : \frac{c^3}{c+d}$$

Solution:

$$\frac{a^2 + b^2}{\frac{a^3}{a+b}} = \frac{c^2 + d^2}{\frac{c^3}{c+d}}$$

As $a : b = c : d$

$$\text{let } \frac{a}{b} = \frac{c}{d} = k$$

then $a = bk$ and $c = dk$

putting these values

$$\frac{(bk)^2 + b^2}{\frac{(bk)^3}{bk+b}} = \frac{(dk)^2 + d^2}{\frac{(dk)^3}{kd+d}}$$

$$\frac{b^2k^2 + b^2}{\frac{b^3k^3}{bk+b}} = \frac{d^2k^2 + d^2}{\frac{d^3k^3}{kd+d}}$$

$$\frac{b^2(k^2+1)}{\frac{b^3k^3}{k+1}} = \frac{d^2(k^2+1)}{\frac{d^3k^3}{k+1}}$$

$$\frac{b^2k^3}{(k+1)} = \frac{d^2k^3}{k+1}$$

$$\frac{(k^2+1)(k+1)}{k^3} = \frac{(k^2+1)(k+1)}{k^3}$$

L. H. S = R. H. S

$$\text{vii. } \frac{a}{a-b} : \frac{a+b}{b} = \frac{c}{c-d} : \frac{c+d}{d}$$

Solution:

$$\frac{a}{a-b} : \frac{a+b}{b} = \frac{c}{c-d} : \frac{c+d}{d}$$

$$\frac{\frac{a}{a-b}}{\frac{a+b}{b}} = \frac{\frac{c}{c-d}}{\frac{c+d}{d}}$$

As $a : b = c : d$

$$\text{let } \frac{a}{b} = \frac{c}{d} = k$$

then $a = bk$ and $c = dk$

putting these values

$$\frac{a}{a-b} \times \frac{b}{a+b} = \frac{c}{c-d} \times \frac{d}{c+d}$$

$$\frac{bk}{bk-b} \times \frac{b}{bk+b} = \frac{dk}{dk-d} \times \frac{d}{dk+d}$$

$$\frac{bk}{b(k-1)} \times \frac{b}{b(k+1)} = \frac{dk}{d(k-1)} \times \frac{d}{d(k+1)}$$

$$\frac{bk}{(k-1)(k+1)} = \frac{dk}{(k-1)(k+1)}$$

L. H. S = R. H. S

Question No.2

if $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ ($a, b, c, d, e, f, \neq 0$) show that

$$\text{i. } \frac{a}{b} = \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}$$

As $a : b = c : d = e : f$

$$\text{let } \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$$

then $a = bk$, $c = dk$ and $e = fk$

putting these values

$$\frac{a}{b} = \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}$$

$$\frac{bk}{b} = \sqrt{\frac{(bk)^2+(dk)^2+(fk)^2}{b^2+d^2+f^2}}$$

$$k = \sqrt{\frac{b^2k^2+d^2k^2+f^2k^2}{b^2+d^2+f^2}}$$

$$k = \sqrt{\frac{k^2(b^2+d^2+f^2)}{b^2+d^2+f^2}}$$

$$k = \sqrt{k^2}$$

$$k = k$$

L. H. S = R. H. S

$$\text{ii. } \frac{ac+ce+ea}{bd+df+fb} = \left[\frac{ace}{bdf} \right]^{\frac{2}{3}}$$

Solution:

$$\frac{ac+ce+ea}{bd+df+fb} = \left[\frac{ace}{bdf} \right]^{\frac{2}{3}}$$

As $a : b = c : d = e : f$

$$\text{let } \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$$

then $a = bk$, $c = dk$ and $e = fk$

putting these values

$$\frac{bkdk + dkfk + fkbk}{bd + df + fb} = \left[\frac{bkdkfk}{bdf} \right]^{\frac{2}{3}}$$

$$\frac{k^2bd + k^2df + k^2fb}{bd + df + fb} = \left[\frac{k^3bdf}{bdf} \right]^{\frac{2}{3}}$$

$$\frac{k^2(bd + df + fb)}{bd + df + fb} = [k^3]^{\left(\frac{2}{3}\right)}$$

$$k^2 = k^2$$

L. H. S = R. H. S

iii. $\frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{fb} = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$

Solution:

As $a : b = c : d = e : f$

let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

then $a = bk$, $c = dk$ and $e = fk$

putting these values

$$\frac{ac}{bd} + \frac{ce}{df} + \frac{ea}{fb} = \frac{a^2}{b^2} + \frac{c^2}{d^2} + \frac{e^2}{f^2}$$

$$\frac{bkdk}{bd} + \frac{dkfk}{df} + \frac{fkbk}{fb} = \frac{b^2k^2}{b^2} + \frac{d^2k^2}{d^2} + \frac{f^2k^2}{f^2}$$

$$\frac{k^2bd}{bd} + \frac{k^2df}{df} + \frac{k^2fb}{fb} = k^2 + k^2 + k^2$$

$$k^2 + k^2 + k^2 = k^2 + k^2 + k^2$$

Exercise 3.7

Question No.1. The surface area A of a cube varies directly as the square of the length l of an edge and $A=27$ square units when $l = 3$ units

Find (i) A when $l = 4$ units

(ii) l when $A = 12$ sq. units

Solution:

$$A \propto l^2$$

$$A = kl^2 \rightarrow (i)$$

When $A = 27$ square units, $l = 3$ units

$$27 = k(3)^2$$

$$27 = k(9)$$

$$\frac{27}{9} = k$$

$$3 = k$$

$$\text{or } k = 3$$

So equation (i) becomes

i. when $l = 4$ units

$$A = 3(4)^2$$

$$A = 3(16)$$

$$A = 48 \text{ sq. units}$$

ii. when $A = 12$ sq. units

$$12 = 3l^2$$

$$\frac{12}{3} = l^2$$

$$4 = l^2$$

l

$= 2$ taking squaring root on both sides

Question No.2

The surface area

S of a surface varies directly as the square of radius

16π when $r=2$.

Find r when $S = 36\pi$

Solution:

$$S \propto r^2$$

$$S = kr^2 \rightarrow (i)$$

$$S = 16\pi \text{ when } r = 2$$

$$16\pi = k(2)^2$$

$$16\pi = 4k$$

$$\frac{16\pi}{4} = k$$

$$4\pi = k$$

$$\text{or } k = 4\pi$$

So, equation (i) becomes

$$S = 4\pi r^2$$

When $S = 36\pi$ units

$$36\pi = 4\pi r^2$$

$$r^2 = 9$$

$$r = 3$$

Question No.3 in Hook's law the force F applied to stretch a spring varies directly as the amount of elongation S and

$F = 32$ lb when $S = 1.6$ inch. find (i) when $F = 50$ lb

Solution:

$$F \propto S$$

$$F = kS \rightarrow (i)$$

$$F = 32 \text{ lb when } S = 1.6$$

$$(32) = k(1.6)$$

$$k = \frac{32}{1.6}$$

$$k = \frac{32}{16} \times 10$$

$$k = 20$$

So equation (i) becomes

$$F = 20S$$

i. when $F = 50 \text{ lb}$

$$50 \text{ lb} = 20S$$

$$S = 2.5 \text{ in}$$

ii. when $S = 0.8 \text{ in}$

$$F = 20(0.8)$$

$$F = 16 \text{ lb}$$

Question No.4 The intensity I of light from a given source varies inversely as the square of the distance d from it. If the intensity is 20 candlepower at a distance of 12ft. from the source, find the intensity at a point 8ft. from the source.

Solution:

$$I \propto \frac{1}{d^2}$$

$$I = \frac{k}{d^2} \rightarrow (i)$$

$$I = 20 \text{ when } d = 12 \text{ ft}$$

$$(20) = \frac{k}{(12)^2}$$

$$20 = \frac{k}{144}$$

$$k = 20 \times 144$$

$$k = 2880$$

So equation (i) becomes

$$I = \frac{2880}{d^2}$$

When $d = 8 \text{ ft}$

$$I = \frac{2880}{8^2}$$

$$I = \frac{2880}{64}$$

$$I = 45 \text{ cp}$$

Question No.5 The pressure p in a body of fluid varies directly as the depth d . if the pressure exerted on the bottom of a tank by a column of fluid 5ft. high is 2.25 $\text{lb}\backslash\text{sq. in}$. In how deep must fluid be to exert a pressure of 9 $\text{lb}\backslash\text{sq. in}$?

Solution:

$$P \propto d$$

$$p = kd \rightarrow (i)$$

$$d = 5 \text{ ft when } P = 2.25 \text{ lb}\backslash\text{sq. in}$$

$$2.25 = k(5)$$

$$k = \frac{2.25}{5}$$

$$k = \frac{225}{5 \times 100}$$

$$k = \frac{45}{100}$$

$$k = 0.45$$

So equation (i)

$$P = 0.45d$$

$$\text{when } P = 9 \text{ lb}\backslash\text{sq. in}$$

$$9 = 0.45d$$

$$\frac{9}{0.45} = d$$

$$\frac{9 \times 100}{45} = d$$

$$\frac{900}{45} = d$$

$$20 = d$$

Question No.6 Labour costs c varies jointly as the number of workers n and the average number of days d , if the cost of 800 workers for 13 days is Rs. 286000, then find the labour cost of 600 workers for 18days.

Solution:

$$c \propto nd$$

$$c = knd \rightarrow (i)$$

$$c = \text{Rs. } 286000, n = 800, d = 13$$

$$286000 = k(800)(13)$$

$$k = \frac{286000}{10400}$$

$$k = 27.5$$

So equation (i) becomes

$$c = 27.5$$

When $n = 600, d = 18$

$$c = 27.5(600)(18)$$

$$c = 297000$$

Question No. 7 The supporting load c of a pillar varies as the fourth power of its diameter d and inversely as the square of its length l . A pillar of diameter 6 inch and of height 30 feet will support a load of 63 tons. How high a 4 inch pillar must be to support a load of 28 tons.?

Solution:

$$c \propto \frac{d^4}{l^2}$$

$$c = \frac{kd^4}{l^2} \rightarrow (i)$$

$$c = 63 \text{ tons}, d = 6 \text{ inch}, l = 30 \text{ feet}$$

$$63 = \frac{k(6)^4}{(30)^2}$$

$$k = 43.75$$

So equation (i) becomes

$$c = \frac{43.75d^4}{l^2}$$

When $d = 4 \text{ inch}, c = 28 \text{ tons}$

$$28 = \frac{43.75(4)^4}{l^2}$$

$$l^2 = \frac{43.75(4)^4}{28}$$

$$l^2 = 400$$

$$l^2 = 20 \text{ feet}$$

Question No.8

The times T required for an elevator to lift a weight varies jointly as the weight w and the lifting depth d varies inversely as the power P of the motor. If 25 sec. are required for a 4-hp motor to lift 500lb through 40ft, what power is required to lift 800lb. through 120 ft in 40 sec?

Solution:

$$T \propto \frac{wd}{p}$$

$$T = \frac{kwd}{p} \rightarrow (i)$$

$$T = 25 \text{ sec}, p = 4 \text{ hp}, w = 500 \text{ lb}, d = 40 \text{ ft}$$

$$25 = \frac{k(500)(40)}{4}$$

$$25 \times 4 = \frac{k(500)(40)}{4}$$

$$K = \frac{500 \times 40}{4}$$

So equation (i) becomes

$$T = \frac{0.005wd}{p}$$

When $w = 800 \text{ lb}, d = 120 \text{ ft}, T = 40 \text{ sec.}$

$$40 = \frac{0.005 \times 800 \times 120}{P}$$

$$P = \frac{0.005 \times 800 \times 120}{40}$$

$$p = 12 \text{ hp}$$

Question No.9

The kinetic energy (K.E) of a body varies jointly as the mass "m" of the body and the square of its velocity "V" if the energy is 4320ft/lb when the mass is 45lb and the velocity is 24ft/sec. Determine the kinetic energy of a 3000 lb automobile travelling 44ft/sec.

Solution:

$$K.E \propto mv^2$$

$$K.E = kmv^2 \rightarrow (i)$$

$K.E = 4320 \text{ ft/lb}, m=45 \text{ lb}, v=24 \text{ ft/sec.}$

$$4320 = k(45)(24)^2$$

$$k = \frac{4320}{45 \times 476}$$

$$k = \frac{4320}{25920}$$

$$k = \frac{1}{6}$$

So equation (i) becomes

$$K.E = \frac{mv^2}{6}$$

When $m = 3000 \text{ lb}, v = 44 \text{ ft/sec.}$

$$K.E = \frac{(3000)(44)^2}{6}$$

$$K.E = 500 \times 1936$$

$$K.E = 96800 \text{ ft/lb}$$

Best wishes

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