

1. What do you understand by measures of central tendency?

Solution: the technique that are used to determine the central value is called is called measure of central tendency.

2. Define Arithmetic mean, geometric mean, Harmonic mean, mode and Median?

Solution:

i. **Arithmetic Means:**

Mean is a measure that determine a value of the variable understudy by dividing the Sum of all valves of the variable by their number of observations.

$$\bar{X} = \frac{\sum X}{n} \text{ (for ungrouped data) and } \bar{X} = \frac{\sum fX}{\sum f} \text{ (for grouped data)}$$

ii. **Geometric Means**

Geometric mean of a variable  $x$  is the  $n$ th positive root of the product of the

$x_1, x_2, x_3, \dots, x_n$  observation.  $G.M = (x_1, x_2, x_3, \dots, x_n)^{\frac{1}{n}}$

iii. **Harmonic Means:**

Harmonic mean refers to the value obtained by reciprocating the mean of the reciprocal of  $x_1, x_2, x_3, \dots, x_n$  observations.

$$H.M = \frac{n}{\sum \frac{1}{x}} \text{ (for ungrouped data) and } H.M = \frac{n}{\sum \frac{f}{x}} \text{ (for grouped data)}$$

iv. **Mode:**

The most repeated value in an observation is called mode.

v. **Median**

Median is the middle most observation in an arranged data set. It divides the data set into equal parts.

3. Find arithmetic mean by direct method for the following set of data:

- i. 12,14,17,20,24,29,35,45  
ii. 200,225,350,375,270,320,290

Solution:

$$\begin{aligned} \text{i. } A.M &= \bar{X} = \frac{\sum X}{n} = \frac{12+14+17+20+24+29+35+45}{8} \\ &= \frac{196}{8} = 24.5 \end{aligned}$$

$$\begin{aligned} \text{ii. } A.M &= \bar{X} = \frac{\sum X}{n} = \frac{200+225+350+375+270+320+290}{7} \\ &= \frac{2030}{7} = 290 \end{aligned}$$

4. For each of the data in Q.No.3 Compute arithmetic mean using indirect method.

Solution:

i. Take any constant say 24 and take deviations from it (24)

$$A = 24$$

| $X$     | $D = X - A$     |
|---------|-----------------|
| 12      | $12 - 24 = -12$ |
| 14      | $17 - 24 = -7$  |
| 17      | $20 - 24 = -4$  |
| 24      | $24 - 24 = 0$   |
| 29      | $29 - 24 = 5$   |
| 35      | $35 - 24 = 11$  |
| 45      | $45 - 24 = 21$  |
| $n = 8$ | $\sum D = 4$    |

$$\begin{aligned}\bar{X} &= A + \frac{\sum D}{n} \\ &= 24 + \frac{4}{8} = 24 + \frac{1}{2} = 24 \times \frac{1}{2} = 24.5\end{aligned}$$

ii. Take any constant say 270 and take deviations from it (270)

$$A = 270$$

| $X$     | $D = X - A$       |
|---------|-------------------|
| 200     | $200 - 270 = -70$ |
| 225     | $225 - 270 = -45$ |
| 350     | $350 - 270 = -80$ |
| 375     | $375 - 270 = 150$ |
| 270     | $270 - 270 = 0$   |
| 320     | $320 - 270 = 50$  |
| 290     | $290 - 270 = 20$  |
| $n = 7$ | $\sum D = 140$    |

$$\begin{aligned}\bar{X} &= A + \frac{\sum D}{n} \\ &= 270 + \frac{140}{7} = 270 + 20 = 290\end{aligned}$$

5. The marks obtained by students of class XI in mathematics are given below.

Compare arithmetic mean by direct and indirect methods.

|         |    |
|---------|----|
| 0 – 90  | 2  |
| 10 – 19 | 10 |
| 20 – 29 | 5  |
| 30 – 39 | 9  |
| 40 – 49 | 6  |
| 50 – 59 | 7  |
| 60 – 69 | 1  |

Solution:

Direct method:

| Classes/<br>Groups | Mid points | f                 | fx                |
|--------------------|------------|-------------------|-------------------|
| 0 – 90             | 4.5        | 2                 | 4.5 × 2 = 9.0     |
| 10 – 19            | 14.5       | 10                | 14.5 × 10 = 145.0 |
| 20 – 29            | 24.5       | 5                 | 24.5 × 5 = 122.5  |
| 30 – 39            | 34.5       | 9                 | 34.5 × 9 = 310.5  |
| 40 – 49            | 44.5       | 6                 | 44.5 × 6 = 267.0  |
| 50 – 59            | 54.5       | 7                 | 54.5 × 7 = 381.5  |
| 60 – 69            | 64.5       | 1                 | 64.5 × 1 = 64.5   |
|                    |            | $n = \sum f = 40$ | 1300              |

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{1300}{40} = 32.5$$

Indirect, short cut method

let  $A = 34.5$

| Classes/ Groups | Mid points | f                 | $D = X - a$       | $U = \frac{D}{10}$ | fD   | $f(U) = -\frac{f(d)}{3}$ |
|-----------------|------------|-------------------|-------------------|--------------------|------|--------------------------|
| 0 – 90          | 4.5        | 2                 | 4.5 – 34.5 = –30  | -3                 | –60  | –6                       |
| 10 – 19         | 14.5       | 10                | 14.5 × 34.5 = –20 | -2                 | –200 | –20                      |
| 20 – 29         | 24.5       | 5                 | 24.5 × 34.5 = –10 | -1                 | –50  | –5                       |
| 30 – 39         | 34.5       | 9                 | 34.5 × 34.5 = 0   | 0                  | 0    | 0                        |
| 40 – 49         | 44.5       | 6                 | 44.5 × 34.5 = 10  | 1                  | 60   | 6                        |
| 50 – 59         | 54.5       | 7                 | 54.5 × 34.5 = 20  | 2                  | 140  | 14                       |
| 60 – 69         | 64.5       | 1                 | 64.5 × 34.5 = 30  | 3                  | 30   | 3                        |
| Total           |            | $n = \sum f = 40$ | 1300              |                    | –80  | –8                       |

$$\begin{aligned}\bar{x} &= h + \frac{\sum fD}{\sum f} \\ &= 34.5 + \frac{-80}{40} \\ &= 34.5 - 2 \\ &= 32.55\end{aligned}$$

$$\begin{aligned}\text{or } \bar{x} &= h + \frac{\sum f(U)}{\sum f} \times h \\ &= 34.5 + \frac{-8}{40} \times h \\ &= 34.5 + \frac{-8}{40} \times 10 \\ &= 34.5 - 2 = 32.55\end{aligned}$$

6. The following data relates to to ages of children in a school. Compute the mean age by direct and short – cut method taking ant provisional mean.

| Class limits | Frequency |
|--------------|-----------|
| 4 – 6        | 10        |
| 7 – 9        | 20        |
| 10 – 12      | 13        |
| 13 – 15      | 7         |
| Total        | 50        |

Also Compute Geometric mean and Harmonic mean.

Solution:

| Class limits | Midpoints | Frequency | fx |
|--------------|-----------|-----------|----|
|--------------|-----------|-----------|----|

|              |                   |    |                      |
|--------------|-------------------|----|----------------------|
| 4 – 6        | 5                 | 10 | $5 \times 10 = 50$   |
| 7 – 9        | 8                 | 20 | $8 \times 20 = 160$  |
| 10 – 12      | 11                | 13 | $11 \times 13 = 143$ |
| 13 – 15      | 14                | 7  | $14 \times 7 = 98$   |
| <i>Total</i> | $n = \sum f = 50$ | 50 | $\sum fx = 451$      |

$$A.M = \frac{\sum fD}{\sum f} = \frac{451}{50} = 9.02$$

Indirect, short cut method

Let  $A = 11$

| Classes/ Groups | $f$      | Midpoint | $D = X - a$    | $U = \frac{D}{10}$ | $fD$ | $f(U) = -\frac{f(d)}{3}$ |
|-----------------|----------|----------|----------------|--------------------|------|--------------------------|
| 4 – 6           | 5        | 5        | $5 - 11 = -6$  | -2                 | -60  | -20                      |
| 7 – 9           | 8        | 8        | $8 - 11 = -3$  | -1                 | -60  | 0                        |
| 10 – 12         | 11       | 11       | $11 - 11 = -3$ | 0                  | 0    | 7                        |
| 13 – 15         | 14       | 14       | $14 - 11 = -3$ | 1                  | 21   | -33                      |
| Total           | $\sum f$ |          |                |                    | -99  | -8                       |

p

$$\begin{aligned} \bar{X} &= A + \frac{\sum fD}{\sum f} & \text{or } \bar{X} &= A + \frac{\sum f(U)}{\sum f} \times h \\ &= 11 - \frac{99}{50} & &= 11 + \frac{-33}{50} \times 3 \\ &= 11 - 1.98 & &= 11 - \frac{99}{50} \\ &= 9.02 & &11 - 1.98 = 9.02 \end{aligned}$$

### Geometric Mean

We proceed as follows:

| Class limits | $f$           | Midpoints | $\log x$        | $f \log x$   |
|--------------|---------------|-----------|-----------------|--------------|
| 4 – 6        | 10            | 5         | 0.6987          | 6.9897       |
| 7 – 9        | 20            | 8         | 0.90309         | 18.0618      |
| 10 – 12      | 13            | 11        | 1.04139         | 13.53807     |
| 13 – 15      | 7             | 14        | 1.14613         | 8.02291      |
|              | $\sum f = 50$ |           | $\sum f \log x$ | $= 46.61248$ |

$$G.M = \text{Antilog} \left( \frac{\sum f \log x}{\sum f} \right)$$

$$G.M = \text{Antilog} \left( \frac{46.61248}{50} \right)$$

$$\text{Antilog}(0.9322496) = 8.553$$

Harmonic means:

| Class limits | $f$           | Midpoints | $\frac{f}{x}$          |
|--------------|---------------|-----------|------------------------|
| 4 – 6        | 10            | 5         | $\frac{10}{5} = 2.0$   |
| 7 – 9        | 20            | 8         | $\frac{20}{8} = 2.5$   |
| 10 – 12      | 13            | 11        | $\frac{13}{11} = 1.18$ |
| 13 – 15      | 7             | 14        | $\frac{7}{14} = 0.50$  |
|              | $\sum f = 50$ |           | $\sum f/x = 6.18$      |

$$H.M = \left( \frac{\sum f}{\sum \frac{f}{x}} = \frac{50}{6.18} = 8.09 \right)$$

7. The following data shows the number of children in which in various familiar. Find mode and median.

9,11,4,5,6,8,4,3,7,8,5,5,8,3,4,9,12,8,9,10,6,1,7,11,4,4,8,4,3,2,7,9,10,9,7,6,9,5

Solution:

Writing the observation in Ascending order

2,3,3,3,4,4,4,4,4,5,5,5,5,6,6,6,7,7,7,7,8,8,8,8,9,9,9,9,9,10,10,11,11,12

Mode: the most frequent observation = 9,4

Number of observation = 38

Therefore, median is the mean of 19<sup>th</sup> and 20<sup>th</sup> observation =  $\frac{7+7}{2} = 7$

8. Find Model number of heads for the following distributive showing of heads when 5 coins are tossed. Also determine median.

| $X(\text{number of heads})$ | Frequency (number of times) |
|-----------------------------|-----------------------------|
| 1                           | 3                           |
| 2                           | 8                           |
| 3                           | 5                           |
| 4                           | 3                           |
| 5                           | 1                           |

Solution:

Mode:

The most frequent observation = 2

For median, we make cumulative frequency column.

| $x$ | frequency | Cumulative frequency |
|-----|-----------|----------------------|
| 1   | 3         | 3                    |
| 2   | 8         | 3+8=11               |
| 3   | 5         | 11+5=16              |
| 4   | 3         | 16 + 3 = 19          |

|   |   |         |
|---|---|---------|
| 5 | 1 | 19+1=20 |
|---|---|---------|

Median = the class containing  $\left(\frac{n}{2}\right)^{th}$  observation

= the class containing  $\left(\frac{20}{2}\right)^{th}$  observation.

= the class containing  $(10^{th})$  observation.

Median = 2

9. The following frequency distribution the weight of boys in kilogram. Compute mean, median, mode.

| Class intervals | frequency |
|-----------------|-----------|
| 1 – 3           | 2         |
| 4 – 6           | 3         |
| 7 – 9           | 5         |
| 10 – 12         | 4         |
| 13 – 15         | 6         |
| 16 – 18         | 2         |
| 19 – 21         | 1         |

Solution:

| Class intervals | frequency | Mid points(x) | $fx$ | Class Boundaries | Cumulative Frequency |
|-----------------|-----------|---------------|------|------------------|----------------------|
| 1 – 3           | 2         | 2             | 4    |                  | 2                    |
| 4 – 6           | 3         | 5             | 15   |                  | 2+3                  |
| 7 – 9           | 5         | 8             | 40   |                  | 5+5=10               |
| 10 – 12         | 4         | 11            | 44   |                  | 10+4=14              |
| 13 – 15         | 6         | 14            | 84   |                  | 14+6=20              |
| 16 – 18         | 2         | 17            | 34   |                  | 20+2=22              |
| 19 – 21         | 1         | 20            | 20   |                  | 22+1=23              |
|                 | 23        |               | 241  |                  |                      |

$$\text{Mean} = \bar{X} = \frac{\sum fx}{\sum f} = \frac{241}{23} = 10.478$$

Median:

Median class = class containing  $\left(\frac{n}{2}\right)^{th}$  observation.

$$= \left(\frac{23}{2}\right)^{th} = (11.5)^{th} \text{ observation}$$

Median class is 9.5 – 12.5

Here  $l = 9.5, c = 10, f = 4, h = 3$

$$\text{Median} = l + \frac{h}{f} \left( \frac{n}{2} - c \right)$$

$$= 9.5 + \frac{3}{4} \left( \frac{23}{2} - 10 \right) = 9.5 + \frac{3}{4} \left( \frac{3}{2} \right) = 9.5 + \frac{9}{8} = 9.5 + 1.125 = 10.625$$

$$\text{Mode: } Mode = l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$$

$$\text{Here } l = 12.5, f_m = 6, f_1 = 4, f_2 = 2, h = 3$$

$$\therefore \text{Mode} = 12.5 + \frac{6 - 4}{2(6) - 4 - 2} \times 3 = 12.5 + \frac{2}{6} \times 3 = 12.5 + 1 = 13.5$$

**10.** A student obtained the following marks at a certain examination: English 73, Urdu 82, Mathematics 80, History 67 and Science 62.

- If the Wight accorded these marks are 4,3,3,4 and 2. *repectively*. what is an appropriate average marks?
- What is the average mark if equal weights are used?

Solution:

| Marks(x)       | Weight(w)     | xw               |
|----------------|---------------|------------------|
| 73             | 4             | 73 × 4 = 292     |
| 82             | 3             | 82 × 3 = 246     |
| 80             | 3             | 80 × 3 = 240     |
| 67             | 2             | 67 × 2 = 134     |
| 62             | 2             | 62 × 2 = 124     |
| $\sum x = 364$ | $\sum w = 14$ | $\sum xw = 1036$ |

$$(i) \quad \bar{X}_n = \frac{\sum Xw}{\sum w} = \frac{1036}{14} = 74$$

$$(ii) \quad \bar{X} = \frac{\sum x}{n} = \frac{364}{5} = 72.8$$

**11.** On a vacation trip a family bought 21.3liters of petrol at 39.90 rupees per liter, 18.7 liters at 42.90 rupees per liter, and 23.5 liters at 40.90 rupees per liter find the mean price paid per liter.

Solution:

| X               | W     | XW                     |
|-----------------|-------|------------------------|
| 21.3            | 39.90 | (21.3)(39.90) = 849.87 |
| 18.7            | 42.90 | (21.3)(39.90) = 849.87 |
| 23.5            | 40.90 | (21.3)(39.90) = 849.87 |
| $\sum x = 63.5$ |       | $\sum xW = 2613.25$    |

$$\text{Mean price} = \frac{\sum XW}{\sum X} = \frac{2613.25}{63.5} = 41.15 \text{ rupees per liter}$$

**12.** Calculator simple moving average of 3 years from the following data;

| Years  | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 |
|--------|------|------|------|------|------|------|------|------|------|------|
| Valves | 102  | 108  | 130  | 140  | 1158 | 180  | 196  | 210  | 220  | 230  |

Solution:

| Years | Values | 3-years moving total | 3- years moving average |
|-------|--------|----------------------|-------------------------|
| 2001  | 102    | -                    | -                       |
| 2002  | 108    | 340                  | 340/3=113.33            |
| 2003  | 130    | 378                  | 378/3=126.00            |
| 2004  | 140    | 428                  | 428/3=142.67            |

|      |     |     |                          |
|------|-----|-----|--------------------------|
| 2005 | 158 | 478 | $\frac{478}{3} = 159.33$ |
| 2006 | 180 | 534 | $534/3=178.00$           |
| 2007 | 196 | 586 | $586/3=195.33$           |
| 2008 | 210 | 626 | $626/3=208.67$           |
| 2009 | 220 | 660 | $660/3=220.00$           |
| 2010 | 230 | -   |                          |

13. Determine graphically for the following data and check your answer by using formulae.

- Median and Quartiles using cumulative frequency polygon.
- Mode using Histogram

| Class Boundaries | Frequency |
|------------------|-----------|
| 10 – 20          | 2         |
| 20 – 30          | 5         |
| 30 – 40          | 9         |
| 40 – 50          | 6         |
| 50 – 60          | 4         |
| 60 – 70          | 1         |

Solution:

| Class Boundaries | Frequency | c. f |
|------------------|-----------|------|
| 10 – 20          | 2         | 2    |
| 20 – 30          | 5         | 7    |
| 30 – 40          | 9         | 16   |
| 40 – 50          | 6         | 22   |
| 50 – 60          | 4         | 26   |
| 60 – 70          | 1         | 27   |

Median Class  $Q_3$  Clas

Median Class =  $\left(\frac{n}{2}\right)^{th}$  observation =  $\left(\frac{27}{2}\right)^{th} = (13.5)^{th}$  observation.

$$\text{Median} = l + \frac{h}{f} \left( \frac{n}{2} - c \right)$$

Here  $l = 30, h = 10, f = 9, n = 27, c = 7$

Thus median  $x = 30 + \frac{10}{9} \left( \frac{27}{2} - 7 \right) = 30 + \frac{10}{9} \left( \frac{13}{2} \right) = 30 + 7.22 = 37.22$

To find  $Q_1$

We have to find  $3 \left( \frac{n}{4} \right)^{th}$  observation.

Next Not Solved. Not important according to exam